Proceedings of the Fall 2004 Astronomy 233 Symposium on

MEASUREMENTS OF THE HUBBLE CONSTANT

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Cornell University

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Editors

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*Astronomy 233 is offered by the Cornell University Astronomy Department and the College of Arts and Sciences under the John S. Knight Institute Sophomore Seminar Program.
Astronomy 233 “From Planets to Galaxies: The Origins of Cosmic Structures” was taught during the fall semester by Professor Don Campbell with the very able assistance of Astronomy and Space Sciences graduate student Julia Deneva. The course is intended to provide students interested in majoring, or concentrating, in astronomy with an introduction to current forefront topics in the field and also to expose them to aspects of a professional research career such as the symposium. It is also intended to hone their writing skills with emphasis on writing for specific audiences. There are four writing assignments of which three, including the symposium paper, are revised by the students based on suggestions and comments from the teaching assistant and instructor on a first draft.

Astronomy 233 was one of the first Knight Institute sophomore seminars to be offered at Cornell. The focus this semester was on a discussion of issues related to the origins of cosmic objects from planets to stars to galaxies to the cosmos. The major emphasis was on the search for extra solar planets, star formation, the evidence for the existence of dark matter and dark energy and issues related to cosmology. Professors Phil Nicholson and Paul Goldsmith gave guest lectures on, respectively, results from the Cassini mission to the Saturn system and star formation. Julia Deneva gave the students a very comprehensive lecture on pulsars. The latter part of the course concentrated on issues related to cosmology and the measurement of the expansion rate of the universe as given by the Hubble constant.

The talks by the students in the symposium ”Measurements of the Hubble Constant” were based on the papers included in these proceedings. The students summarized papers in the scientific literature that discuss different aspects of recent efforts to measure the Hubble constant. The papers in these proceedings represent their original work with, occasionally, very minor editing to conform to the style of the proceedings.

We would like to congratulate the authors for the quality of the papers and compliment them for their enthusiasm and energy.

Donald B. Campbell

Julia Deneva

Ithaca, New York
02 December 2004
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Period-Luminosity and Period-Luminosity-Color Relations of Cepheids in the Magellanic Clouds

Amy Shaw

ABSTRACT Cepheid variables are good standard candles, and therefore it is important to look at their Period-Luminosity-Color and Period-Luminosity relations. These can help determine the distance to a Cepheid and therefore to its host galaxy. For now, we are most concerned with the distances to the Magellanic Clouds. The result obtained for the difference between the mean distance moduli of the Magellanic Clouds is $\mu_{SMC} - \mu_{LMC} = 0.15 \pm 0.03$ mag. In order to calibrate the PLC and PL relations, the LMC distance modulus $\mu_{LMC} = 18.22 \pm 0.05$ mag was used. An upper limit on the absolute magnitude of a Cepheid is also found: $M_{V,10}^C = -3.92 \pm 0.09$ mag.

1.1 Introduction

Long after the initial discovery of the PL relation (Leavitt 1912), its calibration is still being debated. Though the periods of Cepheids can be readily determined, it is difficult to ascertain the brightness of Galactic Cepheids due to their distance from the Earth and their high reddening. Furthermore, the zero-point of the P-L relation depends on metallicity. Yet efforts in calibrating this relation are being continued, especially because the HST has discovered so many Cepheids that can be made use of once calibration of the P-L and P-L-C relations is complete. Their most important use is in the determination of the Hubble constant $H_0$ which has become more accurate over the years but still needs some refinement. $H_0$ can then be used to calculate the age of the universe.

The Magellanic Clouds have advantages over other objects in terms of calibrating the Cepheid P-L-C and P-L relations. It is true that geometric methods using masers can be used to determine an accurate distance to NGC 4285 (Herrnstein et al. 1999). Furthermore, Cepheids have been detected in this galaxy. Therefore, it can be used to test the Cepheid calibrations. However, it is not desirable to use it as the calibrator itself because it only offers a small number of Cepheids. On the other hand, the Magellanic Clouds are nearby and contain many Cepheids. Also, the Clouds are homogeneous in chemical composition, allowing for a smaller error due to metallicity.

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1. Period-Luminosity and Period-Luminosity-Color Relations of Cepheids in the Magellanic Clouds

FIGURE 1.1. A plot of I-band magnitudes of fundamental mode Cepheids. The filled dots represent Cepheids in the LMC, while open circles represent those in the SMC. SMC Cepheids have been lowered by 0.8 mag.

Therefore, the Magellanic Clouds were used in this calibration. Furthermore, red clump stars were the calibrators because Hipparcos obtained accurate parallaxes for them. Results obtained here for the Cepheid P-L-C and P-L relations can be used to calibrate other methods including the Tully-Fisher relation and the Type Ia Supernovae method.

This paper will present findings on the P-L and P-L-C relations that are integral to determining galactic distances. Classical, fundamental mode (FU) Cepheids are concentrated on because they are the type usually found in extragalactic areas and are therefore useful for extending the reach of distance computation. One fortunate result is that P-L relation values for the distance moduli to the Magellanic Clouds agree with the values from other standard candles such as RR Lyr stars (Popowski and Gould 1998). This would imply that population differences among the LMC and SMC do not affect the P-L and P-L-C relations.

1.2 Observations

The 1.3-m Warsaw telescope at the Las Campanas Observatory, Chile was used. Its SITe 2048 x 2048 CCD detector was operated in "slow" driftscan mode. Observations were in the center of the Clouds and were collected in the BVI bands. This was part of the second stage of the OGLE survey. 2140 Cepheid variables from the SMC and 1280 from the LMC were used. These were obtained from 120-360 epochs in the I-band and 15-40 in the B- and V- bands.
1. Period-Luminosity and Period-Luminosity-Color Relations of Cepheids in the Magellanic Clouds

FIGURE 1.2. The P-L relation. Top panel: I-band of LMC Cepheids; FU Cepheids are dark, while first overtone (FO) Cepheids are light. Bottom panel: FU Cepheids. The solid line represents the fit from Fig. 1.4. Rejected spots are light-colored.

1.3 Determination of Relations

Red clump giants were the means of determining the mean extinction in the LMC and SMC: $E(B-V) = 0.137$ and $E(B-V) = 0.087$, respectively. Red clump giants are abundant close to the Sun, which means that their calibration can be double-checked with parallax measurements. Also, they are bright enough to be seen farther way. These stars are present in large numbers in both the LMC and the SMC and their I-band magnitude is largely independent of their age. However, red clump stars could have a different spatial distribution within a Cloud than Cepheids do, so there may exist some systematic error. After each individual Cepheid was corrected for extinction in this manner, this phenomenon was then further compensated for by rejecting certain stars. The criterion for rejection was how well the star fit the equation for each particular relation:

\begin{align}
M &= \alpha \log P + \beta CI + \gamma \\
M &= a \log P + b
\end{align}

where $P$ is the Cepheid’s period, $M$ its magnitude, and $CI$ its color index. Points more than 2.5 standard deviations from the mean were rejected; these turned out to be the shorter period Cepheids. This procedure was repeated a few times with the remaining stars. The closest possible least-squares fit was needed in order to improve upon previous calibrations of the relations, such as those achieved by the Hipparcos satellite (Feast and Catchpole 1997). Accurate results are also important because an error in Cepheid calibration results in errors of the calibration of distances to galaxies which result in error in $H_0$. Unfortunately, error is still present in our results (see Fig.
1.3-1.6), the main source of which, for the P-L-C relation is due to color.

1.4 Calibration of the Relations

Since Cepheids in our own galaxy cannot be used to calibrate the relations because they are all very far from the Sun, other methods must be used. The Magellanic Clouds appear to be the best suited for the calibration. Many different methods have been used to obtain the distance modulus to the LMC. One of the methods that has the potential to be very accurate is the geometric method used with eclipsing binary stars. Another, which is the most reliable is that which uses RR Lyr stars. A third which is the most precisely calibrated is that which uses red clump stars. There is also the method of observing the light from SN1987A’s ring of gas. To calibrate the relations that are the subject of this paper, the average from all of these methods has been used: $\mu_{LMC} = 18.22 \pm 0.05$ mag is obtained.

An upper limit on the magnitude for a Cepheid can also be determined by comparison to RR Lyr stars. For the V-band Cepheid magnitudes, $V_{C,10}^{LMC} = 14.28 \pm 0.03$ mag and $V_{C,10}^{SMC} = 14.85 \pm 0.04$ mag are obtained. This can be compared with the magnitudes of RR Lyr variables: $<V_{RR}^{LMC}> = 18.94 \pm 0.04$ mag and $<V_{RR}^{SMC}> = 19.43 \pm 0.03$ mag. Metallicity differences change this by $\delta V_{RR-C,10}^{LMC} = 4.66 \pm 0.05$ mag and $\delta V_{RR-C,10}^{SMC} = 4.60 \pm 0.05$ mag. We then compare RR Lyr variables and Cepheids and then look at the absolute calibration of RR Lyr stars ($M_{V}^{RR} = 0.71 \pm 0.07$ mag) to obtain $M_{V}^{C,10} = -3.92 \pm 0.09$ mag.

1.5 References

1. Period-Luminosity and Period-Luminosity-Color Relations of Cepheids in the Magellanic Clouds

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FIGURE 1.3. $N$ is the number of Cepheids; $\sigma$ is the error obtained from subtracting the data from the fit presented. $W_1$ stands for the Wesenheit index (Madore 1982).

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FIGURE 1.4. $N$ is the number of Cepheids; $\sigma$ is the error obtained from subtracting the data from the fit presented. $W_1$ stands for the Wesenheit index (Madore 1982).
TABLE 1.5. N is the number of Cepheids; \(\sigma\) is the error obtained from subtracting the data from the fit presented.

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FIGURE 1.5. N is the number of Cepheids; \(\sigma\) is the error obtained from subtracting the data from the fit presented.

TABLE 1.6. N is the number of Cepheids; \(\sigma\) is the error obtained from subtracting the data from the fit presented.

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Measurement of the Distance to Galaxy NGC4258

Greg Vesper

ABSTRACT Much of modern cosmology relies upon an accurate measurement of distances to extragalactic sources; calculating the presence and distribution of dark matter, the flatness and closure of the universe, the Hubble constant and the age of the universe all require this knowledge. Current extragalactic distances are determined using the extragalactic distance ladder (EGL), which is calibrated from distance to Cepheid Variable stars in the Large Magellanic Cloud. This distance estimate is uncertain and so introduces errors into all determinations of extragalactic distances. Several nearby Seyfert galaxies, notably NGC 4258, contain water masers in a disk about the central black hole. By calculating the parameters of their orbits, the first precise independent estimate of an extragalactic distance is achieved: 7.2 ± 0.5 Mpc – and so provides an important calibration of the EGL.

2.1 Introduction

Water masers are produced when a strong source of microwaves illuminates a high density cloud containing a sufficient concentration of H$_2$O. The incident light is absorbed by the water molecules, forcing electrons into higher energy states. The excited molecules subsequently release the added energy by radiation, and in the appropriate conditions, this subsequent radiation will be coherent. By thus concentrating the phase of the microwaves, the cloud effectively amplifies the incident radiation. Theoretical calculations have determined that a dense disk of dust orbiting the super-massive black hole at the center of a Seyfert galaxy can be an effective masing region (Miyoshi et al. 1995, Elmegreen & Morris 1979). The mechanisms which actually serve to form such a region are unknown, but may relate to particularly energetic stellar formations or interactions, and/or to an interaction between the disk and a high-velocity wind emanating from the central source.

Greenhill et al (1990) have discovered at least two dozen nearby galaxies whose nuclei contain water masers. Notable among these are 5 galaxies in which the 22 GHz emission of the water masers has very high apparent luminosity (at least $10\, L_\odot$). NGC 4258 (M106) is one of these galaxies and is particularly interesting because we have sub-arcsecond resolution observations from VLBI. Before the 1990s, astronomers knew about a set of masers with approximately the

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same recessional velocity as the galaxy (472 ± 4 km s\(^{-1}\), Cecil et al. 1992), known as the systemic group. Nakai, Inoue, and Miyoshi (1993) detected a set of high-velocity masers in NGC 4258 (with velocities offset from systemic by about ±900 km s\(^{-1}\)). The power spectrum of emission for NGC 4258 has a pair of central peaks (offset by 50 km s\(^{-1}\)) and outlying smaller peaks corresponding to the high-velocity groups of masers (see Fig. 2.1).

**FIGURE 2.1.** The observed LOS velocities of the masers and the total power spectrum of the galaxy. The arrows indicate the identifiable high-velocity masers to the East (Nakai et al. 1995). Inset is a plot of LOS velocity and impact parameter (Greenhill et al. 1995).

### 2.2 The Disk Model

The VLBI observations of NG4258 have shown that there is an extremely high concentration of matter in the nucleus of the galaxy: over forty times the density of any other proposed location for a super-massive black hole (much denser than other galactic centers and the cores of globular clusters, also much denser than known star forming regions). This evidence implies that there is a super-massive black hole at the center of the galaxy. Within the radius at which the masers are found, there is a mass of \(3.5 \times 10^7 M_\odot\).

For all observations, the masers are found within a rectangle centered on the galactic core (with a small width to length ratio), immediately suggesting that the masers (and thus a dense cloud of particles) form a disk about the black hole which we see edge on. Observations of the masers show that the position of the systemic group shifts slowly with time and that the line-of-sight (LOS) velocity of the high-velocity group of masers is constant to first order. Also, the positions of the high-velocity features show systematic and antisymmetric changes across the systemic group. These results are consistent with Keplerian rotation in a thin, sub-parsec disk about the central black hole. Data taken by Greenhill et al. 1995 shows that there is a linear relation (with low error) between the displacement of the maser from the center and its LOS velocity (see Fig. 2.2),
which indicates that the height of the disk must be small with respect to its radius. Moreover, the highly clumped distributions of the Doppler shifts of the high-velocity maser features show that the radial thickness of the ring must also be small \((\Delta R/R \approx 2\Delta V_{\text{LOS}}/V_{\text{LOS}} \approx 0.3)\)[5].

![Figure 2.2](image)

FIGURE 2.2. These plots of right ascension distance versus LOS velocity and declination versus LOS velocity show a close linear relation between the two features (net offset and LOS velocity). The velocity gradients are \(3.67 \pm 0.02\) km s\(^{-1}\) and \(0.43 \pm 0.07\) km s\(^{-1}\) respectively. These gradients far exceed the possible systematic errors from the observation process and so must be features of NGC 4258 (Greenhill et al. 1995).

