Introduction & Spacetime

Relativity and Astrophysics
Lecture 01
Terry Herter

Outline
- Welcome
- Course Description
- Who am I?
- Expectations
  - Class Schedule
  - Homework, exams and grades
- Definitions we will cover –
  - Spacetime, spacetime interval, and invariance
  - Events
  - Proper time and proper clock (wristwatch time)
  - Timelike and spacelike intervals
Course content

- Rough Course Outline (subject to change)
  - Special relativity
  - Applications of special relativity in astrophysics
  - The equivalence principle and the connection between geometry and gravitation
  - Cosmology and the geometry of the Universe
  - Black Holes

- Goals:
  - Develop “intuition” about SR and GR.
  - Apply this intuition in an astronomical context.
  - Develop modest proficiency/comfort level with calculations in Special Relativity and General Relativity

- Level of course:
  - About the level of A2211 & A2212 – you need appropriate math (exposure through calculus) and physics background
  - Will do some calculus in class but not too much in homework.

Who am I

- Professor in the Astronomy Department
  - On faculty since 1985

- Research interests
  - The center of our galaxy
  - Disks around stars
  - Galaxy formation and evolution
  - Interstellar Medium & Star Formation
  - Exo-planets

- Director of the Center for Radio Astronomy and Space Research

- Graduate Student Advisor

- Undergraduate Advisor
Projects

- Spitzer
  - Spitzer Space Telescope
  - Cornell Instrument
  - Co-Investigator
- SOFIA (2010)
  - Stratospheric Observatory For Infrared Astronomy
  - Building facility instrument
  - 4-5 full time employees
- CCAT (2015?)
  - Cornell-Caltech Atacama Telescope
  - 25-m sub-mm telescope
  - Partnered with Caltech, Colorado, Canada and Germany

Course Structure

- Lectures: MWF 11:15 - 12:05
- Textbooks:
  - Spacetime Physics by Taylor & Wheeler
  - Exploring Black Hole by Taylor & Wheeler
- Homework:
  - Approximately every one to two weeks
  - due one week after assigned
- Prelims:
  - Three during semester (see schedule)
- Final Exam:
  - On scheduled date at end of semester.
- Course web site: http://www.astro.cornell.edu/courses/a2290
  - Look there for lecture notes, readings, and problem assignments
- Academic Integrity:
  - This course is governed by the Cornell Academic Integrity Policy
    http://www.cuinfo.cornell.edu/Academic/AIC.html
  - It is a violation of the University academic integrity code to “knowingly representing the work of others to be one’s own”
Grading

<table>
<thead>
<tr>
<th>Metric</th>
<th>Points</th>
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<tbody>
<tr>
<td>Prelims</td>
<td>300</td>
</tr>
<tr>
<td>Homework</td>
<td>300</td>
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<tr>
<td>Final Exam</td>
<td>200</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>800</strong></td>
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- Homework will count for ~ 3/8 of your grade. It will be somewhat more numerical/abstract than the prelims and final exam.

Expectations -

- What is your background?
  - In math – trigonometry, calculus?
  - In physics – high school?
  - Have you had an astronomy course before?

- What are your interests?
  - Are you a science or a non-science major?

- What are your expectations?
  - Why are you taking this course?

  - (How much work do you expect it to be?)
Coordinates and distances

- Suppose I define an $x$-$y$ coordinate system that specifies coordinates relative to the intersection of East Ave. and Tower Rd. (origin) and take Tower Road to be the $x$-axis.
- Consider stakes placed in front of two buildings on the campus to which I assign coordinates $(x_o,y_o)$ and $(x_1,y_1)$.
  - We'll use Space Science Bldg & Mann Library
  - And the distances to be measured in meters
- The coordinates and distance ($d$) between them are give below.

$$d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$
$$= \sqrt{(520.363 - 139.656)^2 + (130.091 - 149.222)^2}$$
$$= 381.187 \text{ meters}$$

A second frame (of reference)

- Suppose you now measure coordinates relative to a different set of axes $(x',y')$.
  - This coordinate frame is rotated slightly relative to the previous one.
- The coordinates of the buildings will now be different (as illustrated below).
  - Note that $y_1'$ (74.985 m) is quite a bit smaller than $y_1$ (130.091 m).
- However, we haven’t moved the buildings, so the distance between them will not have changed.

