Cosmology & LSS
- Inflation
- CMB Fluctuations
- Large Scale Structure
- Baryon Acoustic Oscillations

Wed: HW#9 presentations
On the horizon: Test#2 (Nov 16)
Main Observational Foundations of Modern Cosmology

1. Olber’s paradox (The night sky is dark); hence the universe cannot be infinite in space and time (1823)
2. The Universe expands (1929); expansion appears to accelerate (2000…)
   … and it got started 13.7 billion yrs ago
3. Cosmic Microwave Background Radiation (1965), of T=2.73 K and fluctuations of predicted amplitude (about 1 part in $10^5$; 1992-2013)
4. Cosmic abundance of light isotopes of H, He, Li (1948)
5. Statistical properties of the large-scale structure of the Universe (how galaxies cluster; 1980’s--)

The “standard” model has been known for a long time and could be falsified by a single observation (e.g., precise shape of CMB, finding a single unprocessed gas cloud with <20% helium, source at $z$ has a single absorption line by a foreground source at $z_{\text{abs}} > z$). However, this has not happened!
Problems in Modern Cosmology

- Finite speed of light causes horizons: if regions are too separated, they could have never exchanged information; why is the CMB temperature in opposite directions on the sky the same, when the horizon at recombination ($z \sim 1100$) spans only $\sim 2^\circ$ on the sky? (horizon problem)

- Universe is nearly flat today, $\Omega_m + \Omega_\Lambda = 1.00 +/− <2\%$ (conservative estimates from recent CMB fluctuation measurements); to achieve this narrow interval, the density must be very fine tuned at much earlier time, otherwise the Universe would have recollapsed by now, or its density had become very very very small by today. What causes this fine tuning? (flatness problem)

- Symmetry problem: why is there matter left but no antimatter?

- Inflationary theory predicts a very fast (exponential) expansion in the first $10^{-32}\ s$ of cosmic history – this inflation solves first two problems mentioned above (and others too).

- Inflation + FL-model form the standard model of the Universe – unrivaled in its achievements!
Inflation as a solution

• The Flatness Problem: Why is the universe so (close to) flat?
• The Horizon Problem: Why is the universe so (close to) being (perfectly) smooth?
• The Symmetry Problem: Where has all the “antimatter” gone?

• Universe suddenly begins to expand exponentially

• Caused by phase transition as strong nuclear force freezes out

• A repulsive “cosmological constant” temporarily dominates the energy density of the universe

• As inflation ends, matter and radiation are (again?) created from the cosm. constant energy density (“reheating”)
Cosmic Inflation

During the inflationary phase, space-time itself expands exponentially (faster than c), ‘stretching out’ curvatures of space, and yielding large horizon – solves flatness problem and horizon problem; ⇒ Only a small part of the universe is visible details of inflationary scenario yet unknown

also: ⇒ “stretches” out early quantum fluctuations to macroscopic scales ⇒ thus produces CMB anisotropies

⇒ and results in structure (inhomogenous galaxy distribution seen today)
⇒ growth of density fluctuations through gravitational instability
Cosmological Parameters

What are the most important cosmological parameters?

- the density parameters $\Omega_m$ and $\Omega_\Lambda$;
- the Hubble constant $H_0$;
- the baryon density $\Omega_b$;
- the shape parameter $\Gamma$ which determines the shape of the density fluctuation spectrum in the early Universe;
- the amplitude $\sigma_8$ of the density fluctuation spectrum in the early Universe

From theory of structure growth, one expects $\Gamma \approx \Omega_m h$; some of these parameters are well determined, e.g., $\Omega_b$. 
### WMAP+BAO+SN parameter summary

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of universe</td>
<td>$t_{\text{H}}$</td>
<td>$13.73 \pm 0.12$ Gyr</td>
</tr>
<tr>
<td>Hubble constant</td>
<td>$H_{0}$</td>
<td>$70.1 \pm 1.3$ km/s/Mpc</td>
</tr>
<tr>
<td>Baryon density</td>
<td>$\Omega_{b}$</td>
<td>$0.0462 \pm 0.0015$</td>
</tr>
<tr>
<td>Dark matter density</td>
<td>$\Omega_{\text{dm}}$</td>
<td>$0.233 \pm 0.013$</td>
</tr>
<tr>
<td>Dark energy density</td>
<td>$\Omega_{\Lambda}$</td>
<td>$0.721 \pm 0.015$</td>
</tr>
<tr>
<td>Age at decoupling</td>
<td>$t_{\text{cmb}}$</td>
<td>$375938 \pm 3148,-3115$ yr</td>
</tr>
</tbody>
</table>

