Luminosity, Flux and Magnitudes

Relativity and Astrophysics
Lecture 13
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Outline

- Blackbody Radiation
- Flux and Luminosity
  - Inverse square law
- Magnitudes

Radiation from objects

- All objects have internal energy which is manifested by the microscopic motions of particles.
- There is a continuum of energy levels associated with this motion.
- If the object is in thermal equilibrium then it can be characterized by a single quantity, it's temperature.
- An object in thermal equilibrium emits energy at all wavelengths.
  - resulting in a continuous spectrum
- We call this thermal radiation.

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Blackbody Radiation

- A black object or blackbody absorbs all light which hits it.
- This blackbody also emits thermal radiation, e.g. photons!
  - Like a glowing poker just out of the fire.
- The amount of energy emitted (per unit area) depends <u>only</u> on the temperature of the blackbody.
- In 1900 Max Planck characterized the light coming from a blackbody.
- The equation that predicts the radiation of a blackbody at different temperatures is known as Planck's Law.

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1} \quad \text{W/m}^2\text{/Hz/sr}$$

  - h = Planck's constant, k = Boltzmann's constant, c = speed of light
  - This is the power radiated per unit area in the frequency range ν to $\nu + d\nu$ into a unit solid angle ($d\Omega = d\phi \sin\theta \, d\theta$ in spherical coordinates)

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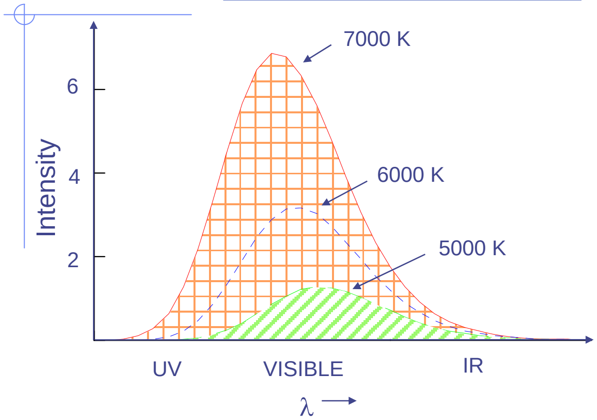
Blackbody Spectra



The peak shifts with T (Wien's law).

$$\lambda_{peak} \propto 1/T$$

The area under the curve increase rapidly with T (Stefan-Boltzmann law).

$$F = \sigma T^4$$

$\sigma = 5.7\text{x}10^{-8}\ \text{Wm}^{-2}\,\text{K}^{-4}$

Object	T (K)	λ_{peak}	F (W/m²)	L (W)	Notes
Person	310	9.4 µm	526	~100	L is net radiation
Sun	5800	0.5 µm	6.5×10^{7}	3.9×10^{26}	$R_{sun} = 6.96\times10^{8}$ m
Neutron Star	10^{6}	29 A	5.7×10^{16}	1.0×10^{26}	R = 12,000 m

Properties of Blackbodies

- The peak emission from the blackbody moves to shorter wavelengths as the temperature increases (Wien's law).

  $$\lambda_{peak} = 2900/T \qquad \lambda \text{ in } \mu\text{m and } T \text{ in K}$$

  - Hot objects look blue, cool ones look red
- The hotter the blackbody the more energy emitted per unit area at all wavelengths (Stefan-Boltzmann law).

  $$F = \sigma T^4 \qquad \sigma = 5.7\text{x}10^{-8}\ \text{Wm}^{-2}\,\text{K}^{-4}$$

  - Note - bigger objects emit more radiation
- Except for their surfaces, stars behaves as a blackbodies

Energy Flux and Luminosity

- The Energy Flux, F, is the power per unit area radiated from an object.

$$F = \sigma T^4 \qquad \text{W/m}^2 \text{ (at all } \lambda)$$

  - The units are energy, area and time.
- Luminosity is the total energy radiated from star of radius R is given by:
- So the luminosity, L, is:

$$L = 4\pi R^2 \sigma T^4 \qquad \text{Watts}$$

  - If stars behave like blackbodies, stars with large luminosities must be very hot and/or very big.