$$d = \sqrt{(x_1' - x_0')^2 + (y_1' - y_0')^2}$$
$$= \sqrt{(531.111 - 154.489)^2 + (74.985 - 133.806)^2}$$
$$= 381.187 \text{ meters}$$
A Matter of Units (or Units Matter)

- Suppose you had measured the “vertical” axis in feet instead of meters.
  - Our distance computation wouldn’t make much sense if one axis were in meters and the other in feet.
- Need to convert to the same units (meters).
  - \(1\ m \times (100\ \text{cm/m}) \times (1\ \text{in/2.54}\ \text{cm}) \times (1\ \text{ft/12}\ \text{in}) = 3.28084\ \text{ft}\)
  - So we need to multiply by a conversion factor, \(k = 0.30480\ \text{m/ft}\)

\[
\begin{align*}
\mathbf{x}_1 & = (154.489\ \text{m}, 438.997\ \text{ft}) \\
\mathbf{y}_1 & = (531.111\ \text{m}, 246.015\ \text{ft})
\end{align*}
\]

\[
d = \sqrt{(x'_1 - x'_0)^2 + (y'_1 - y'_0)^2} = \sqrt{(531.111 - 154.489)^2 + (246.015 - 438.997)^2} = 381.187\ \text{meters}
\]

Spacetime example

- Suppose you pass through my lab on a rocket sled
  - As you travel by sparks are emitted from a knob on the bottom of your sled
  - These sparks create a mark on a steel rail in the lab
  - You and I note the distance and timing between the sparks
- What do we each see?

\[
d_{\text{lab}} = 4\ \text{m} \\
t_{\text{lab}} = 66.7128\ \text{ns}
\]

\[
d_{\text{sled}} = 0\ \text{m} \\
t_{\text{sled}} = 65.3649\ \text{ns}
\]

The distance and time between the two “events” are different for you and I (in our different “reference frames”).
The Spacetime Interval

- The speed of light, \( c = 299,792,458 \text{ m/sec} \) is a fundamental constant of nature.
  - More later when we discuss the fundamental postulates of Special Relativity
- Using \( c \), we can think of any time interval as a distance
  - The distance light would travel in that time \( (d = ct) \)
- Thus we can think of measuring time in units of distance!
  - Perhaps giving some credence to the Han Solo boast in Star Wars of the Millennium Falcon making the Kessel Run in “less than twelve parsecs”
- We use this to define the “spacetime interval”
  - The interval has units of meters (note the minus sign)

\[
\text{(interval)}^2 = (ct)^2 - \text{(space separation)}^2
\]

Note: We could have chosen to use time as our unit of distance. We just need to make sure we are consistent.

Spacetime example - revisited

- Let’s go back to our laboratory example where you and I recorded the distance and time between two sparks (events).
  - I measured: \( d_{\text{lab}} = 4 \text{ m}, \ t_{\text{lab}} = 66.7128 \text{ ns} \)
  - You measured: \( d_{\text{sled}} = 0 \text{ m}, \ t_{\text{sled}} = 65.3649 \text{ ns} \)
- We compute the spacetime interval for each

\[
\begin{align*}
\text{Lab (me):} & \quad \text{(interval)}^2 = (66.7128 \text{ ns} \times 0.299792458 \text{ m/ns})^2 - (4 \text{ m})^2 \\
& = (20.0000 \text{ m})^2 - (4 \text{ m})^2 \\
& = 400.000 - 16.000 \text{ m}^2 = 384.000 \\
\Rightarrow \text{interval} & = 19.596 \text{ m}
\end{align*}
\]

\[
\begin{align*}
\text{Rocket Sled (you):} & \quad \text{(interval)}^2 = (65.3649 \text{ ns} \times 0.299792458 \text{ m/ns})^2 - (0 \text{ m})^2 \\
& = (19.596 \text{ m})^2 - (0 \text{ m})^2 \\
\Rightarrow \text{interval} & = 19.596 \text{ m}
\end{align*}
\]

The interval is the same in both frames!
Properties of Spacetime Interval

- **Invariance**
  - The Spacetime Interval is independent of state of motion of the observer
  - \( \Rightarrow \) space and time cannot be separated! Hence the term “spacetime.”

- **Why the minus sign?**
  - It works!
  - A result of Lorentz geometry – beyond 3-d Euclidean geometry
  - More later

Note: The invariance of the spacetime interval has been demonstrated over and over again in physics. It is a “law of nature” that has never been disproven.