2.2.1 Warping of the Disk

The declinations of the high-velocity features are not colinear with the center of the disk, indicating a warp in the disk. Herrnstein et al. (1996) employed 9 parameters to model the disk, 6 of which model a flat, Keplerian disk: center of mass \((x_0, y_0)\), systemic velocity \(v_0\), inclination \(i_0\), central mass \(M\) and the position angle of the disk \(\xi_0\). The warp is characterized by angle and inclination warping as functions of \(r\) (2 and 1 degree polynomials respectively because the disk appears less sensitive to inclination warping). These polynomials represent the best quadratic and linear approximations to the actual relations involved. Using a \(\chi^2\) minimization on these parameters, an accurate model for the warp in the disk can be found, which gives a centripetal acceleration of \(\sim 0.1\) km s\(^{-1}\) yr\(^{-1}\) and a mass interior to the disk of \(3.5 \times 10^7 M_\odot\), both of which agree with observations (Herrnstein et al. 1996).
2. Measurement of the Distance to Galaxy NGC4258

2.2.2 Correlation of Theoretical Model

The simple model of a rotating, masing disk orbiting a central microwave emitter will produce a power spectrum with a central peak and two subsidiary ones, which does not match the double central peak that we observe. However, with the assumption of an inner absorbing layer and outer masing layer, Watson and Wallin (1994) have shown that the spectrum of a Keplerian disk can be reconciled to NGC 4258. Since there is an upper threshold on density at which a particle cloud can be masing, such a model seems realistic for a disk which decreases in density with increasing radius. We consider such a system viewed almost head-on, where the region between source and absorbing disk is transparent (and the velocity of particles at $b = r_o$ is $v_0$). The optical depth of the disk ($-r_o \leq b \leq r_o$) can be expressed (in the simplified case) as

$\tau_v(b) = \frac{\tau_0}{r_o} \int g(r) \exp\left\{ -\frac{[\Delta v - v_o b(r_o)^{1/2}]^2}{v^2}\right\} ds$, \hspace{1cm} (2.1)

where we integrate along the path of the ray, $\bar{v}$ is the average velocity of particles in the medium, $\Delta v$ is related to the Doppler velocity by the usual equation $v = v_0(1 - \frac{\Delta v}{\bar{v}})$, and the function $g(r)$ is an amplification function such that $g(r) = 1$ in the masing region, meaning $g(r) \equiv k < 0$ in the absorbing region and 0 everywhere else. $\tau_0 = \tau_{un}(0)/2$, so the intensity at $r_o$ is simply

$I_v(b) = I_v^0(b) e^{\tau_v(b)}$, \hspace{1cm} (2.2)

where $I_v^0(b)$ is the intensity of incident radiation at $r_a$ when $b > r_c$ and the intensity of the radiation created by the opaque core otherwise. Finally, the observer measures a radiation flux

$F_v = \frac{\alpha \beta}{2} \int I_v(b) \frac{I_v^0(b)}{r_0} db$, \hspace{1cm} (2.3)

where $\alpha$ and $\beta$ are the angular thickness and diameter of disk as seen by the observer.
2. Measurement of the Distance to Galaxy NGC 4258

By adjusting the parameters $r_i, r_a, r_c, v_0, \tau_0$ and $g(r)$, Watson and Wallin (1994) were able to match the results to equation 2.3 to the spectrum of NGC 4258. This gave the results: $r_i = 0.85r_0, r_a = 0.7r_0, r_c = 0.1r_0, g(r) = -0.4$. The central double peak in the spectrum can be understood as follows. At $b = 0$, there is no Doppler shift, so the absorptive region has the greatest effect and greatly reduces the optical depth (OD). As $|b|$ increases, the optical depth caused by the masing gas will decrease, while the OD due to the absorptive region will decrease even faster. Combining these effects gives an increase and subsequent decrease in optical depth (and thus power) as $|b|$ goes from 0 to $r_c$.

2.2.3 CONFIRMATION OF THE DISK MODEL BY OBSERVATIONS

As further confirmation that the rotating disk model is accurate, we consider the variation in time of the LOS velocity (as determined through Doppler shift) of specific masers (which can be modelled as small irregularities in the gas)[14]. For $|b| < r_c$, the peak of emission should occur where $\Delta \tau \approx \frac{v_0}{r_0}$ and so:

$$\frac{d(\Delta \tau)}{db} = \frac{v_0}{r_0}.$$  \hspace{1cm} (2.4)

For an irregularity, the time variation should model a circular orbit:

$$\frac{d(\Delta \tau)}{dt} \approx \frac{v_0^2}{r_0}.$$  \hspace{1cm} (2.5)

From observations, we know that $\frac{d(\Delta \tau)}{dt} = 6 \text{ km s}^{-1} \text{ yr}^{-1}$, so we get that $v_0 = 700 \text{ km s}^{-1}$, a good agreement with the observed 900 km s$^{-1}$.[14]

2.3 Distance to NGC 4258

Now that we understand the central region of the galaxy and have quantitative results for the important parameters of the disk, we can proceed to determine the distance to the galaxy. Although perhaps 25–35 trackable masers exist in NGC 4258, they are so densely packed as to be untrackable individually. To determine the rotation in bulk, Herrnstein et al. (1999) developed a statistical method for calculating the overall rotation of the disk.[8] The algorithm assumes that the masers have a random distribution about $\langle r_s \rangle \approx 3.9 \text{ mas}$ (from the accuracy of the disk model). This algorithm, when applied to sets of data spanning 1994–1997, yields a bulk proper motion $\langle \dot{\theta}_x \rangle = 31.5 \pm 1 \mu\text{as yr}^{-1}$ and an acceleration $\langle \dot{v}_{\text{LOS}} \rangle = 9.3 \pm 0.3 \text{km s}^{-1} \text{ yr}^{-1}$.[7] This is the first detection of a bulk proper motion and matches expectations. From the disk model, we can obtain estimates of the parameters: $M_e = \frac{v_0^2}{2GM_e} \approx 3.9$ represents the mass inside the disk (as calculated from the high-velocity rotation curve. $i_s = 82.3^\circ$ is the inclination angle of the disk. $\alpha_s \approx 80^\circ$ is the angle from our line of sight to the clump of systemic masers and is obtained from the warped disk model. $r_s$ is the distance to the systemic group of masers from the center of the disk. Elementarily, we find that the angular velocity $\omega$ of the disk: $\omega^2 = \frac{GM_e}{r_s^2}$, and so by
straightforward orbital mechanics and trigonometry:

\[
\langle \dot{\theta}_x \rangle D \sin i_s = (\omega M_e)^{1/3} \cos \alpha_s
\]  
(2.6)

\[
\langle \dot{v}_{\text{LOS}} \rangle D \sin i_s = \omega (\omega M_e)^{1/3}
\]  
(2.7)

Solving these equations gives the independent estimates \( D = 7.2 \text{Mpc} \) and \( D = 7.1 \text{Mpc} \).

2.3.1 Errors in the estimate

There is an error of \( \pm 0.2 \text{Mpc} \) on each of the distance estimates due to statistical concerns from the tracking of maser features. Since there are two estimates, the systematic error in disk model parameters is reduced, and so we get a combined estimate of \( D = 7.2 \pm 0.3 \text{Mpc} \). A final, major source for error is possible ellipticity of the disk, which was included neither in the warped disk model nor in the distance calculations. The symmetry of maser emissions helps to constrain this error, but it nevertheless introduces an uncertainty of \( \pm 0.4 \text{Mpc} \). This yields a final estimate of \( D = 7.2 \pm 0.5 \text{Mpc} \).

2.4 References

[13] Rowan-Robinson, M. \textit{The Cosmological Distance Ladder} (Freeman, New York, 1985)
Measuring the Hubble Constant through the Tully-Fisher Relationship

Edward Damon

ABSTRACT This paper offers a calibration of the Tully-Fisher (TF) relation by examining Cepheids in distant spiral galaxies. This distance scale is then used to evaluate the Hubble constant. Primary evaluation takes place in the I-band, with data supplied by Giovannelli et al. (1997). Comparison is then provided with an H-band survey by Aaronson et al. (1982), from which we derive alternate values of $H_0$, the Hubble constant. A weighted average is then taken from the data, which results in an $H_0$ of $71 \pm 4$ (random)$\pm 7$ (systematic).

3.1 Introduction

One of the most powerful tools for the determination of galactic distances is the Tully-Fisher relationship, which relates the maximum rotational velocity of a spiral galaxy to its luminosity. Even though the method by which this process functions is a matter of some debate, the Tully-Fisher relationship holds, especially in the infrared, where the relationship is morphology-independent. Therefore, it follows that, through use of the Tully-Fisher method, distances to galactic clusters could be accomplished with a high degree of accuracy. This could then allow the Hubble constant to be determined with unprecedented accuracy. However, for the required precision, a much clearer study of the Tully-Fisher relationship is required. This was exactly the goal of the Hubble Space Telescope Key Project, and, in this paper, we present our findings about the distances to galaxies and, consequently, the Cepheid variables contained therein.

3.2 Data

Table 3.1 contains the relevant information on the spiral galaxies used in the study. This included the fifteen galaxies viewed as a part of the Key Project, along with three galaxies which were found by other groups also using the HST, and, finally three galaxies that were analyzed using

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2Steinmetz & Navarro (1999); Mo, Mao, and White (1998) and Eisenstein & Loeb (1996)
ground-based telescopes. This table excludes those galaxies which were observed by the HST, but were unsuitable for use in the study, due to, primarily, the inclination angle of the galaxy $i$. For reasons detailed later, galaxies with too high or too low of an inclination angle could not be used.

In the table, all distances are taken directly from the references indicated. Error estimates, however, were altered to fit the error estimates from galaxy to galaxy, especially since, during the course of the project, we became much more familiar with the uncertainties in measuring Cepheid distances.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Type</th>
<th>m-M (Mag)</th>
<th>Reference</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 224</td>
<td>3</td>
<td>24.44 ± .10</td>
<td>Freedman 1990</td>
<td>C/F/H</td>
</tr>
<tr>
<td>NGC 598</td>
<td>6</td>
<td>24.64 ± .09</td>
<td>Freedman 1990</td>
<td>C/F/H</td>
</tr>
<tr>
<td>NGC 925</td>
<td>7</td>
<td>29.84 ± .08</td>
<td>Silbermann et al. 1996</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 1365</td>
<td>3</td>
<td>31.39 ± .20</td>
<td>Silbermann et al. 1999, Ferrarese et al. 2000b</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 1425</td>
<td>3</td>
<td>31.81 ± .06</td>
<td>Ferrarese et al. 2000b, Mould et al. 2000a</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 2090</td>
<td>5</td>
<td>30.45 ± .08</td>
<td>Phelps et al. 1998</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 2403</td>
<td>6</td>
<td>27.51 ± .24</td>
<td>Madore and Freedman 1991</td>
<td>C/F/H</td>
</tr>
<tr>
<td>NGC 2541</td>
<td>6</td>
<td>30.47 ± .08</td>
<td>Ferrarese et al. 1998</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3031</td>
<td>2</td>
<td>27.80 ± .08</td>
<td>Freedman et al. 1994</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3198</td>
<td>5</td>
<td>30.80 ± .06</td>
<td>Kelson et al. 1999</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3319</td>
<td>3</td>
<td>30.78 ± .12</td>
<td>Sakai et al. 1999</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3351</td>
<td>3</td>
<td>30.01 ± .08</td>
<td>Graham et al. 1997</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3368</td>
<td>2</td>
<td>30.20 ± .10</td>
<td>Tanvir et al. 1995, Gibson et al. 2000</td>
<td>HST</td>
</tr>
<tr>
<td>NGC 3621</td>
<td>7</td>
<td>29.13 ± .11</td>
<td>Rawson et al. 1997</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 3627</td>
<td>3</td>
<td>30.06 ± .17</td>
<td>Gibson et al. 2000, Saha et al. 1999</td>
<td>HST</td>
</tr>
<tr>
<td>NGC 4414</td>
<td>5</td>
<td>31.41 ± 1</td>
<td>Turner et al. 1998</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 4535</td>
<td>5</td>
<td>31.10 ± .07</td>
<td>Ferrarese et al. 2000b, Macri et al. 1999</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 4536</td>
<td>4</td>
<td>30.95 ± .08</td>
<td>Gibson et al. 2000, Saha et al. 1996</td>
<td>HST</td>
</tr>
<tr>
<td>NGC 4548</td>
<td>3</td>
<td>31.04 ± .23</td>
<td>Graham et al. 1999</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 4725</td>
<td>2</td>
<td>30.57 ± .08</td>
<td>Ferrarese et al. 2000b, Gibson et al. 2000</td>
<td>$H_0$KP</td>
</tr>
<tr>
<td>NGC 7331</td>
<td>1</td>
<td>30.89 ± .10</td>
<td>Hughes et al. 1998</td>
<td>$H_0$KP</td>
</tr>
</tbody>
</table>

Table 3.1. Galactic/Cepheid distance moduli. C/F/H indicates the Canada/France/Hawaii telescope, and $H_0$KP stands for the Hubble Key Project.

### 3.2.1 Photometry of Tully-Fisher Calibrators

This section focuses on the results and analysis of the photometry data obtained as a part of this project. To begin, all BVRI observations of Tully-Fisher calibrators were made using the Fred L. Whipple Observatory 1.2 m telescope and the Mount Stromlo and Siding Springs Observatory’s 1 m telescope. In the analysis, we used an exponential disk fit for the surface brightness profiles, which allowed us to extrapolate the total magnitude of the galaxy in question. This, of course, was similar to the method used in Han (1992), but the fit was done interactively, which is the method used by Giovanelli et al. (1997a). As for the IR data, all of it comes from the $H_{-0.5}$ data.
3. Measuring the Hubble Constant through the Tully-Fisher Relationship

(H-band data with various corrections that will be detailed later) data given by Aaronson, M. et al (1982), as it is the only other work currently available that has a consistent set of magnitudes between calibrators and cluster galaxies, apart from work done by Bernstein et al. (1994), which has an insufficient number of examined galaxies.