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Luminosity and Flux

- Luminosity, L
  - The total energy radiated from an object per second.
  - Measured in Watts
- Emitted Flux, F
  - The flow of energy out of a surface.
  - Measured in Watts/m^2
- Observed flux, f
  - The power per unit area we receive from an object
  - Depends on the distance to the object.
  - Measured in W/m^2 e.g. f_{sun} = 1 kW/m^2
  - Also called flux or apparent brightness
- Meaning of Observed Flux
  - Make a sphere of radius, r, around an object (such as the Sun or a light bulb) which is radiating power.
  - All energy radiated from the object must pass through this sphere
    - The size of the sphere does not matter!

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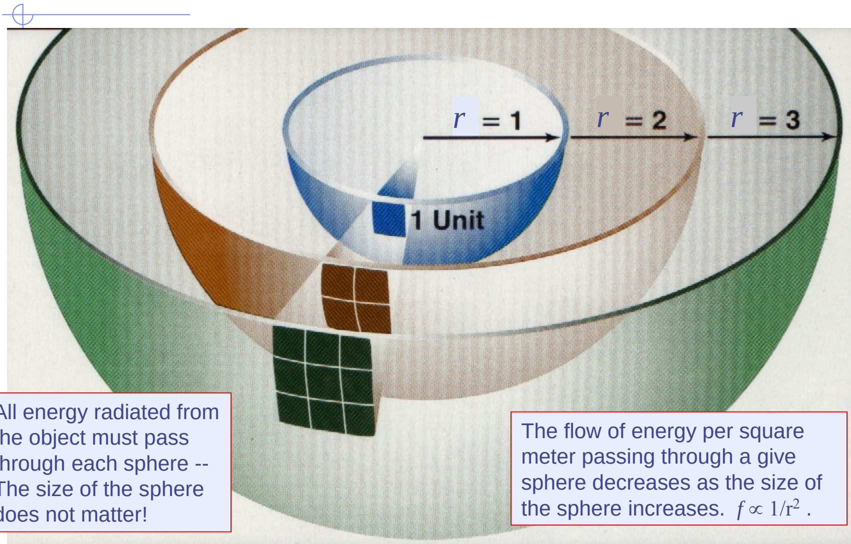
Flux falls off with distance



All energy radiated from the object must pass through each sphere -- The size of the sphere does not matter!

The flow of energy per square meter passing through a give sphere decreases as the size of the sphere increases. $f \propto 1/r^2$.

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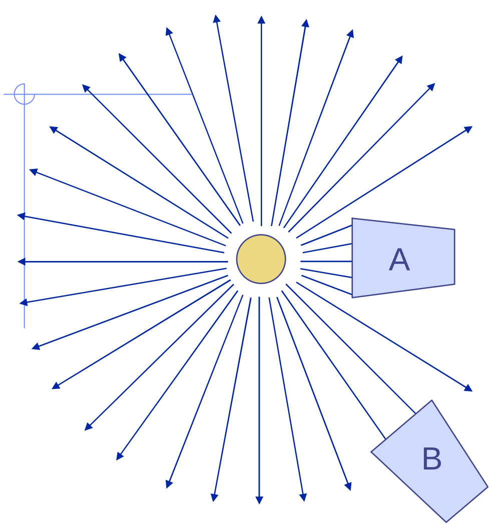
Flux: Sprinkler and Bucket Analogy



If bucket B is twice as far away as bucket A, it collects 1/4 as much water.

The sprinkler is the star, one of the buckets is the telescope (or your eye), and the water jets are the photons.

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Inverse square law

- The flux, f, of energy through a sphere of radius, r, is given by

$$f = \frac{L}{4\pi r^2} \quad (\mathrm{W/m^2})$$

Inverse square law

where L is the luminosity of the object

- Why do we care about flux?
  - The flux is what we measure.
  - We use a telescope (or our eye) and measure a small fraction of the light passing through this sphere.

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An illuminating example?

