Astronomy 2299
Assignment 1 (due in class Thursday Feb 15, 2018)

1. The dark night sky: an analogy:
The darkness of the night sky has a cosmological explanation as discussed in class. Here you will consider a very hypothetical analogy:

Suppose that you don’t know anything a priori about the prior history of our species nor its distribution on the Earth or on other planets in the universe. For some bizarre reason, every person ever born has been instructed to write a letter to be received by you on 2018 Feb 6.

Identify and discuss the circumstances in which you would receive an infinite number of letters vs. a finite number. You can and probably should consider space, time, and the delivery speed of such letters.

2. Orbital speeds: Calculate the orbital speed $V$ of an object with mass $m$ assuming it is in a perfect circular orbit around the Sun. Use

$$\frac{GM\odot m}{r^2} = \frac{mv^2}{r}$$

where the gravitational constant is $G = 6.67 \times 10^{-8}$ dyne cm$^2$ g$^{-2}$, the Sun’s mass is $M\odot = 2 \times 10^{33}$ g, and one astronomical unit is 1 AU = $1.5 \times 10^{13}$ cm. Express your results in km/s.

You can use SI units if you prefer, but still convert your answer to km/s; in those units the gravitational constant is $6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ and the Sun’s mass is $M\odot = 2 \times 10^{30}$ kg. One astronomical unit is 1 AU = $1.5 \times 10^{11}$ m.

(a) In general, how does the speed $v$ depend on orbital radius $r$?
(b) Earth ($r = 1$ AU)
(c) Jupiter ($r = 5.2$ AU)
(d) a Kuiper belt object ($r = 40$ AU)
(e) a comet in the Oort cloud of comets ($r = 50,000$ AU)

3. Escape velocities: In class we derived the escape velocity from a central object with mass $M$ and radius $R$

$$v_{\text{esc}} = \left(\frac{2GM}{R}\right)^{1/2}$$

by finding the kinetic energy that cancels the gravitational potential energy. Invert this equation for the case where the escape velocity is the speed of light, $c$, by solving for the radius in that case. For $M = 1 M\odot$, what is $R$ expressed in kilometers. Interpret your result.

4. Black holes: Suppose you are in orbit around a black hole, well outside its event horizon with radius $R_s$. For example, suppose your orbital radius is $5 \times R_s$ and you decide that you want to go home. What velocity does your spacecraft need to have in order to escape the gravitational pull of the black hole? Assume that your spacecraft has thrusters that operate in the radial direction from the black hole. If your spacecraft has mass $m$, what fraction of its rest mass energy $mc^2$ does its kinetic energy represent once the thrusters have brought it up to speed?

5. Black hole masses: Look on the web and identify one of the methods used to determine masses of supermassive black holes in the centers of galaxies. Write a paragraph that describes the method and what some of the mass determinations are.