3.2.2 Corrections Applied to the Photometric Data

The corrections placed on the data came from three primary sources, Galactic extinction \( (A_{G,\lambda}) \), internal extinction \( (A_{Int,\lambda}) \), and k-correction. These values were estimated as follows: Galactic extinction was estimated through the reprocessing of IRAS ISSA and COBE DIRBE data on the 100 m\(^3\). The cosmological \( k \)-term in the I-band was adopted from Han (1992): \( k_1 = 0.16 \). However, internal extinction is still under debate, as the current morphological classification system creates large uncertainties. Therefore, we have opted to use a system which does not reference morphology; for our internal extinction calculations we will be using the corrections derived by Tully et al. (1998)(T98). These corrections operate as a function of major to minor axis ratio and line width, with no dependence on morphology, which can be found using \( A_{T98, BRIK}^{\lambda} = \gamma_{\lambda} \log a/b \), where \( \gamma_{\lambda} \) is a function of line width:

\[
\begin{align*}
\gamma_{\lambda} &= 1.57 + 2.75(\log W_{20} - 2.5), B \\
\gamma_{\lambda} &= 1.15 + 1.88(\log W_{20} - 2.5), R \\
\gamma_{\lambda} &= 0.92 + 1.63(\log W_{20} - 2.5), I \\
\gamma_{\lambda} &= 0.22 + 0.40(\log W_{20} - 2.5), K
\end{align*}
\]

In these equations, \( W_{20} \) is the 20 percent line width, whose significance will be explained in Section 3.2.4. The reason for this dependence on line width is that light needs to travel through more dust and gas on average through a large galaxy than a small galaxy, which, in turn, requires more correction for extinction. Strangely enough, however, an examination by Willick et al. (1996), showed no relationship between internal extinction and line width or internal extinction and luminosity. However, other methods, most notably, Han (1992) and Giovanelli et al. (1994) have severe disadvantages. The methods depend on the morphological classification of the galaxy, which produces large uncertainties in the final computation. Therefore, Tully et al. (1998) will be the method used for determining internal extinction. However, one more correction needs to be applied to the chosen method: it needs to be recalibrated for the H and V bands. This leads us to adopt the following ratios \( A_V = 1.5 A_I \) and \( A_H = 0.5 A_I \). These are both reasonable assumptions, arrived through interpolation of the Tully et al. (1998) paper. The final corrected apparent magnitude is expressed by

\[
m_{\lambda}^c = m_{\text{obs,}\lambda} - A_{G,\lambda} - A_{Int,\lambda} - k_{\lambda}.
\]

These might not be the optimum corrections. However, this is acceptable so long as they are the most consistent corrections.

\footnote{see Schlegel, Finkbeiner, and Davis (1998)}
3. Measuring the Hubble Constant through the Tully-Fisher Relationship

3.2.3 Inclination

The largest potential source of error in the TF relation comes from the inclination angle of the galaxy. To ensure accuracy we derived the inclination in each of the four bands, all of which were within 1 standard deviation of each other. These angles were determined from photometric analysis and will be used throughout the rest of the paper when calibrating the TF relationship.

3.2.4 Line Widths

Line widths of the calibrator galaxies have been measured outside of the HST Key Project. However, for the sake of consistency, we have re-measured the various line widths. Furthermore, it is important to define precisely what is meant by line width in order to avoid systematic errors. The most commonly used line width in TF applications is the 20 percent line width, $W_{20}$. $W_{20}$ represents the width in $kms^{-1}$ of the of the $H_I$ profile measured at 20 percent of the peak of the $H_I$ flux, where for a two-horned profile this is taken as the mean of each horn’s peak flux. Also used in TF calculations is the $W_{50}$ value, which is the width at 50 percent of each horn’s maximum. $W_{50}$ works best with low S/N profiles and with standard profiles. However, all the Key Project galaxies are close and have high S/N profiles; in addition, many are large and overfill the telescope’s beam. Finally, there are a number of non-standard profiles and so the use of $W_{50}$ would result in a systematic underestimation of the data and large uncertainties. Therefore, we used $W_{20}$ as much as possible.

3.2.5 Corrections Applied to the Line Widths

The raw line width data in our study is subject to the following corrections: first, line widths must be adjusted for redshift and inclination angle through

$$W^c = \frac{W}{\sin(i)(1 + z)}$$  \hspace{1cm} (3.6)

The inclination angle was derived through photometric analysis of the eccentricity measured via CDD frames with the following formula:

$$i = \cos^{-1} \left( \frac{(b/a)^2 - q_0^2}{1 - q_0^2} \right)$$  \hspace{1cm} (3.7)

This depends, of course, on $q_0$, which is the intrinsic minor to major axis ratio for spiral galaxies. We use a value of $q_0$ of 0.13 for all $T > 3$ and 0.20 for all other types. We do not correct for turbulence, as the basis for doing so remains subject to debate, except when applying internal extinction corrections. Moreover, since we are examining galaxies with a width of greater than 300 $kms^{-1}$ so any turbulence correction will be negligible to our calculation of $H_0$.

\[4\] Most notably, NGC 3031, NGC 3319, NGC 3368
3.3 Calibration of Multiwavelength Tully-Fisher Relations

This section deals with the derivation of TF calibrations. The method that will be used relies on utilizing the line width and inclination angles and then incorporating the cluster galaxy data. This process will yield the TF slopes, which can then be used in the calculation of the Hubble constant.

3.3.1 Tully-Fisher Calibration Using Calibrators and Spiral Galaxies

This method makes use of only the I-band data and is derived through the following procedure: To begin with, it must be noted that the sample, consisting of only local galaxies with Cepheid distances, was not large enough to constrain the slope with a high degree of accuracy. Furthermore, the sample, consisting as it does of nearby galaxies, is very vulnerable to error, as an individual error in distance will cause a large shift in the final dispersion. In order to minimize the first problem, we used an iterative determination of slope and zero point. The method used in the determination of the slope and zero point was as follows: (1) Calculation of slope and zero point using only the calibrator sample; (2) Using the results from step one, determine the distances to distant cluster galaxies in such a fashion that all lie on the same \( \log(W - 2.5) - M \) plane, where \( W \) is the line width, and \( M \) is the appropriate \( BVRI \) or \( H - 0.5 \), where \( BVRI \) are the total magnitudes from the photometric data and \( H - 0.5 \) is an aperture magnitude; (3) Determine the slope using this combined cluster sample; (4) Find the zero point through the minimization of dispersion in the local calibrating sample via the slope determined in step three; (5) Iterate steps 2-4 until the zero-point and slope converge. This gives us the following I-band TF relationship:

\[
I_T^c = -(10.00 \pm 0.08) \log(W_{50}^c - 2.5) - 21.32
\]  

This equation utilizes \( W_{50}^c \), as the Giovanelli et al. (1997) (G97) database, from which most of the I-band data was culled, uses the 50 percent line width. To correct for this, we merely utilized the data from an \( H \)-band survey by Aaronson et al. (AHM82), which gives the mean ratio for appropriate cluster galaxies, namely, \( W_{20} = (1.1 \pm 0.03)W_{50} \)

3.3.2 Uncertainties and Sources of Dispersion

To more accurately understand the error in our derived TF relationship, we generated a group of 5000 random points with a dispersion of 0.43 mag. We then selected a group of 500 random sub samples of N galaxies, and calculated zero point and slope. We used three different cases, \( N = 25, 500, \) and 2000. With \( N=25 \) we arrived at a slope distribution that was about 5 larger than the other two samples, which suggested that our own sample of 21 galaxies could result in something far different from the true value. Analysis using a number of galaxies that more closely resembled our own database still resulted in a slope distribution with a 1σ rms scatter of 0.09 mag. Then we ran the analysis again, this time examining the distribution of zero points. From the smallest sample, we arrived at a standard deviation of 0.13 mag, which gave us our final declared error bounds for slope and zero point of 0.09 and 0.13 respectively. The fundamental reason for the
dispersion of the TF relationship is three-fold. First, the relationship itself is subject to error as galaxies deviate from perfect correlations like Freeman’s exponential disk law or the mass to light ratio. Second, we cannot ignore observational errors in magnitude and line widths. Finally, for a cluster TF relation, the depth of the cluster or uncertainties in the distance estimates of galaxies can result in error. Thankfully, however, the error totals to a small value, confirming that the intrinsic dispersion of the TF relation is fairly small.

3.4 Application of the Tully-Fisher Relation to the Distant Cluster Samples and the Value of the Hubble Constant

3.4.1 I-band Survey

There have been many galactic surveys focusing on the TF relationship in various wavelengths. However, one of the most recent and complete surveys was the Giovanelli survey. Based on photometry and radio line widths, this survey looked at over 2000 spiral galaxies in clusters far enough away as to have recessional of over 10,000 $\text{km s}^{-1}$. Therefore, in this paper, we will focus on the data given by Giovanelli et al. (1997a), as it not only has all of the advantages previously described, but it also used a CCD for observation, rather than utilizing aperture magnitudes. In the previous section, we discussed the absolute calibration of $BVRIH_{0.5}$. Furthermore, the I-band relation was further recalibrated to the G97 database. This calibration allows us to survey distant clusters, out to 10,000 $\text{km s}^{-1}$ recessional velocity in order to determine $H_0$. We sorted the database with the following criteria in mind:

1. Galaxies should not deviate by more than twice the dispersion of the TF relationship.
2. Galaxies must not be close to face-on or edge-on ($40 \leq i \leq 80$ is desired), in order to reduce internal extinction and uncertainty in velocity measurements.
3. The distribution of line-width among the galaxies should be close to that of the calibrators (log $W$ cutoff at 2.35).
4. The internal extinction correction should not exceed 0.6 mag ($A_{\text{int,} \lambda} < 0.6$).

This gave us an I-band sample of 276 galaxies. Then we examined the clusters with five or more members and took the TF distances to them, using the equation derived in section 3.1. These results are given in Table 3.2. We then took clusters with $v_{\text{cmb}} \geq 20,000\text{km s}^{-1}$ whose velocities with respect to the CMB and flow field model were within 10 percent of each other. This left us with 15 clusters and groups of galaxies. Taking the average of these 15 clusters, we arrived at $H_0 = 73 \pm 2$ (random) $\pm 9$ (systematic) through our I band survey. Our results can be seen in Fig. 3.1.
Cluster | N | m - M | D | $V^{lg}/D$ | $V^{CMB}/D$ | $V^{model}/D$
--- | --- | --- | --- | --- | --- | ---
A1367... | 28 | 34.84 | 93.0 | 67.9 ± (11.0) | 72.2 ± (11.7) | 73.6 ± (11.9)
A2197... | 3 | 35.56 | 129.3 | 71.8 ± (11.4) | 70.5 ± (11.1) | 73.9 ± (11.7)
A262... | 22 | 34.18 | 68.7 | 75.3 ± (12.2) | 68.9 ± (11.1) | 74.2 ± (12.0)
A2634... | 20 | 35.36 | 118.0 | 80.6 ± (12.6) | 75.7 ± (11.8) | 77.5 ± (12.1)
A3574... | 14 | 34.03 | 64.1 | 7.2 ± (10.5) | 74.1 ± (11.6) | 72.0 ± (11.2)
A400... | 18 | 34.81 | 91.8 | 79.1 ± (12.3) | 76.5 ± (11.9) | 76.1 ± (11.8)
Antlia... | 16 | 33.30 | 45.6 | 54.9 ± (8.8) | 68.1 ± (10.9) | 61.8 ± (9.9)
Cancer... | 17 | 34.40 | 75.8 | 61.4 ± (9.8) | 65.8 ± (10.5) | 65.2 ± (10.4)
Cen 30... | 22 | 33.25 | 44.7 | 61.7 ± (10.2) | 73.2 ± (12.1) | 99.5 ± (16.4)
Cen 45... | 8 | 34.24 | 70.5 | 61.1 ± (10.0) | 68.4 ± (11.2) | 62.5 ± (10.3)
Coma... | 28 | 34.74 | 88.6 | 77.7 ± (12.2) | 80.6 ± (12.6) | 83.4 ± (13.0)
Eridanus... | 14 | 31.66 | 21.5 | 76.4 ± (12.4) | 74.8 ± (12.1) | 75.8 ± (12.3)
ESO 508... | 9 | 33.07 | 41.1 | 64.8 ± (10.3) | 76.7 ± (12.1) | 70.5 ± (11.2)
Fornax... | 14 | 30.93 | 15.3 | 86.6 ± (14.0) | 90.0 ± (14.5) | 89.5 ± (14.4)
Hydra... | 17 | 33.90 | 60.2 | 57.4 ± (8.9) | 67.4 ± (10.5) | 64.5 ± (10.0)
MD L59... | 10 | 32.56 | 32.5 | 80.7 ± (12.6) | 71.0 ± (11.1) | 82.1 ± (12.8)
N3557... | 9 | 33.02 | 40.2 | 67.2 ± (11.1) | 81.9 ± (13.5) | 73.5 ± (12.1)
N383... | 15 | 34.19 | 68.9 | 78.7 ± (12.3) | 71.5 ± (11.2) | 77.3 ± (12.1)
N507... | 7 | 33.86 | 59.1 | 90.6 ± (14.0) | 82.4 ± (12.8) | 89.0 ± (13.8)
Pavo... | 5 | 32.71 | 34.8 | 112.6 ± (17.4) | 115.7 ± (17.9) | 121.3 ± (18.7)
Pavo 2... | 13 | 33.62 | 52.9 | 81.3 ± (13.0) | 83.2 ± (13.4) | 87.9 ± (14.1)
Pegasus... | 14 | 33.73 | 55.8 | 73.6 ± (11.5) | 63.5 ± (9.9) | 69.4 ± (10.9)
Ursa Major... | 16 | 31.58 | 20.7 | 46.3 ± (7.1) | 52.6 ± (8.1) | 52.6 ± (8.1)

Table 3.2 (From Sakai et al. 2000)

FIGURE 3.1. (From Sakai et al. 2000)
3.4.2 H-band Survey

The $H_0$ value derived in Section 3.4.1 is fairly close to the accepted value for the Hubble constant. However, as a part of our investigation we also examined the H-band and preformed a similar survey. Our H-band data comes from two sources, AHM82 and Aaronson et al. (1986). These surveys featured measurements of rotational velocities at the 20 percent level, and an H-band isophotal aperture correction referred to before as $H_{-0.5}$. Furthermore, we will not include the 3 term additive term for inclination angles used by Aaronson et al. (1980), as we use the RC3 axis ratios. Cluster membership was difficult to determine in the H-band, as the survey was not as thorough in assigning cluster membership to various galaxies. For this reason, we split several clusters (most notably, A1367, Z74-23, Hercules, and Pegasus) into subgroups. The same selection criteria were used, which gave a sample of 163 Galaxies in 26 clusters. We then utilized the previously described methodology to evaluate TF relationship data, using the aforementioned correction to the I-band analysis. This resulted in $H_0 = 67 \pm 3$ (random) $\pm 10$ (systematic). This is significantly different from the value obtained by the I-band search.

3.5 Conclusion

In order to explain the discrepancies in the value of $H_0$, we examined the relationship between the two measurements' line widths, and, when that showed no room for large error, we examined the photometry and then color distribution.

3.5.1 Line Width

The I-band and H-band utilize different distance scales, and because the TF relationship is quite steep, it’s important to measure the widths accurately. However, by cross-referencing the galaxies featured in both the H-band and I-band surveys, along with a survey that includes many of the I-band galaxies in the H-band width scale, we confirmed that $H_0$ is insensitive to width scale. This means that photometric analysis is required.

