Basics of Stars

A star is a self-gravitating object that radiates significant amounts of electromagnetic energy (e.g. visual light, but also radio, infrared, X-rays and \(\gamma\)-rays).

“Normal” stars (like the sun) are those that are luminous due to thermonuclear fusion of hydrogen into helium. The sequence of such hydrogen burning stars as a function of mass is called the main sequence on the Hertzsprung-Russell (HR) diagram, which is a plot of luminosity vs. temperature or color.

**Hydrostatic equilibrium:** Stars are usually in a state of equilibrium or balance between gravity and gas pressure:

- gravity pulls inward
- gas pressure due to heat (= random kinetic energy) pushes outward
- the star is in balance: gravity causes high temperature in the core of the star needed to drive thermonuclear reactions; thermonuclear reactions in turn keep the temperature high to produce pressure that balances gravity.
- once thermonuclear reactions cease, the star cools and must collapse.
- the extent of collapse (how small does the star get?) depends on other mechanisms of gas pressure, such as quantum mechanical (“degenerate”) gas pressure.
- Degenerate electron pressure balances gravity in white dwarf stars (about the size of the Earth \(\approx 10^4 \text{ km}\)).
- Degenerate neutron pressure balances gravity for neutron stars (about \(\approx 10 \text{ km in radius}\)).
- Stars too massive to be supported by degenerate neutron pressure cannot be supported: gravity wins and a black hole is formed.

**Elemental abundances:** The sun is about 74% hydrogen, 25% helium and 1% trace elements. The relative abundance of H and He was determined in the early universe (first 3 minutes). The trace elements originate from previous generations of star formation, with recycling of heavy elements into the interstellar medium (gas clouds) from which new stars form.
Virial Theorem

The virial theorem ('virial' has a Latin root meaning energy) states that for an object that is bound by gravity (as opposed to mechanical forces, as in rock) and is stable because of counteracting pressure forces, that kinetic energy $W$ and potential energy $U$ satisfy

$$2W + U = 0.$$  \hspace{1cm} (1)

The total energy is

$$E = W + U = \frac{U}{2} < 0.$$  \hspace{1cm} (2)

It can be shown that the VT holds for a planet of mass $m$ in a circular orbit of radius $r$ around a star of mass $M$:

$$W = \frac{mv^2}{2} \quad \text{and} \quad U = -\frac{GMm}{r}.$$  \hspace{1cm} (3)

For a circular orbit $mv^2/r = GMm/r^2$ from which the kinetic energy $W$ can be calculated in terms of $U$.

**Application to stars:** For a star, the VT is also satisfied statistically. Consider a star of mass $M$ and radius $R$ having uniform density $\rho = M/[(4\pi/3)R^3] = 3M/4\pi R^3$. The mass interior to a radius $r$ is $M(r) = M \times (r/R)^3$. Using this, the potential energy of the entire star is

$$U = -\int_0^R dr \frac{GM(r)4\pi r^2\rho}{r} = -\int_0^R dr \frac{GM4\pi r^2}{r} \cdot \frac{3M}{4\pi R^3} \cdot \frac{r^3}{R^3} = -\frac{3GM^2}{5R}.$$  \hspace{1cm} (4)

For the kinetic energy we consider only thermal energy of mono-particles. We have

$$W = \left\langle N \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}NkT = \frac{3}{2}\frac{M}{\mu m_H}kT,$$  \hspace{1cm} (5)

where $k =$ Boltzmann’s constant $= 1.38 \times 10^{-16}$ erg K$^{-1}$, $m_H =$ mass of the hydrogen atom (i.e. the proton) $= 1.67 \times 10^{-24}$ g, and $\mu$ is the mean atomic weight, slightly greater than 1, that takes into account that the stars are predominantly hydrogen combined with helium. The total energy is

$$E = W + U = -\frac{U}{2} = -\frac{3GM^2}{10R}.$$  \hspace{1cm} (6)

**Mean temperature of a star:** Using the VT we can solve for the temperature $T$ using $W = -U/2$,

$$T = \frac{GM\mu m_H}{5kR}.$$  \hspace{1cm} (7)
Evaluating for the Sun with $M_\odot = 2 \times 10^{33}$ g and $R_\odot = 7 \times 10^{10}$ cm we get a mean density that is about that of liquid water

$$\bar{\rho} = \frac{3M}{4\pi R^3} = 1.4 \text{ g}$$

and a temperature

$$T = \frac{(6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2})(2 \times 10^{33} \text{ g})(1.67 \times 10^{-24} \text{ g})\mu}{5(1.38 \times 10^{-16} \text{ erg s}^{-1})(7 \times 10^{10} \text{ cm})} = 4.6 \times 10^6 \mu K.$$  

(9)

This temperature is an average over the star. For a real star, the mass density is not uniform so that the mass is more centrally condensed than a uniform star. This means that the ratio $M_c/R_c$ is larger in the core of a star and so the core temperature $T_c$ is larger than the value derived assuming a uniform star. Nonetheless, the value we get is comparable to what is needed to drive nuclear reactions given the density inside the Sun.

*Kelvin-Helmholtz contraction time and the age of the Sun:* The VT applies when there is balance between pressure and gravity. If a star were to contract to smaller $R$, its potential energy becomes more negative and half of the increase in its magnitude goes into thermal motions. To conserve energy, another half of the potential energy needs to escape the star by radiation. Stated better, in order for a star to get smaller it needs to radiate as much energy as goes into increased thermal motion inside the star. For example, if $R \rightarrow R/2$, the potential energy $U$ gets more negative by a factor of two. The kinetic energy $W$ therefore also increases by a factor of two and the energy radiated is equal to the original amount of kinetic energy.