- A 100 W light bulb
  - about 1/5 of power goes into light
- It's total power output is always 100 W.
- It's apparent brightness to us depends upon how far away it is.

- For instance at 1 m the flux is:
  - Flux = 0.08 $\mathrm{W/m^2}$ [$f = 100\ \mathrm{W}/(4\pi(1\mathrm{m})^2)\ \mathrm{W/m^2}$]
- If we double the distance away from the light bulb, the flux drops by a factor of 4.
  - At 2 m, the flux is 0.02 $\mathrm{W/m^2}$

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Observed Flux – Distance Example:

- A star like the sun has an observed flux of 2.4×10^{-10} W/m^2. If the flux of the sun at the Earth is 1 kW/m^2, how far away is the star?

$$L_{sun} = 4\pi d^2 f \quad \Rightarrow \quad L_{sun} = 4 \times 3.14 \times \left(1.5 \times 10^{11}\,\mathrm{m}\right)^2 1000\ \mathrm{W/m^2}$$

$$\Rightarrow \quad L_{sun} = 3 \times 10^{26}\,\mathrm{W}$$

- Now

$$d = \sqrt{L_{sun} / 4\pi f} \quad \Rightarrow \quad d = \sqrt{3 \times 10^{26}\,\mathrm{W} / \left(4 \times 3.14 \times 2.4 \times 10^{-10}\,\mathrm{W/m^2}\right)}$$

$$\Rightarrow \quad \boxed{d = 3 \times 10^{17}\,\mathrm{m} = 10\mathrm{pc}}$$

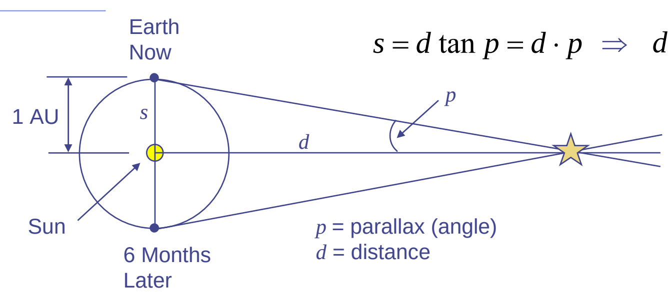
- We could also have used ratios rather than compute $L_{\rm sun}$ first.

$$\frac{d_{star}}{d_{sun}} = \sqrt{\frac{f_{sun}}{L_{sun}} \frac{L_{star}}{f_{star}}} = \sqrt{\frac{f_{sun}}{f_{star}}} \quad \Rightarrow \quad d_{star} = d_{sun} \sqrt{\frac{f_{sun}}{f_{star}}}$$

- Where we know everything but $d_{\rm star}$.

Distances to stars: Stellar Parallax

$$s = d \tan p = d \cdot p \quad \Rightarrow \quad d = \frac{s}{p}$$



- As stars get further away, their parallax becomes smaller.
- Parallax can not be measured to better than ~0.02" from the ground (d < 50 pc).
  - Interferometry is improving on this for selected applications
- Parallax is measured in arcseconds.
- Equations are for distances in AU and parsecs (pc), respectively

$$d(\mathrm{AU}) = \frac{206265}{p(\mathrm{arcsec})} \quad \text{or} \quad d(\mathrm{pc}) = \frac{1}{p}$$

1.0 arcsec => 1 pc,
0.5 arcsec => 2 pc

The closest stars

Star	Parallax (arcsec)	Distance (pc)	Luminosity (L_{sun}=1)
Proxima Centauri	0.763	1.31	5×10^{-5}
α Centauri A	0.741	1.35	1.45
α Centauri B	0.741	1.35	0.4
Barnard's Star	0.522	1.81	4×10^{-4}
Wolf 359	0.426	2.35	2×10^{-5}
Lalande 21185	0.397	2.52	5×10^{-3}
Sirius A	0.377	2.65	23
Sirius B	0.377	2.65	2×10^{-3}