3.5.2 Photometry

We tested the zero point of the I-band survey by comparing it to the Han and Mould (1992) survey. Of the fifty-nine galaxies that were present in both works, they agree to within about 0.01 mag of the mean. Therefore, the I-band photometry data cannot be responsible for the large discrepancies. Similar work was done on the H-band and revealed much the same information, leading to the conclusion that the data itself is not at fault for the difference between H-band and I-band However, in an interview with A. Mander et al. (1999, private communication), we found that the consistency in isophotal diameters, upon which we based our value of $H_{-0.5}$, is lacking. The diameter has been found to differ by 0.1 dex (Tormen and Burstien 1995). While this might apply to both calibrators and cluster galaxies, without further work we cannot guarantee that
there are no systematic differences of the same order.

3.5.3 Color Distribution of Calibrators and Cluster Galaxies

The aforementioned results suggest a systemic difference between calibrator and cluster samples. Analysis of the data shows that the calibrators are redder than the cluster sample galaxies, which is probably the reason for the disagreement between the two values. The reddening could be due to environmental effects, but the color distributions show no inconsistency. K-S tests lead us to conclude that the chance that the two color distributions are constant is quite low.

3.5.4 Reasons for Discrepancy

It seems clear that the most likely source of error in the calculation of the Hubble constant is systematic differences in the H-band due to isophotal diameters, with color distribution playing a supporting role. Unfortunately, these problems are not likely to be solved without a consistent set of H-band magnitudes, quite difficult to come by, as the sky background is more than 100 times brighter at H than at I. This background then needs to be carefully subtracted from the low surface brightness profiles in the outer parts of the galaxy. Until better photometric H-band databases become available, the systematic error we have found is likely to cause overestimation of distance, and will set limits on the accuracy of any \( H_0 \) measurement from a Tully-Fisher study.

3.6 References

3. Measuring the Hubble Constant through the Tully-Fisher Relationship

A Derivation of the Hubble Constant
Using the Fundamental Plane and $D_n - \sigma$ Relations

Kyle T. Story

ABSTRACT The distance to galaxies in the Leo I, Virgo, and Fornax clusters is calculated using the fundamental plane and $D_n - \sigma$ relations. The value of the Hubble constant can then be derived from these distances. These calculations are to be compared with other methods for measuring the Hubble constant by the HST Key Project in order to derive the most accurate value for $H_0$. Published period-luminosity (PL) relations of Cepheid variables are used to establish the relationship between angular size and metric distance for galaxies in close clusters. Using photometry and astrometry techniques, fundamental plane and $D_n - \sigma$ relationships are calibrated using local galaxies. This calibration is then extended to the Leo I, Virgo, and Fornax clusters to establish distances to galaxies in these clusters and derive a value of $H_0 = 78 \pm 5 \pm 9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the local expansion rate. Corrections are made for uncertain metallicities in Cepheid PL relations, for sampling bias that arises from observing the brightest stars in the galaxies, and for spatial coincidence of spiral and elliptical galaxies. Systematic errors resulted from uncertainty in distance to the Magellanic Cloud, uncertainties in the WFPC2 calibration, and several small uncertainties in the fundamental plane calculations. These add up to a total uncertainty of 11%.

4.1 Introduction

The purpose of the Hubble Space Telescope Key Project is to measure the distances to galaxies where it is expected that the Hubble flow dominates local velocity irregularities, and from these distances measure the value of the Hubble constant. We hope to reduce the systematic error in the Hubble constant to less than 10%. Due to the discrepancies in secondary distance indicators, several techniques will be employed and combined to establish the value of $H_0$, though only the fundamental plane and $D_n - \sigma$ techniques will be discussed thoroughly in this paper. We use data from the literature to calibrate the fundamental plane (FP) and $D_n - \sigma$ relationships in order to establish them as secondary distance indicators and use them as measuring sticks to derive distances to galaxies in the Leo I, Virgo, and Fornax clusters. Several Cepheid distances to galaxies in Leo I, Virgo, and Fornax have been published previously by the Key Project. These galaxies also have data in the literature concerning their internal kinematics and photometric properties that will be used in combination with the Cepheid distances for this calibration. The distances to these galaxies can then be used in combination with Doppler-shift spectrometry to derive local expansion rates, and calculate a value for the Hubble constant. These results are
4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations

compared to studies done using the Tully-Fischer relations for spiral galaxies (Tully & Fisher 1997; Aaronson, Mould, & Huchra 1979), $D_n - \sigma$ technique (Dressler et al. 1987, Lynden-Bell et al. 1987), surface brightness fluctuations (Tonry & Schneider 1988), and type Ia supernovae (Gibson et al. 2000). It is hoped that all these results can be reasonably combined to derive a value for the Hubble Constant with a systematic error of less than 10%.

4.2 Finding Distances with the Fundamental Plane and $D_n - \sigma$ Relations

The fundamental plane relation seeks to establish a relation between the radius, luminosity, and average kinetic energy of the stars in an elliptical galaxy. Elliptical galaxies can be described and characterized using relatively few parameters. By making use of the dynamical equilibrium and the virial theorems, which govern the balance of energy in many-particle systems, along with the fact that the velocities of the stars within stable elliptical galaxies are correlated to their binding energies, and with the assumption that mass and luminosity have a predictable relationship for all elliptical galaxies, a relationship between luminosity ($L$) and the velocity distribution of stars in a galaxy ($\sigma$) should be able to be established. This was made a reality by Faber & Jackson (1976), who discovered this relationship. This allows early-type galaxies to be used as measuring rods to more distant clusters by calibrating the FP relation using close early-type galaxies which have pre-measured, accurate Cepheid distances, and extrapolating the relationship to massive galaxies in distant clusters.

An improvement on this type of calibration was made with the introduction of surface brightness considerations (Dressler 1987). This is accomplished by defining $D_n$ as the diameter within which the surface brightness is some fixed value. For galaxies with similarly shaped growth curves, $D_n$ is related to the half-light diameter, $D_e$, and the average surface brightness within $D_e$, such that $D_n = f(D_e, <I>e)$. The half-light diameter is defined such that half of the total light from the galaxy comes from within half-light diameter. This leads to the $D_n - \sigma$ relation, which uses galaxy size and surface brightness in addition to the parameters used to define the Faber-Jackson relation, thereby reducing scattering of results about the mean by around 50%.

It was proposed that the $D_n - \sigma$ relation was merely an edge-on projection of the fundamental plane (Djorgovski & Davis 1987). However, by using galaxies from the Coma cluster, Jørgensen, Frans, & Kjaergaard (1993) showed that the $D_n - \sigma$ relation was close to but not exactly a near edge-on projection of the FP, and established the relation as follows:

$$r_e \propto \sigma^{1.24} <I>_e^{-0.82}.$$  \hspace{1cm} (4.1)

This relation had only 14% scatter in $r_e$ (which is analogous to distance) within the Coma cluster.

Early-type galaxies in dynamical equilibrium with fairly homogeneous stellar populations give a good approximation to the observed FP. In order to define a FP, we must make the assumption that early-type galaxies form a homologous group. This, in combination with the virial theorem, implies that the ratio between mass and luminosity is strongly correlated to the structural parameters of the galaxy such that

$$M/L \propto \sigma^{0.49} r_e^{0.22} \sim M^{1/4}.$$  \hspace{1cm} (4.2)
Therefore, in order to use FP and \(D_n-\sigma\) relations as distance indicators, two important assumptions must be made: 1) M/L ratios correspond to structural parameters in the same way everywhere, and 2) early-type galaxies have similar stellar populations for a given galaxy mass. Though these assumptions may not be valid, they have been supported by empirical data (Burstein, Faber, & Dressler 1990; Jørgensen et al. 1996).

In order for FP and \(D_n-\sigma\) relations to be used as secondary distance measurement indicators, we need the slope and scatter of the mean FP or \(D_n-\sigma\) relations to be consistent with local, calibrated samples. This is necessary for us to be able to use our calibrations to understand distant galaxies in a meaningful way. There are reasons that this relationship holds: 1) the FP is a combination of a systematic relationship between stellar population and galactic mass with correlated observational errors and selection biases, which will be discussed thoroughly in section five, 2) the stellar population to galaxy mass trend is the same in the Virgo and Coma clusters, and 3) residuals from the FP relation for cluster E/S0’s are uncorrelated with other stellar population indicators, suggesting that this is not a systematic error in that sense.

The FP relation and \(D_n-\sigma\) relation have been successfully used by several other groups in measuring distances to elliptical galaxies (e.g. Dresser et al. 1987; Wegner et al. 1996). Based on this, the Key Project has attempted to use the Cepheid distances to calibrate the fundamental plane and from that derive distances to galaxies, thereby allowing for the calculation of the value for \(H_0\).

### 4.3 Data Used in Analysis

The data used in the FP and \(D_n-\sigma\) analysis was acquired completely from published sources. Calculations rely on two types of data: photometry and spectroscopy. The photometry data must have sufficient signal-to-noise ratios (S/N) in order to derive the radii of galaxies (\(\theta_e\) in radians or \(r_e\) in metric units) and the mean surface brightness within that radius. It is absolutely imperative that the calibration be consistent for each set of galaxies that are placed on the FP since they are being directly compared to each other. It was found that the CCD photometry of Tonry et al. (1997) met these requirements. For the spectroscopy measurements, it was necessary to find a S/N ratio that was high enough to measure internal kinematics from absorption widths in order to determine \(\sigma\). Also, because of the nature of the calculations, a direct comparison between distant and very different sources is being made. This required measuring the calibrators very carefully using the same procedures that were used on distant samples.

#### 4.3.1 Velocity Dispersion

Data for the velocity dispersion (\(\sigma\)) calculations was taken from Dressler et al. (1987), who obtained spectroscopy measurements at Las Campanas Observatory (LCO). Twenty galaxies from Virgo and eight from Fornax were used. Data on four galaxies in the Leo I cluster was taken from Faber et al. (1989) and Fisher (1997). Because early-type galaxies have radial gradients in velocity dispersion, \(\sigma\), and radial velocity, \(v_r\), the measured value of \(\sigma\) depends on the aperture used when taking data. In early-type galaxies, the velocity dispersion is a mix of pressure and
rotational support. The measurement of $\sigma$ therefore depends on the distribution of orbits as well as of light. A suitable correction for this effect was given by Jørgensen et al. (1995a). The effect is corrected by creating simulated galaxies from the literature and "observing" them with varying apertures. Jørgensen et al. discovered a power-law function to relate $\sigma_{\text{corrected}}$ to $\sigma_{\text{observed}}$ as follows:

$$\log \sigma_{\text{cor}} = \log \sigma_{\text{obs}} + 0.04(\log D_{\text{cor}} - \log D_{\text{obs}}),$$

(4.3)

where $D_{\text{cor}}$ is the nominal aperture and $D_{\text{obs}}$ is the aperture of observation. This correction was applied to the Leo I, Virgo, and Fornax data.

### 4.3.2 Photometric Structural Parameters

Tonry et al. (1997) included circular aperture photometry on several galaxies in Virgo and Fornax in their study. Their images were adjusted to remove image defects such as saturated pixels and overlapping objects, thus achieving extremely high S/N ratios. Their data was fitted to parameterized growth curves and compared to the Jørgensen et al. (1996) fundamental plane. This comparison was accomplished using the photometric transformation of Jørgensen (1994) from V-band surface brightness to Gunn-r surface brightness. Errors in $\theta_e$ were between 9% and 30% for each galaxy. However, the error in the individual parameters is much larger than the resulting error in the FP. By experimenting with surface brightness profiles, it was concluded that systematic errors in the FP due to galaxy profile shapes are insignificant in the final result, and we expect the resulting systematic error in the FP to be less than 1%.

### 4.4 The Cepheid Distances to Leo I, Virgo, and Fornax

Cepheid stars are crucial in the calibration of secondary distance indicators. An important assumption is that the Cepheid distances to spiral galaxies in these clusters are appropriate for early-type galaxies also. Though we have no way of being sure, this is supported by the fact that there is little scatter among the Cepheid distances to the Virgo cluster. The published Cepheid distances that are used are given by Ferrarese et al. (1999). From these published results, we take the distance mean moduli in mag, $10.4 \pm 0.6 \pm 0.8$ Mpc, $16.1 \pm 0.3 \pm 1.2$ Mpc, and $20.9 \pm 1.4 \pm 1.6$ Mpc for Leo I, Virgo, and Fornax respectively. These distances are related to the LMC distance modulus, which was adopted as $\mu_{\text{LMC}} = 18.5 \pm 0.13$ mag. The systematic errors in these values arise from uncertainties in the distance to the LMC, uncertainties in the LMC PL relations, and uncertainties in the WFPC2 photometric calibration. A dependence on metallicity of Cepheid PL relations was measured by Kennicutt et al. (1998). The appropriate correction is $\mu_{\text{VC}} = -0.24 \pm 0.16$ mag dex$^{-1}$ in [O/H]. The correction for this effect will be discussed in section 8.
4.5 Calculation of the Hubble Constant from the Fundamental Plane

The FP was calculated for the three clusters Leo I, Virgo, and Fornax. The units of surface brightness were converted to \( L_0/\text{pc}^2 \), and \( r_e \) was calculated in Kpc from \( \theta_e \) using the Cepheid distances. The slope of the FP was fixed at the value found by Jørgensen et al. (1996). Using this, the zero point is defined as \( \gamma = \log r_e - 1.24 \log \sigma + 0.82 \log \langle I \rangle_e \). While Jørgensen et al. used the mean value to set their zero point due to the large sample size, we set the zero point using the median because our sample is small.

![Figure 4.1](image.png)

**FIGURE 4.1.** Figure 2a from Kelson et al. (1999)

Fig. 4.1 shows the fundamental plane from these three clusters, as well as Jørgensen’s data from the Coma cluster. Due to imaging problems or other errors, our final sample was reduced to four galaxies in Leo I, 14 in Virgo, and eight in Fornax. Ideally all galaxies should line up perfectly along the FP. Any scatter is due to: 1) errors in Cepheid distances; 2) the failure of the assumption that Cepheid distance applies to the early-type galaxies in the group or cluster; 3) errors in the FP parameters themselves; or 4) differences in the M/L ratio from galaxy to galaxy. We can see that the three cluster samples agree well, implying that the systematic errors are small.