Suppose the Sun’s measured luminosity $L_\odot = 4 \times 10^{33} \text{ erg s}^{-1}$ was fueled by gravitational potential energy. Then the radiation lifetime of the Sun (defined as the time needed for the Sun to shrink in radius by 1/2) is

$$\tau = \frac{|U|}{L_\odot} = \frac{3}{10} \frac{GM^2}{RL_\odot} \approx 2.9 \times 10^{14} \text{ s} \approx 9 \text{ Myr.}$$

(10)

This time is much shorter than the known age of the solar system. Historically, the discrepancy between the Kelvin-Helmholtz contraction time and the estimated ages of fossils was a conundrum. Once nuclear reactions were discovered and understood, the luminosities of stars were rapidly understood to derive from nucleosynthesis rather than from gravity.

We will find, however, that other objects in the universe do radiate by virtue of their gravitational potential.
Main sequence stars: Thermonuclear reactions for hydrogen burning, $4H \rightarrow He^4 + $ photons + neutrinos, include the proton-proton chain and the CNO (Carbon-Nitrogen-Oxygen) cycle:

- the efficiency of energy release is 0.7%: for each gram of hydrogen processed, about 0.007 gm of equivalent energy ($E = mc^2$) is released.
- nucleosynthesis occurs only in the core of a star.
- the core temperature of the Sun is about $10^7$ K, compared to 6000 K at the surface.
- the core density of the sun $\sim 100 \text{ gm cm}^{-3}$ while at the surface (the photosphere) it is $\ll 1 \text{ gm cm}^{-3}$.
- the minimum mass for hydrogen fusion is $\sim 0.07M_{\odot}$.
- the maximum observed mass for hydrogen burning stars is $\sim 100M_{\odot}$.
- if/when nucleosynthesis is shut off, the star collapses to a more compact form.
- the main sequence lasts until about 10% of core hydrogen is converted to helium.
- the main sequence lifetime is the duration of core hydrogen burning.
- after the main sequence stage, a star becomes a red giant and ultimately turns into a white dwarf, neutron star, or black hole, usually through violent evolution involving novae or supernovae. Some stars explode completely, leaving behind no remnant.
Main sequence luminosities & lifetimes: The more massive a star, the faster it burns hydrogen, because the core temperature is higher. Roughly, the luminosity (units = energy/time in erg s\(^{-1}\) or joules s\(^{-1}\) ≡ watts) of the star (erg s\(^{-1}\)) scales with mass as

\[L(M) \propto M^3\]

or

\[L(M) = L_\odot \left(\frac{M}{M_\odot}\right)^3\]

The strong dependence on mass results from the much higher central temperatures of more massive stars, which can drive nucleosynthesis reactions much faster. Accordingly, a star with mass \(10M_\odot\) has a luminosity of \(10^3L_\odot\) while a star with \(100M_\odot\) has luminosity \(10^6L_\odot\). Recall that

- \(M_\odot = 2 \times 10^{33}\) g = \(2 \times 10^{30}\) kg.
- \(L_\odot = 4 \times 10^{33}\) erg s\(^{-1}\) = \(4 \times 10^{26}\) watts.

The main sequence lifetime of a star is much shorter if it is much more massive than the sun:

- The ms lifetime is the time it takes to process about 10% of the star’s hydrogen.
- The energy released is \(E = 0.1\epsilon M c^2\) where the efficiency is \(\epsilon = 0.007\).
- the MS lifetime is the time it takes to radiate this energy at the luminosity of the star, which also depends on the mass:

\[t_{MS} = \frac{E}{L} \approx \frac{0.1\epsilon M c^2}{L_\odot (M/M_\odot)^3} \approx \frac{0.1 \times 0.007 \times M_\odot c^2 (M/M_\odot)}{L_\odot (M/M_\odot)^3} \approx \frac{0.1 \times 0.007 \times 2 \times 10^{33} \text{ gm } \times (3 \times 10^{10} \text{ cm s}^{-1})^2}{4 \times 10^{33} \text{ erg s}^{-1}} \left(\frac{M_\odot}{M}\right)^2 \approx 3 \times 10^{17} \left(\frac{M_\odot}{M}\right)^2 \text{ sec} \approx 10^{10} \left(\frac{M_\odot}{M}\right)^2 \text{ years.}\]
Star Formation: New stars are formed continuously in the Galaxy (and in other galaxies) in dense clouds of atomic and molecular gas that become unstable to collapse of condensations. As usual, gravity must win over gas pressure, requiring that the clouds cool by radiating. Magnetic fields that thread the clouds also contribute to the pressure. Collapse is therefore a slow process. Clouds can be driven unstable when a shock wave collides with it, such as shock waves from supernovae. Thus the “death” of some stars lead to the birth of new ones.

- small, less massive stars are more common than very massive stars
- the prevalence of low-mass stars is consistent with star formation in molecular clouds involving turbulence.
- most stars form in multiple systems (binaries, triple-star systems, etc.)
- the Sun is in the minority of single stars.
- if the Sun were in a binary system, the orbits of the planets would not have been stable; either the planets would plunge into one of the stellar components or the climates on any surviving planets would be too volatile for life to evolve to any significant complexity.
- star formation has been relatively steady in the Galaxy over the last 10 Gyr.
- steady star formation may have been punctuated by bursts of intense star formation (starbursts).
- starburst galaxies are seen where the rate of star formation is much larger than in our Galaxy.
- spiral galaxies (like ours and Andromeda = M31) have higher rates of star formation than in elliptical galaxies, which have much less mass in gas clouds.
- star formation in all galaxies appears to have been more prevalent at redshifts $z \approx 1$ to 3 than at the present time.
- all galaxies will eventually run out of gas to form new stars.