Parallax is motion of a star on the sky due to the Earth orbiting around the Sun. 1" parallax corresponds to 1 pc. The more distant a star the smaller the parallax.
(1 pc = 3.26 lyr)

$$d(\text{pc}) = \frac{1}{p(\text{arcsec})}$$

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Current status

- The Hipparcos Satellite (1989 – 1993)
  - Astrometry mission, produced two catalogs
- Hipparcos catalog: ~120,000 stars
  - Measured parallaxes to better than 0.002" => d < 500 pc
- Tycho catalog: ~ 1,100,000 stars
  - Measured parallaxes and proper motions to ~ 0.025" (40 pc)
- Tycho 2 catalog: 2,500,000 stars
  - Update version of Tycho catalog
  - Reprocessed raw Tycho data & used 144 other catalogs to obtain proper motions
  - Proper motions to 0.0025"/yr

- Parallaxes are the key to knowing distances in the universe.
- Nearby stars are the stepping stone to measuring distance to everything else in the universe.
- We can now compute the luminosity of stars!

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Magnitudes

- We would like a way of specifying the relative brightness of stars
- Hipparchus
  - Devised a the magnitude system 2100 years ago to classify stars according to their apparent brightness.
  - He labeled 1080 stars as class 0, 1,.. 6.
  - 0 was the brightest, 1 the next brightest, etc.
- The magnitude scale is logarithmic.
- An increase in magnitude by 2.5 means an object is a factor of 10 dimmer, e.g.
  - a 0 mag star is 10 times brighter than a 2.5 mag star.
  - a 0 mag star is 100 times brighter than a 5 mag star.

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Example magnitudes

Star	m_v	Designations
Sun	-26.8	
Sirius	-1.47	α CMa
Canopus	-0.72	α Car
Arcturus	-0.06	α Boo
Vega	0.03	α Lyr
Betegeuse	0.45	β Ori
Altair	0.77	α Aqu
Deneb	1.26	α Cyg

- A dark adapted person with good eyesight can see to ~ 6^{th} magnitude.
- Hubble Space Telescope can observed objects fainter than 30 mag.
  - 4×10^9 times fainter than the eye!

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Fluxes and Magnitudes

- Flux is the power per unit area received from an object, e.g. f_{sun} = 1 kW/m^2
- If two stars, A and B, have fluxes, f_A and f_B, their magnitudes are related by

$$m_A - m_B = -2.5\log(f_A / f_B)$$

  - Thus if $f_B / f_A = 10$, then $m_A - m_B = 2.5$
- We can also write the inverse relation

$$\frac{f_B}{f_A} = 10^{\frac{m_A - m_B}{2.5}}$$

  - So that if $m_A = 5$ and $m_B = 0$, $f_B / f_A = 100$.

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Absolute & Bolometric Magnitudes

- m_v – apparent magnitude
  - How bright a star appears in the sky.
- M_v – absolute magnitude
  - Brightness if the star were at 10 pc
  - This is an intrinsic property of the star!
- M – absolute bolometric magnitude
  - Brightness at ALL wavelengths (and 10 pc).
- To get M_v or M we must know the distance to the star.
- Example:
  - Suppose a star has m_v = 7.0 and is located 100 pc away.
  - It is 10 times the standard distance, thus, it would be 100 times brighter to us at the standard distance.
  - Or 5 magnitudes brighter => M_v = 2.0

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Example Absolute Magnitudes

Object	m_V	M_V
Sun:	-26.8	4.77
Full Moon:	-12.6	(32)
Sirius:	-1.47	1.4
Canopus:	-0.72	-3.1
Arcturus:	-0.06	-0.3
Deneb:	1.26	-7.2

The Distance Modulus Equation

- The relation between m_v and M_v is written in equation form as:

  $$m_v - M_v = -5 + 5 \log_{10}(d) \qquad (d \text{ in pc})$$

- $m_v - M_v$ is called the distance modulus.
- Examples:
  - Deneb: $m_v = 1.26$ and is 490 pc away.