The mean FP zero points, their formal errors and the external systematic and random errors are shown in Fig. 4.2. The zero points agree remarkably well, yielding a weighted mean of \( \langle \gamma \rangle = -0.173 \pm 0.013 \). We note that the scatter among the three zero points is consistent with the uncertainties in the individual zero points and the Cepheid distances, implying that variations in the M/L ratio do not contribute much to the scattering and are therefore not large among the three. Also in Fig. 4.1 the values for the Coma sample have been included as dots. It is apparent that the dots agree well with the fundamental plane of Leo I, Virgo, and Fornax, which validates our assumption that fundamental plane relations can be extrapolated to galaxies where Cepheids cannot be distinguished. This derived zero point is used to explicitly relate angular size, \( \theta_e \), to metric scale and find \( r_e \). Distance can be directly calculated from the zero point of the metric fundamental plane \( (\log r_e) \) and the angular fundamental plane \( (\log \theta_e) \). This can be
4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations

demonstrated in an example. In the Coma cluster, Jørgensen et al. used 81 galaxies to find a zero point for Coma of $\gamma_C = -5.129 \pm 0.009$. The distance to Coma can therefore be written as: $\log d = 5.129 \pm \langle \gamma \rangle - 3$ (the additional constant converts the units to Mpc), which gives a distance of $90 \pm 6$ Mpc. Using a recessional velocity for Coma of $7143$ km s$^{-1}$, the value of $H_0$ is calculated at $H_0 = 79 \pm 6$ km s$^{-1}$ Mpc$^{-1}$.

In order to reduce the effect of small-scale peculiarities in velocities and individual measurement errors, we include the measurements of 11 clusters compiled by Jørgensen et al. and take a weighted average value of $H_0$ over the entire sample. The information for all 11 galaxies is listed in Fig. 4.3. The weighted average is $H_0 = 82 \pm 5$ km s$^{-1}$ Mpc$^{-1}$. Because the value of $H_0$ decreases by only 2.5% when only the clusters with $\geq 20$ galaxies are used, it is concluded that the results do not depend greatly on the sample size of the distant clusters. Also, by observing that $H_0$ changes by 1% when only the clusters with recession velocities of $\geq 5000$ km s$^{-1}$ are used, we conclude that the peculiar velocities of galaxies do not contribute greatly to the uncertainties of $H_0$.

### Table 4 from Kelson et al. (1999)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$m - M^a$ (mag)</th>
<th>$\langle \gamma \rangle^b$</th>
<th>$\langle \delta \rangle^c$</th>
<th>Cepheid Random (dex)</th>
<th>Cepheid Systematic (dex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leo I</td>
<td>$30.08 \pm 0.11 \pm 0.16$</td>
<td>$-0.106 \pm 0.033$</td>
<td>$-2.241 \pm 0.033$</td>
<td>$\pm 0.022$</td>
<td>$\pm 0.032$</td>
</tr>
<tr>
<td>Virgo</td>
<td>$31.03 \pm 0.04 \pm 0.16$</td>
<td>$-0.182 \pm 0.012$</td>
<td>$-2.403 \pm 0.012$</td>
<td>$\pm 0.008$</td>
<td>$\pm 0.032$</td>
</tr>
<tr>
<td>Fornax</td>
<td>$31.60 \pm 0.14 \pm 0.16$</td>
<td>$-0.173 \pm 0.033$</td>
<td>$-2.388 \pm 0.033$</td>
<td>$\pm 0.028$</td>
<td>$\pm 0.032$</td>
</tr>
<tr>
<td>Weighted mean</td>
<td>$\ldots$</td>
<td>$-0.173 \pm 0.011$</td>
<td>$-2.395 \pm 0.011$</td>
<td>$\pm 0.007$</td>
<td>$\pm 0.032$</td>
</tr>
</tbody>
</table>

$^a$ Weighted mean distance modulus from Table 2 with random and systematic uncertainties. The errors in the Cepheid distance scale are detailed in Table 3

$^b \gamma = \log r_n - 1.24 \log \sigma + 0.82 \log \langle \gamma \rangle$, Internal random errors given.

$^c \delta = \log r_n - 1.24 \log \sigma$, Internal random errors given.

4.6 The $D_n - \sigma$ Relation

Though the $D_n - \sigma$ relation has been thought of as an edge-on projection of the fundamental plane, Jørgensen et al. (1993) argue convincingly to the contrary. However, it will still be useful to calculate the $D_n - \sigma$ relation and its zero points for comparison with the FP relation. This will help us to understand the systematic differences between the two methods. In Leo I, Virgo, and Fornax, a zero value of $\delta \equiv \log r_n - 1.24 \log \sigma$ is defined. As seen in Fig. 4.1b and 4.2, the zero points agree closely with each other. This gives a weighted mean of $\langle \delta \rangle = -2.395 \pm 0.013$. This calibration was compared with the 81 galaxies compiled by of Jørgensen et al. (1996). These have a mean zero point of $\delta_C = -47.373 \pm 0.009$, so the distance can be written as $\log \delta = 7.373 \pm \langle \delta \rangle - 3$. This gives a distance of $95 \pm 7$ Mpc, which in turn implies that $H_0 = 75 \pm 5$ km s$^{-1}$ Mpc$^{-1}$. Averaged over the 11 clusters, $H_0 = 79 \pm 6$ km s$^{-1}$ Mpc$^{-1}$, which is only 4% different from the average value calculated by the FP.

Based on the given equations, for the FP and $D_n - \sigma$ relation to be equivalent, $r_n = r_e <I>_e^{0.82}$. This was tested using 4 galaxies from Leo I, 14 from Virgo, 8 from Fornax, and 81 from Coma.
It was found that
\[
\log(r_n/r_e) = (0.73 \pm 0.01) \log \frac{<I>_e}{<I>_r} \tag{4.4}
\]
with a scatter of around 0.01 dex. This illustrates that the \(D_n - \sigma\) scaling relation is different from that of the fundamental plane. Thus the accuracy of \(D_n - \sigma\) as a distance indicator is related to the range of galaxy sizes used in the scaling relation. While both methods are related to the correlation of error in \(r_e\) and \(<I>_e\), error in measurements does not affect the FP very much because the underlying scaling relation is somewhat parallel to the error correction vector. This means that large errors in \(r_e\) do not necessarily translate into large errors in distance. Because of this, the FP is less likely to be biased by the error correlation and is therefore more reliable as a distance indicator.
4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations

4.7 The Scatter in the Fundamental Plane and $D_n - \sigma$ Relations

The Virgo FP and $D_n - \sigma$ relations have very low observed scatter, so that there is a $\pm 10\%$ uncertainty to any individual galaxy. This scatter is a combination of uncertain velocity dispersion, which is estimated at $\pm 9\%$, errors in the FP parameters, which are $\geq 5\%$, and an approximate rms scatter on the sky that adds an estimated $\pm 5\%$ scatter in distance. Given these various sources of scatter and the $\pm 10\%$ uncertainty in distance, there is very little room for internal scatter within Virgo. This implies a very homologous stellar population. It is observed that either Virgo has very similar conditions in its inner regions to those of Coma, or that the M/L ratio for a galaxy is not particularly dependent on environment. Fornax has slightly more scatter, equivalent to $21\%$ uncertainty in distance to individual galaxies, though there is no obvious reason why this is the case. Two possible explanations are 1) that the cluster is quite elongated along the line of sight, or 2) that the galaxies in Fornax have slightly more internal scattering in their stellar populations. Fortunately, these variations have limited effect on the value of the Hubble constant because they are weighed out by Virgo and Leo I.

4.8 Systematic Corrections to the Hubble Constant

Three potential sources of systematic error will now be discussed. These include the metallicity correction to the Cepheid distances, galaxy selection bias in the clusters, and the spatial coincidence of the spiral and elliptical galaxies. From Kennicutt et al. (1998), it is known that Cepheid distances have a small but relevant dependence on the $[O/H]$ abundance in the stars. This corresponds to an error in two-color VIC distance determinations of $\Delta \mu_{VIC} = -0.24 \pm 0.16$ mag dex$^{-1}$, such that metal-rich Cepheids appear closer and brighter than metal-poor ones. This result is insignificant in the value of $H_0$, and is therefore not included in its derivation. However, it is a source of systematic error. The corrections increase the adopted distances to Cepheids in all cases, decreasing $H_0$ by $6\% \pm 4\%$.

Another source of potential error is the cluster population incompleteness bias. This sampling bias occurs when the bright objects are selected over the dim ones because they are more easily observed. To test the effects of this bias, a Monte Carlo experiment was completed using the Coma sample. First, a random sub-set of galaxies was selected with the criterion in $r_e$ such that the sample had the same size and depth in each of the 11 clusters. 500 iterations were performed, and the systematic bias was found to be less than $1\%$.

Another test of this effect was completed by carrying out calculations on only the galaxies for which $\sigma < 200$ km s$^{-1}$. For the entire Coma sample, $H_0 = 82 \pm 6$ km s$^{-1}$. For the sub-set, $H_0 = 81 \pm 6$ km s$^{-1}$. When the absolute residuals were minimized in finding the zero points of the FP, $H_0 = 81 \pm 6$ km s$^{-1}$, which is less than a $1\%$ deviation and is consistent with the Monte Carlo simulations. We therefore conclude that this effect is not statistically significant in the calculation of $H_0$. Using the information from Coma, the introduced uncertainties are at most $\pm 2\%$.

Lastly, a correction is made for the uncertainties due to the spatial coincidence of spiral and elliptical galaxies. This source of error is introduced when galaxies are systematically selected from the near side of the cluster because they are the ones that can be most easily seen. To
test the extent of this bias, simulations of the same type done by Gonzalez & Faber (1997) were carried out using 11 Cepheid distances to three clusters that had distance errors of between ±2% and ±7%. From these simulations, we make a 5% downward correction to the value of $H_0$. The assumed uncertainties in this correction, which are due to uncertainties in the line-of-sight structure of Virgo and Fornax, are also less than ±5%.

4.9 Systematic Errors and the Final Value of $H_0$

We will now analyze the various sources of systematic uncertainty in order to determine a total error for the derivation of the Hubble constant. It is the goal of the Key Project to keep this total error under 10%. The various errors are shown in detail in Table 3 in Kelson et al. (1999). Uncertainties in the Cepheid distances account for the bulk of the total error. Uncertainties in their measurements correspond directly to error in the Hubble measurement. This error is estimated at ±7%. It is assumed to be uncorrelated from galaxy to galaxy, so it decreases as $N^{-1/2}$ per cluster for $N$ galaxies in a given cluster.

4.9.1 Velocity Dispersion

The largest source of systematic error in the FP component of the analysis comes from errors in comparing the velocity dispersion of calibrated galaxies to those of distant ones. Fortunately, the exponent on $D_n - \sigma$ is small (close to 1), so that large errors in $\sigma$ do not necessarily translate to large errors in $H_0$. Using data from Dressler et al. (1987), we find that the rms scatter causes an uncertainty in FP zero points of ±2%. Averaging over the galaxies in Leo I from Fisher (1997) under similar analysis, we expect error due to velocity dispersion to be $\approx \pm 3-4\%$. Small aperture corrections are also necessary, because small uncertainties can still affect the value of $H_0$ noticeably. For Virgo and Fornax, the uncertainties in the correction that was applied are less than 1%. In the two Fisher (1997) galaxies, the applied correction was $\approx 9\%$ with uncertainty of ±6% per galaxy. Dividing by $N^{1/2}$ ($2^{1/2}$) gives a net uncertainty of ±4%. These results are then added to the Faber et al. (1989) galaxies, yielding a net uncertainty of ±3%. When all of the galaxies are put together and a weighted uncertainty is calculated, the uncertainties in $H_0$ due to aperture corrections are small, about ±2\%.

4.9.2 Structural Parameters

Despite the possibility of severe measurement errors in the FP parameters, the combination of $\theta_e$ and $<I>_e$ in the FP is very stable, in that the coefficient between log $\theta_e$ and log $<I>_e$ is -0.73. The precision of information depends on the quality of the photometry/imaging data. The errors are typically around a few percent.
4.9.3 Photometry

Several sources of uncertainty in the derivation of the Gunn r surface brightness show up and are discussed below, including the calibration technique of Tonry et al. (1997), the transformation to Gunn r, and errors in galaxy colors. Tonry et al. (1997) used the same reference system of Landolt (1992) as Jørgensen (1994). The systematic errors were explicitly calculated by Tonry et al. and are very small. According to Tonry, uncertainties in the V photometric zero point are ±2%, and errors in (V-I_C) are ±2%. These will probably propagate into the value of H_0. Taking the value reported by Jørgensen (1994) of the rms scatter in photometric transformation to Gunn r of ≈ ± 0.02 mags and reversing the transformation, we find that the net effect of these errors on the Hubble constant is less than 2%.

To compare consistency of colors, we use a subset of 28 Coma galaxies for which we have both Gunn r and Johnson B photometry. From this we find that the mean V-r colors of these early-type Coma galaxies is ⟨V−r⟩ = 0.198 ± 0.002 mag. The mean color values for Leo I, Virgo, and Fornax are computed using the equation

\[ (V−r) = −0.273 ± 0.396(V−I_C). \]  

The uncertainties of the colors themselves are therefore less than ±0.02 mag.

4.9.4 Systematic Uncertainties due to the Velocity Field

First we note that the same values for H_0 are derived whether we use the CMB reference frame or the flow-field model of Mould et al. (2000). In order to assess the uncertainties of peculiar velocities of clusters on top of the smooth Hubble flow, we now look at only the clusters with more than 20 galaxies, which have a standard deviation of 4% in their implied Hubble constants. This is due to two effects: 1) random uncertainties in the zero points of clusters due to a finite number of galaxies in the clusters, and 2) the random motion of clusters on top of the smooth Hubble flow. We find that there is a 4% uncertainty per cluster, which decreases as a factor of N^{-1/2}, where N is the number of galaxies in the cluster. This gives a final error of about 1% in H_0.

4.9.5 Uncertain Slope of the Fundamental Plane and D_n − σ Relations

The FP relation is in essence an association between M/L ratios and galaxy masses. For the predicted FP to be correct, the M/L scaling must be constant between clusters. To see what the effect of a non-constant scaling would have, we re-calculate the FP using the ±1 σ range of Jørgensen et al. (1996). Taking their Coma data, and implementing a least squares fit to those galaxies with log σ > 2, we find the D_n − σ relation

\[ \log \theta_n \propto (1.22 ± 0.08) \log \sigma. \]

This corresponds to a change in H_0 of -2.5%. By averaging over Leo I, Virgo, and Fornax, the uncertainties in H_0 are diminished, primarily due to the small scatter in Virgo. We therefore conclude that the contributed uncertainties are small in H_0.
4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations

4.9.6 Total Error in $H_0$

From all these sources of systematic error, shown in Table 3 of Kelson et al. (1999), it is found that the goal of a systematic error of less than $\pm 10\%$ cannot be achieved. The largest sources of error currently are: 1) LMC distance, 2) the photometric calibration of WFPC2, 3) the Cepheid metallicity correction, and 4) the depth effects between spiral and elliptical galaxies. The uncertainties in Cepheid distances, which, including uncertainties in the LMC distance, are a total of $\pm 7\%$, do not contribute greatly to the uncertainty in $H_0$. Most of this error comes from a compilation of small errors in the FP, which contribute $\pm 6\%$ error. When this is combined with other systematic errors in the FP analysis, a total error of $\pm 11\%$ is calculated for $H_0$. However, when combined with the other distance indicators that are a part of the Key project as a whole, the goal of $\pm 10\%$ may still be achieved.

