Astronomy 2299
Assignment 4 (due in class Thursday May 3, 2018)

1. **Planetary Transits:** For transiting planets there are requirements for a primary transit or a secondary transit to be detectable. Assume the star and planet have radii $R_*$ and $r_p$ respectively; that the planet is in a circular orbit with radius $a_p$, and that the star is a distance $D$ from Earth. Consider a star exactly like the Sun.

(a) What are the requirements for a transit to occur along our line of sight?

(b) How deep is the primary transit in terms of the defined quantities?

(c) Suppose the star is seen at its distance $D$ with a signal to noise ratio of $S/N = 10^4$. This means that in the light curve for the star (i.e. flux vs. time), the ‘noise’ is 1/10000 the mean of the stars flux). What is the smallest planet that is detectable? (Hint: assume that the noise has a standard deviation $\sigma$ and that detection of the transit requires at least a $3\sigma$ dip in the flux.) Evaluate your expression in kilometers by using the known radius of the Sun $(5 \times 10^5 \text{ km})$.

(d) Three different flux levels can be measured (assuming the stars luminosity does not vary): (1) when neither the star nor the planet block one another; (2) when the planet transits the star; and (3) when the star occults the planet. Sketch the three geometries, showing the line of the sight to the observer. Flux from the planet is reflected or re-radiated starlight. Assume it does not vary with orbital phase. Let $F$ be the flux incident on the Earth from the star and $F_p$ that from the planet. For the three cases, write expressions for the fluxes using these quantities and, as needed, other quantities. Sketch a time sequence where you show schematically these flux levels vs. time over one orbit of the planet.

(e) In some cases the planet’s flux does vary with orbital phase because the planet is tidally locked to the star, presenting the same side to the star. When in the time sequence would you then expect the planet’s observable flux to be maximum?

2. **Number Density:** In several topics we have discussed the number of a particular kind of object per unit volume, such as the number of stars per cubic parsec or the number of galaxies per cubic Mpc. At the same time we often talk about the typical distance between objects. In this problem you will explore the relationship between typical distance and the number density.

(a) If the number density per unit volume is $n_3$, what is the typical distance? Hint: what is the radius of a sphere that contains exactly one object?

(b) Analogously, we sometimes consider the number of objects per unit area when the space of objects is a surface rather than a volume. If $n_2$ now represents the number of objects per unit area, what is the typical spacing?

(c) Consider a galactic disk with radius much larger than its thickness. Let $R$ be the radius and the thickness be $2H$ where $H$ is the ‘scale height’ of the disk. For a population of objects with large $n_3$, the mean distance is small and a 3-dimensional approach can be taken. For a sparse population with large mean distance, a 2-dimensional (area) approach is appropriate where $n_2$ is the integral of $n_3$ through the disk vertically. At what mean distance does the transition from a 2D to 3D approach occur?
3. **Estimating Numbers**: Use the Drake equation and your own assessment for the various factors used in it to estimate the number of technological civilizations there might be in the Galaxy. Using a similar approach, estimate how many planets in the Galaxy might have microbial life. There are no right answers for this problem but your estimates should be well reasoned and described.

4. **Detecting Technological Civilizations**: This problem is about detectability of a transmitter on interstellar distances. Let $p_t$ be the transmitter power (watts). It is analogous to the luminosity of a star so if the transmitter radiates isotropically, the flux (watts m$^{-2}$) at a distance $D$ would be

$$F = \frac{p_t}{4\pi D^2}.$$  

However, the bandwidth $B$ into which the transmitter radiates also matters, so we use the flux density (watts m$^{-2}$ Hz$^{-1}$),

$$F_\nu = \frac{p_t}{4\pi D^2 B}.$$  

In practice transmitters do not radiate isotropically. Instead they radiate into a solid angle $\Delta \Omega \leq 4\pi$ that can be quite small, $\Delta \Omega \ll 1$. In this case the flux density at a distance $D$ in a direction that the transmitter is pointed to is:

$$F_\nu = \frac{p_t}{\Delta \Omega D^2 B}.$$  

The world’s largest radio telescope (Arecibo) can transmit $10^6$ watts into a narrow bandwidth $B = 1$ Hz and into a narrow angular beam that is $\theta = 3$ arcmin wide (or 1/20 degree). The corresponding solid angle is $\Delta \Omega = (\pi/4) \theta^2$. To use this you will have to convert the solid angle from degrees$^2$ to steradians. (Hint: there are $4\pi$ steradians over the sky, corresponding to $129,600/\pi \approx 41,253$ deg$^2$.)

A similar telescope at distance $D$ could detect the radar signal from Arecibo if it produces a flux density of about $F_{\nu, \text{minimum}} = 10^{-27}$ watts m$^{-2}$ Hz$^{-1}$. (Note: **Fixed missing minus sign**.) Using this information, calculate how far away the Arecibo signal could be detected.