    $m_v - M_v = -5 + 5 \log_{10}(d)$

    $1.26 - M_v = -5 + 5 \log_{10}(490) = -8.5$

    $\Rightarrow M_v = -7.2$
  - Sun: $m_v = -26.8$, $d = 1$ AU

    $-26.8 - M_v = -5 + 5 \log_{10}(1/206265)$

    $\Rightarrow M_v = 4.8$

Bonus: Deriving the S-B Law

- We get the Stefan-Boltzmann law by integrating the Planck function over all frequencies (area under the curve)

$$B = \int_0^\infty B_\nu d\nu = \int_0^\infty \frac{2h\nu^3}{c^2} \frac{d\nu}{\exp(h\nu/kT)-1}$$

Let

$$x = \frac{h\nu}{kT} \quad \Rightarrow \quad \nu = \frac{kT}{h}x \quad \& \quad d\nu = \frac{kT}{h}dx$$

$$\Rightarrow \quad B = \frac{2h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3 dx}{\exp(x)-1} \quad \Rightarrow \quad B \propto T^4 \qquad \mathrm{W/m^2/sr}$$

- The total power emitted from a surface is proportional to the temperature to the fourth power – just as the S-B law
  - The constant of proportionality is not quite the S-B constant because we need to integrate over all solid angles to get it (which gives an additional factor of π). The integral is related to the Riemann zeta function giving

$$\sigma = \pi \frac{2k^4}{c^2h^3}\frac{\pi^4}{15} \quad \Rightarrow \quad \sigma = 5.67\times10^{-8} \qquad \mathrm{W/m^2/K^4}$$

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Another form of Planck function

- Let's rewrite the Planck function in terms of power per unit wavelength rather than frequency interval
  - Note that
  - $$B_\nu d\nu = B_\lambda d\lambda \quad \& \quad \nu = \frac{c}{\lambda} \quad \Rightarrow \quad d\nu = -\frac{c}{\lambda^2}d\lambda$$
  - where B_λ is over the interval λ to $\lambda + \Delta\lambda$ rather than ν to $\nu + \Delta\nu$.
  - This relationship must be true since the integral over frequencies and over wavelengths much be the same.
- Substituting gives

$$B_\lambda = B_\nu \frac{d\nu}{d\lambda} = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}\frac{c}{\lambda^2}$$

- Which yields for B_λ

$$\Rightarrow \quad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1} \qquad \mathrm{W/m^2/m/sr \ or \ W/m^2/\mu m/sr}$$

- Note that the units are now per m rather than per Hz.

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Bonus: Deriving Wien's Law

- To find the peak we differentiate with respect to λ and setting this equal to zero to get the peak

$$\frac{dB_\lambda}{d\lambda} = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}\left(-\frac{5}{\lambda}+\frac{\exp(hc/\lambda kT)}{\exp(hc/\lambda kT)-1}\frac{hc}{\lambda^2 kT}\right)=0$$

$$\Rightarrow \quad \left(-5+\frac{1}{1-\exp(-hc/\lambda kT)}\frac{hc}{\lambda kT}\right)=0$$

- Letting $x = hc/\lambda kT$ and solving iteratively solving for x,

$$x = 5(1-\exp(x)) \quad \Rightarrow \quad x = 4.965$$

- Putting back into the equation defining x

$$\lambda T = \frac{hc}{xk} = \frac{6.626\times10^{-34}\,\mathrm{J\text{-}s}\times 3\times10^{8}\,\mathrm{m/s}}{4.965\times1.38\times10^{-23}\,\mathrm{J/K}}\times\frac{10^{6}\,\mu\mathrm{m}}{\mathrm{m}} \quad \Rightarrow \quad \boxed{\lambda T = 2900\ \mu\mathrm{m\,K}}$$

- which is Wien's law. Note that If we had solved for the peak in B_ν (rather than B_λ) this would yield, x = 2.82 and the form of Wien's law is

$$\lambda T = 5100\ \mu\mathrm{m\,K} \qquad B_\nu \text{ space}$$

- Both forms is correct since the peak is different between B_ν and B_λ space.