4.9.7 Final Value for the Hubble Constant

From our raw derived value of $H_0 = 82 \pm 5 \pm 10$ km s$^{-1}$ Mpc$^{-1}$, we subtract off corrections for the assumption that Key Project spirals can be used to set distances and for the metallicity correction of Kennicutt et al. (1998), we adopt a final value of $H_0 = 73 \pm 4 \pm 9$ km s$^{-1}$ Mpc$^{-1}$.

4.10 Comparison with the Literature

This value of the Hubble Constant can be compared to that of four groups. Our results for Leo I can be directly compared to the results of Hjorth and Tanvir (1997). Their distance was found to be $5\%$ larger. This must have arisen from systematic differences in the aperture-corrected velocity dispersions. It is possible that our correction is more uncertain for apertures as small as those used by Fisher (1997). It is also possible that the uncertainties assumed by the authors were too small. Gregg (1995) calculated distances using the K-band $D_n - \sigma$ relation to the Coma and Virgo clusters that are well within our error limits. Faber et al. (1989) studied Leo I, Virgo, and Fornax also. They only reported two galaxies for Leo I for which their distance calculation falls outside our error boundaries. Because of the small sample size, it is difficult to find systematic differences. Their Virgo distances are offset by only $-3\%$, which is statistically insignificant and agrees well. Fornax, on the other hand, has an offset of $-11\% \pm 4\%$. Uncertainty estimates of the photometry of Faber et al. are not available, so it is difficult to analyze systematic errors. Based on the analysis of D’Onofrio et al. (1997), we conclude that the difference between our data and that of Faber et al. may point to larger uncertainties than anticipated in the Faber et al. (1989) data set.
4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations

4.11 Conclusion

By analyzing FP relations for Leo I, Virgo, Fornax, and incorporating the 11 clusters from Jørgensen et al. (1996), a raw value of $H_0 = 82\pm 5\pm 10$ km s$^{-1}$ Mpc$^{-1}$ is calculated. This is reduced by a 5%±5% correction for depth effects in the nearby cluster, and 6%±4% for metallicity corrections in Cepheid distances. The final adopted value of the Hubble constant is $H_0 = 73\pm 4\pm 9$ km s$^{-1}$ Mpc$^{-1}$. Our cluster sample spans a wide range of recession velocities, from 1100 km s$^{-1}$ to 11,000 km s$^{-1}$ with a mean of $cz \simeq 6000$ km s$^{-1}$. The value of $H_0$ is found to be the same relative to the CMB or the flow-field model of Mould et al. (2000). $H_0$ has a value that was smaller by 4% from the $D_n - \sigma$ relation. However, this relation is a curved representation of the FP relation, and therefore the FP relation is to be preferred. While our uncertainties in $H_0$ are greater than 10%, these results will be combined with the results from Mould et al. (2000) and W.L. Freedman et al., in preparation, with results from the Type Ia supernovae (Gibson et al. 2000), the Tully Fisher relation (Sakai et al. 2000), and surface brightness fluctuation method (Ferrarese et al. 2000) to produce a more accurate value of the local expansion rate of the Universe.

4.12 References

4. A Derivation of the Hubble Constant Using the Fundamental Plane and $D_n - \sigma$ Relations


A Recalibration of Cepheid Distances to Type Ia Supernovae

Daniel H. Rasolt

ABSTRACT The standard Hubble Space Telescope (HST) Key Project on the extragalactic distance scale has derived Cepheid-based distances to seven type Ia supernovae (SNe) host galaxies. The significant brightness and precision in distance measurements of type Ia SNe make them tools for Hubble constant and cosmological expansion measurements. Type Ia SNe are easier to detect with current instruments due to their brightness and are considered to have a precision in reference to distances of approximately 8%. Data from the Key Project, the Sandage et al. type Ia SNe program, and the Tanvir et al. (1995) Leo I group study, were compared. The Sandage et al. (1996) galaxies and the Tanvir et al. (1995) cluster give a mean offset in true distance moduli of $0.12 \pm 0.07$ mag. By analyzing the red-shift from Hubble relations from a zero point based on SNe 1990N, 1981B, 1989B, 1972E, and 1960F, we derive a Hubble constant of $H_0 = 68 \pm 2(\text{random}) \pm 5(\text{systematic}) \text{km s}^{-1} \text{Mpc}^{-1}$. This result is consistent with the findings based on the Tully-Fisher relation and surface brightness fluctuations.

5.1 Introduction

With an 8% precision in distance measurements, and the ability to see SNe Ia’s very far away due to their tremendous brightness, type Ia SNe have become one of the most precise methods for measuring relative distances, both in the nearby and the distant universe. Type Ia SNe, which are thought to be thermonuclear explosions of white dwarfs, are found at redshifts of $z = 0.01 - 0.1$, with the Calan-Tololo Survey of 29 type Ia SNe helped demonstrate.

Type Ia SNe form when a degenerate white dwarf has explosive nuclear burning occurring within its core with iron-group elements (Ni, Co, Fe). This explosive nature is a result of unstable degenerate material under high gravitational compression, whose electrons are nearly relativistic. This process occurs when the degenerate dwarf is at or near the Chandrasekhar mass (1.4 solar masses). Theoretical calculations of type Ia SNe explosions give predictions very close to the peak luminosities of those observed.

Type Ia SNe are no longer considered standard candles of constant luminosity, with definitive evidence of dispersion of their peak luminosities recently uncovered (e.g. Phillips et al. 1999).

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The same groups that made this discovery also uncovered a relationship between the SNe light curve and the peak luminosity. The observed differences in peak luminosities of type Ia SNe are very closely correlated with observed differences in the shapes of their light curves: dimmer type Ia SNe decline more rapidly after maximum brightness, while brighter type Ia SNe decline more slowly. This relationship corresponds to a decline rate, which can be used to make corrections due to dispersion effects on a Hubble diagram (Hubble diagrams measure the distance modulus vs. the red-shift). Hubble diagrams are very useful in finding relative distances to objects, and the correlation between peak luminosity and decline-rate allow a more accurate relative distance to be calculated.

Hubble diagrams help to find relative distances, but in order to calculate $H_0$, a zero point must be provided. The HST has the potential to provide this zero point by way of nearby type Ia SNe host galaxies. The HST Key Project on the Extragalactic Distance Scale has found distances to 18 galaxies, including one type Ia SNe host galaxy, and aims to use these galaxies to move deeper into the Hubble flow, with calibrating type Ia SNe's being one major part.

What this paper will focus on is the recalibration of Cepheid data in galaxies host to type Ia SNe that were observed by the HST, both from the Key Project, and those observed that were not part of the Key Project. The seven galaxies previously specified are not part of the Key Project, but will be analyzed in the same manner that the 18 Key Project galaxies were, with the main goal being to calculate $H_0$. Previous results will be introduced, followed by data specific to each of the seven Type Ia host galaxies and a determination and description of the Hubble constant. The general potential of type Ia SNe measurements in determining cosmological parameters will also be presented. Unless otherwise noted, all information in this paper is based on Gibson et al. (2000).

### 5.2 Past Results of Type Ia SNe Measurements

Type Ia SNe have been cosmological distance indicators since the early 1980's, but were believed to be standard candles with similar light curve shapes, spectral time series and absolute magnitudes (Perlmutter et. al 2003). It was not until the Calan-Tellolo survey that it was determined that the appearance of type Ia SNe is not predictable. Phillips et al. (1999) discovered a correlation between type Ia SNe absolute magnitude and the rate at which its luminosity declined. Phillips plotted the absolute magnitudes of nearby type Ia SNe versus $\Delta m_{15}(B)$, a parameter that analyzes the decrease in brightness in the B-band over 15 days of the SNe. This plot is shown below in Fig. 5.3. Hamuy et al. (1996) used this correlation to find a distance modulus $\sigma < 0.2$ mag in the V-band for a sample of 29 Type Ia SNe from the Calan-Tellolo survey (Perlmutter et al. 2003).

A variety of new methods and surveys have been used to calculate $H_0$, with seemingly more accurate measurements with each successive method. In 1997, Saha et al. calculated $H_0$ to be $58 \pm 3$ km s$^{-1}$ Mpc$^{-1}$ based on Calan-Tololo measurements and ignoring external systematic effects. In 1996 Humay et al. and Riess et al. found $H_0 = 56 \pm 2$ km s$^{-1}$ Mpc$^{-1}$, which agrees with Saha et al. within the error. Adjustments have been made in many experiments, some large, some very subtle, with $H_0$ measurements rising towards our measurement of $68 \pm 2$(random) $\pm 5$(systematic) km s$^{-1}$ Mpc$^{-1}$. Fig. 5.2 shows previous determinations of $H_0$ from type Ia SNe.
5. A Recalibration of Cepheid Distances to Type Ia Supernovae

The empirically calculated $H_0$ use methods similar to ours, while the physical determinations are based on purely theoretical models of type Ia SNe explosions, as opposed to observations. It is encouraging that these physical models agree so closely with recent empirical measurements and calculations. Included also in the table are the number of SNe included in the adopted Hubble diagrams, as well as the number of local calibrators employed in deriving the associated zero points. The associated mean peak B-band magnitude for each sample is also provided, as well as the number of SNe calibrated.

5.3 The Data

Of the seven type Ia SNe host galaxies analyzed, six were from the Sandage & Saha team (NGC 4496A, 4536, 4639, 5253, 3627 and IC 4182), and one from Tanvir et al. (1995; NGC 3368). Only Cepheids with high quality V and I-band photometry were considered, making these Cepheid numbers smaller than some previous projects, such as Saha et al. (1997). High-quality Cepheids are recorded and plotted from each galaxy based on their period-luminosity relationships in the V and I-bands (Fig. 5.5-5.11). The top plots of each figure are from V-band observations and the bottom plots represent I-band observations. The Key Project’s ALLFRAME (photometry; Stetson 1994) and TRIAL (variable finding; Stetson 1996) help process the data in the vertical (luminosity) and horizontal (period) axis respectively. Based on these graphs, distance moduli can be calculated for each galaxy, and these results are shown in Fig. 5.3 for the seven select galaxies.
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5.4 Determination of $H_0$

From the distance moduli listed in Fig. 5.3, we are able to calculate $H_0$ for B, V and I-bands, which is listed in Fig. 5.4. The three relations used are:

$$\log H_0(B) = 0.2M_{B}^{max} - 0.720(\pm 0.459)$$
$$\times [\Delta m_{15}(B)_t - 1.1] - 1.010(\pm 0.934)$$
$$\times [\Delta m_{15}(B)_t - 1.1]^2 + 26.685(\pm 0.042)$$

(5.1)

$$\log H_0(V) = 0.2M_{I}^{max} - 0.672(\pm 0.396)$$
$$\times [\Delta m_{15}(B)_t - 1.1] - 0.633(\pm 0.742)$$
$$\times [\Delta m_{15}(B)_t - 1.1]^2 + 28.590(\pm 0.037)$$

(5.2)

$$\log H_0(I) = 0.2M_{V}^{max} - 0.853(\pm 0.214)$$
$$\times [\Delta m_{15}(B)_t - 1.1] + 28.219(\pm 0.034),$$

(5.3)

where

$$\Delta m_{15}(B)_t = \Delta m_{15}(B)_{obs} + 0.1E(B-V)_t.$$  

(5.4)

$\Delta m_{15}(B)$ is a parameter that analyzes the decrease in brightness in the B-band over 15 days for the SNe. $M_{BVI}^{max}$ refer to the associated mean peak B, V, and I-band magnitudes. $E(B-V)$
5. A Recalibration of Cepheid Distances to Type Ia Supernovae

5.5 Conclusion

Type Ia SNe are at present the best and most reliable tool for calculating extragalactic distances and the Hubble constant. This paper has focused on seven specific galaxies that host these type Ia SNe, as well as given an overview of the potentials and previous achievements in observing type Ia SNe. The final result was a Hubble constant of $H_0 = \pm 2$ (random) ±5 (systematic) km s$^{-1}$ Mpc$^{-1}$, which is consistent with previous studies of this topic.
5.6 References


FIGURE 5.5. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 17 high-quality NGC 4639 Cepheid candidates found by TRIAL. The solid lines are least-squares fits to this entire sample, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2σ dispersion. The inferred apparent distance moduli are then $V = 31.92 \pm 0.08$ mag (internal) and $I = 31.87 \pm 0.06$ mag (internal). From Gibson et al. (2000).

FIGURE 5.6. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 39 high-quality NGC 4536 Cepheid candidates found by TRIAL. The solid lines are least squares fits to the 27 $P > 20$ day candidates, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2σ dispersion. The inferred apparent distance moduli are then $V = 31.20 \pm 0.06$ mag (internal) and $I = 31.10 \pm 0.04$ mag (internal). From Gibson et al. (2000).
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FIGURE 5.7. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 36 high-quality NGC 3627 Cepheid candidates found by TRIAL. The solid lines are least-squares fits to the 17 \( P > 25 \) day candidates, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2\( \sigma \) dispersion. The inferred apparent distance moduli are then \( V = 30.40 \pm 0.08 \) mag (internal) and \( I = 30.26 \pm 0.07 \) mag (internal). From Gibson et al. (2000).

FIGURE 5.8. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 11 high-quality NGC 3368 Cepheid candidates found by TRIAL. The solid lines are least-squares fits to the seven \( P > 20 \) day candidates, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2\( \sigma \) dispersion. The inferred apparent distance moduli are then \( V = 30.55 \pm 0.10 \) mag (internal) and \( I = 30.41 \pm 0.08 \) mag (internal). From Gibson et al. (2000).
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FIGURE 5.9. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the seven high-quality NGC 5253 Cepheid candidates found by TRIAL. The solid lines are least-squares fits to this entire sample, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2σ dispersion. The inferred apparent distance moduli are then $V = 27.95 \pm 0.10$ mag (internal) and $I = 27.82 \pm 0.08$ mag (internal). From Gibson et al. (2000).

FIGURE 5.10. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 28 high-quality IC 4182 Cepheid candidates found by TRIAL, in common with Saha et al. (1994). The solid lines are least-squares fits to this entire sample, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2σ dispersion. (internal). From Gibson et al. (2000).
FIGURE 5.11. Period-luminosity relations in the V (top) and I (bottom) bands, based on the Stetson (1998) calibrated ALLFRAME photometry. The filled circles represent the 94 high-quality NGC 4496A Cepheid candidates found by TRIAL. The solid lines are least-squares fits to the 51 $P > 25$ day candidates, with the slope fixed to be that of the Madore & Freedman (1991) LMC PL relations, while the dotted lines represent their corresponding 2σ dispersion. The inferred apparent distance moduli are then $V = 31.12 \pm 0.05$ mag (internal) and $I = 31.08 \pm 0.04$ mag (internal). From Gibson et al. (2000).
Determining the Hubble Constant with the Sunyaev-Zeldovich Effect

David Eisler

ABSTRACT A general review of the theoretical and observational methods for using the Sunyaev-Zeldovich Effect to determine the Hubble constant is presented. An overview of the effect and how it can be used to determine \( H_0 \) is given as well as a more quantitative approach to finding the distance to a galaxy cluster from observationally determined quantities. Results from recent surveys are also presented for different cosmologies, depending on the study that was conducted. Lastly, future prospects for increased accuracy and improvements in the overall method and technique are discussed.

6.1 Introduction

The Sunyaev-Zeldovich Effect (SZE) was first discovered by analyzing the effects of the interactions between hot electrons in the dense intracluster medium (ICM) with photons from the cosmic microwave background (CMB) (Sunyaev & Zeldovich 1969, 1972). Galaxy clusters contain hot (\( \approx 6 \times 10^7 \) K) gas trapped in their potential wells, much hotter than the CMB radiation (\( \approx 3 \) K). On average approximately 1% of the CMB photons passing through a dense cluster of galaxies are inverse Compton scattered from low to high frequency, boosting the energy of a scattered photon and causing a small (\( \approx 1 \) mK) distortion in the CMB temperature, altering its spectrum. A quantitative description of this effect can be found in the original Sunyaev & Zeldovich papers (1969, 1972).

The change in intensity of the CMB radiation is redshift independent, making it possible to obtain direct distance measurements to high redshift (\( z \approx 1 \)) clusters, as opposed to the standard "distance ladder". With calculated distances to many clusters, a measurement of the Hubble constant (\( H_0 \)) can be obtained by comparing the X-ray emission data of a cluster with the thermal effect. This paper will outline the general technique for determining \( H_0 \) from these observations and present some recent results obtained from various clusters. The final section will discuss practical limitations of the SZ method and future possibilities for more accurate determinations of \( H_0 \).

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6.2 Theory

As photons from the CMB pass through the hot ICM in a galaxy cluster, some of them interact with high energy electrons and are scattered in any number of directions. When this happens, energy from the hot IC gas is transferred to the CMB radiation and a fraction of the photons are shifted from the Rayleigh-Jeans (low frequencies) side of the Planck spectrum to the Wien side (high frequencies). This process is known as Comptonization. Because of the homogeneity and general isotropy of the CMB, the photons passing through the cluster gas gain energy, consequently distorting the CMB spectrum. Because the CMB spectrum is observed as a near perfect blackbody, variations in it can be measured, though no current detectors can sense the small scale SZE. Fig. 6.1 relates the original CMB spectrum (with temperature $T = 2.73 \, K$, solid line) to the distorted spectrum (dashed line):

\[ \text{FIGURE 6.1. A graph of the change in the CMB spectrum due to the Compton interaction between the photons and the electrons. Note the frequency dependence of the distortion. For low frequencies the temperature decreases but for high frequencies it increases. Courtesy of http://www.mpifr-bonn.mpg.de/staff/mthierbach/sz.html} \]

Sunyaev & Zeldovich interpreted this spectral distortion as a relative change in the CMB temperature. The original quantitative description of the SZE was based on a solution to the Kompaneets equation, a nonrelativistic diffusion approximation to the exact kinetic equation for scattering (Rephaeli 1995). This equation took the form

\[
\frac{\partial N}{\partial t} = \frac{k T_e}{mc} \sigma_T n_e \frac{\partial}{\partial x} \left( x^4 \frac{\partial N}{\partial x} \right)
\]

where $n_e$ and $T_e$ are the electron number density and temperature, $N$ is the photon occupation number, $\sigma_T$ the Thomson cross section, and $x$ is defined as $h\nu/kT$ with $T$ being the temperature of the radiation.
A simple expression for the change of CMB intensity induced by scattering the CMB photons can be obtained from this nonrelativistic calculation, yielding a thermal velocity distribution of

$$\Delta I_t = i_0 y g_0(x)$$  \hspace{1cm} (6.2)

where $i_0 = 2(kT)^3/(\hbar c)^2$, and $y$ is the Comptonization parameter defined as

$$y = \int dl(kT_e/\hbar c)^2 n_e \sigma_T$$  \hspace{1cm} (6.3)

integrating over $n_e$ and $T_e$ along the line of sight through the cluster, having assumed that the cluster is spherically symmetric.

The final factor in equation 6.2 is $g_0(x)$, which is the spectral form of the thermal SZE, expressed as a function of the frequency and CMB temperature:

$$g_0(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left[ \frac{x(e^x + 1)}{e^x - 1} - 4 \right].$$  \hspace{1cm} (6.4)

With the above information, it is now possible to write an expression for the magnitude of the temperature change in the CMB due to the thermal effect:

$$\Delta T_t / T = -2y = -2 \int dl(kT_e/\hbar c)^2 n_e \sigma_T$$  \hspace{1cm} (6.5)

In addition to the thermal effect, there is also a kinematic (Doppler) effect present because of the motion of the cluster. The kinematic SZ component is given by

$$\Delta I_k = -i_0 h_0(x) \beta_c \tau,$$

where $\beta_c = v_r/c$, $v_r$ is the line of sight component of the cluster peculiar velocity and $\tau$ is the Thomson optical depth of the cluster. There is an associated temperature change of this component given by $\Delta T_k / T = -\beta_c \tau$ (Sunyaev & Zeldovich 1980).

There is one relevant detail here that can affect the calculations. The energy of the IC gas spans a range of $3-15\text{ keV}$, and so the velocities of the IC electrons are near relativistic. As a consequence, the nonrelativistic Kompaneets-based solution is only accurate at low frequencies on the far Rayleigh-Jeans side of the spectrum. Fig. 6.2 shows the distortion of the spectral intensity versus the frequency $\nu$.

To deal with this discrepancy, a relativistic correction was derived for the total intensity change $\Delta I = \Delta I_t + \Delta I_k$ using a power series expansion for the variable $\theta$ defined as $kT_e/\hbar c^2$:

$$\Delta I = i_0 h_0(x) \int d\tau \left[ \theta g_1(x) - \beta_c + R(x, \theta, \beta_c) \right]$$  \hspace{1cm} (6.7)

where $g_1(x) = x(e^x + 1)/(e^x - 1) - 4$ and $R(x, \theta, \beta_c)$ is the relativistic correction factor.

All of the variables have now been defined by the properties of the cluster, the IC electrons, and the CMB radiation. To determine the Hubble constant, measurements from the thermal SZ effect described above and X-ray measurements of thermal emission from IC gas can be used to
determine the angular diameter distance - defined as $d_A = l/\theta$, where $l$ is the distance through the cluster (having assumed spherical symmetry so that the angular diameter of the cluster is equal to the line of sight distance through it) and $\theta$ is the angle between the earthbound observer and the edges of the cluster - to a specific cluster of galaxies. In addition, this method can be used to determine the deceleration parameter ($q_0$) if measurements are made of distant ($z \approx 1$) clusters and a value for $H_0$ is assumed (Silk & White 1978).

Once the distance to the cluster is determined, a Hubble diagram can be constructed from measured redshifts of each cluster. To determine the distance to the cluster, it is necessary to solve for the angular diameter distance ($d_A$) in terms of observable quantities and make certain assumptions about the cluster, specifically spherical symmetry, in order to determine the line of sight distance to the cluster. The first step is to observe the X-ray surface brightness of the IC gas in some energy band. The equation is given by (Birkinshaw et al 1991)

$$b_x = \frac{\Lambda_0 n_e^2 d_A}{4\pi(1+z)^4} \int d\zeta w_n^2 w_\Lambda.$$

(6.8)

The electron density has been written as $n_e = n_0 w_n$, and the temperature/energy dependent coefficient of the bremsstrahlung emissivity has been written as $\Lambda = \Lambda_0 w_\Lambda$. The quantities $w_n$ and $w_\Lambda$ are profile functions of the spatial coordinates, relating the quantities $n_0, \Lambda_0$ (electron density and emissivity coefficient independent of position) to their coordinates in space (Rephaeli 1995). Rather than integrating along $l$ as was done earlier, the line of sight crossing the cluster has been expressed in terms of the nondimensional angular variable $\zeta = l/d_A$ in order to have the angular diameter distance present in the equation.

Returning to the intensity change in the CMB given by Equations 6.2 and 6.3 and noting the
spatial dependence of $T_e$, the full expression becomes

$$\Delta I = i_0 g_0(x) \frac{kT_e}{mc^2} \sigma_T n_0 d_A \int d\zeta w_n w_T,$$

(6.9)

where $T_e = T_{eo} w_T$ and again $w_T$ is a function of the spatial coordinates. For now it will be assumed that only the lower frequencies are being considered so that we can avoid using the relativistic expression, though it should be noted that relativistic corrections have been developed and refined, either analytically or numerically, e.g., Rephaeli (1995a), Sazonov & Sunyaev (1998), Itoh et al 2000, and Shimon & Rephaeli (2002). Continuing with the method for determining $H_0$, solve for $n_o$ in Equation 6.6 and substitute the result into the expression for the X-ray surface brightness (Equation 6.5). This will yield an expression that can be rearranged to give $d_A$ in terms of quantities that can be observationally determined:

$$d_A = \frac{1}{4\pi(1+z)^4} \left( \frac{\Lambda_o}{\sigma_T^2 b_x} \right) \left( \frac{\Delta I}{i_0 g_0(x)} \right)^2 \left( \frac{mc^2}{kT_e} \right)^2 \left( \frac{Q_x}{Q_m^2} \right),$$

(6.10)

with

$$Q_x = \int d\zeta w_n^2 w_A, \quad Q_m = \int d\zeta w_n w_T$$

(6.11)

The above expressions relate the angular diameter distance to measurable variables. The theoretical expression for $d_A$ is given by

$$d_A = \frac{c \left[ zq_o + (q_o - 1) \left[ (1 + 2zq_o)^{1/2} - 1 \right] \right]}{H_0 q_o^2 (1+z)^2}.$$

(6.12)

By assuming a value for the deceleration parameter $q_o$, (e.g., $q_o = 1$), the equation can be simplified further and $H_0$ can be calculated by comparing the measured value to the predicted value. However, dependence on $q_o$ in the theoretical expression introduces an error of up to 12% for $z \leq 0.2$ due to the uncertainty in the value of $q_o$ (Rephaeli 1995a).

In order to actually carry out this calculation, several assumptions must be made. First, the IC gas is assumed to be uniform and spherically distributed to avoid anisotropic complications in the equations. Also, the gas is assumed to be generally isothermal. Such assumptions introduce an amount of uncertainty that must be considered with the formal and systematic uncertainties of the measurements. However, over a large enough statistical sample, many of assumptions average out and still yield accurate results, though the bias in the measurements should not be ignored when comparing the results with other observationally determined values for $H_0$.

There is one final factor that affects the values for $H_0$ as determined from the SZE. Since the analyzed galaxy clusters can be (and have been) at high redshifts ($z \approx 0.55$), the geometry of the universe itself can affect the observations and thus the deduced mean value depends on the cosmological model (Rephaeli 2002). The two general models relating $\Omega_M$ and $\Omega_{\Lambda}$ (SCDM and $\Lambda$CDM) give different results for $H_0$, as will be discussed in the next section.
6. Observations

Since the original measurements were made improved methods and instrumentation have yielded more and more accurate measurements of the Hubble constant. Recent observations and data have given a Hubble constant in the vicinity of $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$, with published results ranging from $\approx 40$ to $80 \text{ km s}^{-1} \text{ Mpc}^{-1}$, in relative agreement with the value obtained by the Hubble Key Project, $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Freedman et al 2001), though the range of values does not give a solid value for precise comparison. Such a large deviation can be explained by astrophysical complications that give rise to a number of uncertainties in the SZ measurement, as has been previously mentioned. The main factors affecting this uncertainty are due to assumptions made about the clusters and the ICM to facilitate calculations. For instance, the gas distribution in clusters was assumed to be uniform, though this is not necessarily the case. If the gas were to clump together significantly, the overall value for $H_0$ would decrease. In addition, the assumption of spherical symmetry and a generally uniform distribution of the galaxies in the clusters can give a lower value for $H_0$ if for example the cluster were observed edge on from earth. Also, galaxy clusters themselves are not always uniform, with local groups forming higher density regions in various parts of the cluster. This assumption will average out, however, over a statistical sample of enough clusters due to the general uniformity that they possess, and so these isolated regions of galactic groups can be ignored on the whole.

It is difficult to quantify exactly the combined errors from all of these effects and assumptions, though as a higher data sample of clusters is collected the method seems to improve its overall accuracy. With higher resolution X-ray and thermal SZ data, the results become more and more reliable.

The following is a brief overview of some recent results given by the SZ effect as given in a review by Rephaeli (1995a) and some more recently published values from Mason, Meyers, & Readhead (2001), Reese et al (2002), and an improved method carried out by Schmidt, Allen, and Fabian (2004).

6.3.1 Rephaeli (1995a)

Before 1995, the practical complications were the main cause in the varied range in values for $H_0$. In a review of the literature that had been published from 1990 to 1995, Rephaeli listed a table with the measured values of $H_0$ as determined from 7 separate clusters, with some duplication. There was a very wide range of values, from approximately 24 to 82 $\text{ km s}^{-1} \text{ Mpc}^{-1}$, with an average of $H_0 = 54 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Table 1).

6.3.2 Mason, Myers, & Readhead (2001)

Seven clusters with $z < 1$ were studied using the same method as had been done in the previous review with a range from 36 to 102 $\text{ km s}^{-1} \text{ Mpc}^{-1}$ and an average of 64 $\text{ km s}^{-1} \text{ Mpc}^{-1}$ for a standard cold dark matter (SCDM) cosmology or 66 $\text{ km s}^{-1} \text{ Mpc}^{-1}$ for a flat $\Lambda$CDM cosmology (Table 2).
6.3.3 **Reese et al (2002)**

Distances were determined to 18 galaxy clusters (0.14 < z < 0.78) and the final result was $H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for an $\Omega_M = 0.3, \Omega_\Lambda = 0.7$ cosmology (Table 3).

6.3.4 **Schmidt, Allen, & Fabian (2004)**

In one of the most recent papers published on the SZE, an improved method was presented for predicting the effect and carrying out the observations. The main advantage to their new method was extrapolating a pressure profile of the X-ray gas, allowing the Comptonization parameter to be predicted precisely. Applying their method to Chandra observations of three clusters, they found $H_0 = 69 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for an $\Omega_M = 0.3, \Omega_\Lambda = 0.7$ cosmology, very close to the value of the Hubble Key Project.

6.4 **Conclusion**

From the above data and observations it is obvious that a more precise method for measuring the SZE is needed before the most accurate measurements of the Hubble constant can be obtained. For the method outlined in Section 6.2, it is necessary to have detailed spectral and spatial X-ray measurements in order to determine $n_0, T_0$ and the integrals of the spatial profile functions $w_n, w_T$. These quantities depend on the details of the gas models constructed for data analysis, any deviations from uniformities of $n_e$ and $T_e$, and the presence of any gas not detected by the X-ray measurements (Rephaeli 1995a). Because of the differences in models used for the gas distribution, it is not surprising that the values of $H_0$ are different among the individual measurements made by each group. However, prospects for better modeling and higher resolution X-ray data are on the horizon with the most modern satellites (*Chandra* and *XMM-Newton*) as well as radio observatories such as the Owens Valley Radio Observatory (OVRO), the Berkley-Illinois-Maryland Association (BIMA), and the Very Large Array. When both the thermal and X-ray data reach a high enough level of accuracy, extremely precise measurements of the Hubble constant will be made by using the Sunyaev-Zeldovich effect.

6.5 **References**

6. Determining the Hubble Constant with the Sunyaev-Zeldovich Effect


Table 1  The Hubble constant from S-Z and X-ray measurements

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$H_0$ (km s$^{-1}$ Mpc$^{-1}$)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A478</td>
<td>$32^{+19}_{-15}$</td>
<td>Myers et al (1995)</td>
</tr>
<tr>
<td>A665</td>
<td>$40 \pm 9$</td>
<td>Birkinshaw et al (1991)</td>
</tr>
<tr>
<td>A1656</td>
<td>$74^{+29}_{-24}$</td>
<td>Herbig et al (1994)</td>
</tr>
<tr>
<td>A2142</td>
<td>$57^{+61}_{-39}$</td>
<td>Myers et al (1995)</td>
</tr>
<tr>
<td>A2163</td>
<td>$82^{+35}_{-22}$</td>
<td>Holzapfel et al (1995)</td>
</tr>
<tr>
<td>A2218</td>
<td>$24 \pm 11$</td>
<td>McHardy et al (1990)</td>
</tr>
<tr>
<td>A2218</td>
<td>$65 \pm 25$</td>
<td>Birkinshaw &amp; Hughes (1994)</td>
</tr>
<tr>
<td>A2218</td>
<td>$38^{+18}_{-16}$</td>
<td>Jones (1994)</td>
</tr>
<tr>
<td>A2256</td>
<td>$76^{+22}_{-19}$</td>
<td>Myers et al (1995)</td>
</tr>
</tbody>
</table>

Table 2  $H_0$ RESULTS ON FIVE CLUSTERS

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$H_0$ (km s$^{-1}$ Mpc$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A359</td>
<td>$102^{+14}_{-13}$</td>
</tr>
<tr>
<td>A401</td>
<td>$48^{+14}_{-16}$</td>
</tr>
<tr>
<td>A478</td>
<td>$61^{+23}_{-20}$</td>
</tr>
<tr>
<td>A1651</td>
<td>$36^{+15}_{-13}$</td>
</tr>
<tr>
<td>Coma</td>
<td>$62^{+14}_{-12}$</td>
</tr>
<tr>
<td>A2142</td>
<td>$79^{+24}_{-23}$</td>
</tr>
<tr>
<td>A2256</td>
<td>$67^{+28}_{-25}$</td>
</tr>
<tr>
<td>Sample</td>
<td>$64^{+14}_{-11}$</td>
</tr>
</tbody>
</table>

Table 3

\[
H_0 = \begin{cases} 
60^{+4}_{-3} +13 \text{ km s}^{-1} \text{ Mpc}^{-1}, & \Omega_M = 0.3, \Omega_\Lambda = 0.7 \\
56^{+4}_{-3} -12 \text{ km s}^{-1} \text{ Mpc}^{-1}, & \Omega_M = 0.3, \Omega_\Lambda = 0.0 \\
54^{+4}_{-3} -16 \text{ km s}^{-1} \text{ Mpc}^{-1}, & \Omega_M = 1.0, \Omega_\Lambda = 0.0 
\end{cases}
\]
Combining the Constraints on the Hubble Constant

Artin Teymourian

ABSTRACT The Hubble Space Telescope Key Project was designed to calibrate new techniques for measuring distances to galaxies. This involved using Cepheid variables to determine distances to 18 galaxies within 25 Mpc and calibrating the Tully-Fisher relation, fundamental plane, surface brightness fluctuations, and Type Ia supernovae techniques. The measurements then must be modified in order to account for velocities not typical of the Hubble flow, that is velocities of galaxies with respect to the cosmic microwave background, velocities of galaxies towards attractors, and the velocity of our own galaxy. Each technique for determining distances also has a certain percent error associated with it. Using a computer simulation, the errors were followed in the process of determining the distances and the final error for each method was determined. Finally, all of the methods were combined, with more weight given to the supernova Ia method, and a final value of $H_0 = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ was determined. However, it should be noted that the distances obtained with Cepheid variables could be affected by metallicity values. If this were to be taken into consideration, the Hubble constant becomes $H_0 = 68 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The largest contributor to the uncertainty in the Hubble constant is the distance to the Large Magellanic Cloud, which was assumed to be $50 \pm 3 \text{ kpc}$.

7.1 Introduction

One of the main goals of the Hubble Space Telescope is to determine the Hubble constant to an accuracy of $<10\%$. To do this, 18 galaxies, all within $\sim25$ Mpc were observed and the distances to these galaxies were determined using the Cepheid period-luminosity relation. These distances were then used to calibrate the Tully-Fisher relation for spiral galaxies (TF), the fundamental plane for elliptical galaxies (FP), surface brightness fluctuations (SBF), and Type Ia supernovae (SNe Ia) (Mould et. al., 2000). These methods will be able to determine distances of galaxies far enough so that the Hubble flow dominates over other types of motion. All of these techniques however, use implicit assumptions about the stellar populations of the galaxies being observed. Because of this, it is better to combine the constraints from four separate measurements using different assumptions than to analyze only one technique. This paper will combine the constraints on $H_0$ and thus determine a value for $H_0$ to an accuracy $<10\%$ at the 1 $\sigma$ confidence level.

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7.2 The Velocity Field Model

In order to accurately calibrate the various methods of distance determination, Cepheids from several nearby galaxies were observed. Because of the relative proximity of the observed galaxies to our own, large-scale motions due to factors other than the Hubble flow must be considered. The various motions affecting the observed velocities of galaxies include the rotation of our own galaxy, the movement of our galaxy with respect to the centroid of the Local Group, the velocity of the Local Group with respect to the cosmic microwave background (CMB), the infall of the Local Group into the core of the Local Supercluster, and other larger scale flows (Mould et. al., 2000).

In order to compensate for these velocities, several modifications must be made to the observed velocities. First, the velocity must be corrected for our movement with respect to the CMB. This is done by adding a correction that is our CMB velocity ($\sim 630$ km s$^{-1}$) multiplied by the cosine of the angle between the direction of motion of the galaxy with respect to the CMB. Not doing so may result in a bias of $H_0$ of about 6%.

The movement with respect to the CMB is the main concern for velocity determinations of galaxies with relatively high redshifts ($cz = 10,000$ km s$^{-1}$). At lower redshifts, there are many more motions to take into account. For these closer objects, it may be necessary not to apply a simple $\cos \theta_{\text{cmb}}$ term because these objects are closer to being at rest with respect to the Local Group frame than with respect to the CMB frame (Mould et. al., 2000).

In order to correct for the various velocities, a linear multiattractor model was developed based on the Han & Mould (1990) and Han (1992) models. The model corrects for flows towards each attractor (e.g., Virgo, the Great Attractor, Shapely supercluster), flows due to our own velocities towards each attractor, and essentially a cylindrical volume around each attractor that forces objects to the attractor’s velocity. Thus, to determine the velocities of objects characteristic of the expansion of the universe, the following equation must be employed:

$$V_{\text{cosmic}} = V_H + V_{c,\text{LG}} + V_{\text{in,Virgo}} + V_{\text{in,GA}} + V_{\text{in,Shap}} + ..., \quad (7.1)$$

where $V_H$ is the observed heliocentric velocity, $V_{c,\text{LG}}$ is the correction for the velocity of the Milky Way around the centroid of the Local Group, and each of the $V_{\text{in}}$ components refer to the infall velocities of the galaxy towards an attractor. The details of how the velocities are determined will not be discussed in this paper, however they are easily found using trigonometry and some assumptions on the attractor’s density profile. The specifics on determining the velocities are described by Mould, et. al. (2000).

7.3 The Virtual Key Project

In order to fully investigate the propagation of errors in distance estimates, a simulation code was designed to recreate the Key Project on a computer. Uncertainties in various measurements were observed in the simulation 500,000 times and error distributions for each of the four techniques were created. Based on this data a value of $H_0$ was determined.
Of the various uncertainties in observations, the most prominent one is that of the distance to
the Large Magellanic Cloud (LMC). The distance is adopted to be 50 kpc \( (m - M = 18.50 \pm 0.13 \text{ mag}) \) however this incorporates a 6.5% uncertainty in our measurements. Also, from Tanvir (1999) we obtain a 0.02 mag zero-point uncertainty for the LMC Cepheid PL relation in the simulation, further contributing to the error. Another important error to note is that of the instruments used to observe the targets. The HST’s WFPC2 has a residual uncertainty in correcting for charge
transfer efficiency (Mould et. al., 2000). This uncertainty is amplified to a 0.09 mag uncertainty in the distance modulus because of the approach used to correct for reddening.

For each of the galaxies analyzed in the simulation, a Cepheid distance was generated assuming a 0.05 mag intercept uncertainty in its PL- \( V \) and PL- \( I \) relations. Although the values for individual galaxies vary (e.g. galaxies with more Cepheids have better accuracies), we have assumed that the reddening law is universal and have adopted an uncertainty in its slope of \( R_V = 3.3 \pm 0.3 \). Also, it should be noted that PL measurements are dependent on metallicity values, however such values have been difficult to constrain. In order to account for the metallicity dependence,

FIGURE 7.1. Distribution of uncertainties in \( H_0 \) for each of the four secondary distance indicators calibrated and applied in the virtual Key Project
a value of $\gamma = d(m - M)/d[O/H]$ was drawn from each simulation. These values were then used to generate two values of $H_0$, one neglecting the metallicity dependence and one incorporating it.

In calibrating the TF relation for galactic distances, Sakai et al. (2000) found an rms scatter, which was included in the simulation. In the simulation, the velocities were drawn from a normal distribution with a zero mean and $\sigma = 300$ km s$^{-1}$ (Giovanelli et al., 1998). Sakai et al. (2000) reported discrepancies between the I-band and H-band photometry, along with other errors. These discrepancies and errors resulted in a 0.18 mag uncertainty in the simulation. When using the SBF calibration techniques, Ferrarese et al. (2000) took six galaxies and derived a zero point for the relation between SBF magnitude and color. The simulation followed the same procedure, and an rms scatter of 0.11 mag was assumed. From this, and from the errors noted by Ferrarese et al. (2000), an overall uncertainty was calculated. Next, the effects of reddening and uncertainties in observed magnitudes were analyzed for the SN Ia calibrations. The uncertainties noted by Gibson et al. (2000) were included in the simulation. Finally, the errors noted by Kelson et al. (2000) using the FP technique were accounted for. The FP technique also assumes that the Leo ellipticals lie within 1 Mpc of the respective mean distances of their Cepheid-bearing associates, further adding to the uncertainties (Mould et al., 2000).

The simulation uses the aforementioned data and creates 500,000 realizations. The results are simulated error distributions for each of the four calibrations. Fig. 7.1 shows an error distribution width of $\sigma \approx 12\%$ for the TF, SBF, and FP techniques, while the SN technique shows a relatively narrow distribution of $\sigma \approx 9\%$.

![FIGURE 7.2. Uncertainty distribution for the combined constraints on $H_0$.](image)
7.4 Combining the Constraints

In combining the constraints of the various techniques, a weighted average was chosen rather than a simple mean of the four. The weight of each method was the inverse of $\sigma^2$ from each technique shown in Fig. 7.1. Because of this, the SN technique is weighted about 1.7 times more than the other three. Combining $H_0^{TF} = 71 \pm 4$ (random) $\pm 7$ (systematic) (Sakai et. al., 2000) with $H_0^{SBF} = 69 \pm 4 \pm 6$ (Ferrarese et. al., 2000), $H_0^{FP} = 78 \pm 8 \pm 10$ (Kelson et. al., 2000), and $H_0^{SN Ia} = 68 \pm 2 \pm 5$ (Gibson et. al., 2000), we obtain $H_0 = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Mould et. al., 2000). The error distribution of the combined constraints is shown in Fig. 7.2, and the width of the distribution is $\pm 9\%$ ($1\, \sigma$).

It is important to note that all of the calibrations included an assumption for the value of the LMC. The probability distribution for the LMC is depicted in Fig. 7.3, and when combined with the error distribution for the combined constraints on $H_0$, we obtain the error distribution shown in Fig. 7.4. Here, the error has skewed the value of $H_0$ to an underestimate of 4.5% and broadens the error to a width of $\sigma = 12\%$. If we were to calculate $H_0$ in terms of the distance to the LMC, we obtain a value of $H_0 = 3.5 \pm 0.2 \text{ km s}^{-1} \text{ per LMC distance}$.

Thus, after combining all the constraints and weighing them on the basis of the widths of the error distributions, we come up with a final value of $H_0 = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. By using oxygen abundances measured spectroscopically in each Cepheid field, we were able to find the impact of metallicity on the overall value. Taking metallicity into consideration, the value drops by 4% to $H_0 = 68 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. 

FIGURE 7.3. Distribution of published LMC distance moduli from the literature.
7. Combining the Constraints on the Hubble Constant

7.5 References