![Pie chart showing the composition of the universe today](image)

- Atoms: 4.6%
- Dark Matter: 23%
- Dark Energy: 72%
Planck: New Cosmological Parameters (21 Mar 2013)
From the CMB to Galaxies

- The cosmology determines the *timescales* on which structures (BH’s, galaxies, clusters, super-clusters) form and evolve.

- *Observing* the electromagnetic radiation from *objects* (CMB, galaxies, QSOs, clusters) at different redshifts allows us to see how (luminous) matter was distributed at different times in the history of the universe.

- Galaxies/clusters can constrain the *cosmological parameters*, but we need to understand how galaxies/clusters evolve.
The Inhomogenous Universe

Cosmological Principle:
To first order, the Universe is homogenous and isotropic

Of course, homogeneous world models only yield gross properties of the Universe; on small scales, the Universe is inhomogeneous, as seen, e.g., in local galaxy distribution and CMB fluctuations (they signify inhomogeneities at the time of recombination);

Galaxies are clustered, and form dense structures like clusters of galaxies and groups; there exist voids in the Universe, large regions nearly empty of galaxies.
Gravitational Instability Picture

Structure in the Universe evolves via gravitational instability: starting from very tiny initial fluctuations (e.g., inflated quantum fluctuations), slightly over-dense regions

• have higher gravity field,
• expand slower than rest of Universe,
• so their density relative to rest of Universe increases further,
• slows relative expansion down more,
• etc.

⇒ Initial density fluctuations will only grow, due to the attractive force of gravity
⇒ Regions that start slightly over-dense will grow to the densest regions in the universe seen today (galaxy clusters/super-clusters)

This gravitational instability is predictive; density evolution in the Universe can be calculated, today with an impressive accuracy
Cosmological simulations:

Understand state and growth of large scale structure in the universe

Galaxies trace the structure of the universe

Zoom-in: LSS
⇒ Galaxy halos

The Millennium I/II simulations
http://www.mpa-garching.mpg.de/galform/millennium-II/
Formation of Large Scale Structure

Galaxies are not distributed randomly in space.

If one galaxy has comoving coordinate $\mathbf{x}$, then the probability of finding another galaxy in the vicinity of $\mathbf{x}$ is not random. They are correlated.

Courtesy of Michael Blanton.
Different models yield different structure

Cold dark matter (CDM) models

Simulations of Structure Formation

“cold”: $v \ll c$

$\Lambda\text{CDM}$: $\Omega_m=0.3$, $\Omega_\Lambda=0.7$

SCDM, $\tau\text{CDM}$: $\Omega_m=1$, $\Omega_\Lambda=0$

(they have different $\Gamma$'s).

$\Gamma=\Omega_m h$: shape parameter

$\Omega_m=0.3$, $\Omega_\Lambda=0.0$

From the Virgo Consortium Simulations

$\Rightarrow$ more on DM in a later lecture
The Cosmic Microwave Background Radiation

1990’s: Cosmic Background Explorer (COBE)
Smoot and Mather 2006 Nobel prize

- Current temperature is 2.725K, and uniform to better than 1 part in 100,000.
- Uniformity best described by Big Bang Cosmology.
- Small “wiggles” are seeds of galaxies (i.e., structure formation).
From there to here

$t = 3.8 \times 10^5 \text{ yr}$

very smooth: $\delta \rho/\rho \sim 10^{-5}$

$\delta \rho/\rho \propto t^{2/3}$

$t = 1.4 \times 10^{10} \text{ yr}$

very lumpy: $\delta \rho/\rho \sim 1$
Characterizing structure

If one galaxy has comoving coordinate, \( \mathbf{x} \), then the probability of finding another galaxy in the vicinity of \( \mathbf{x} \) is not random. They are Correlated.

Consider two comoving points, \( \mathbf{x} \), and \( \mathbf{y} \). If \( \bar{n} \) is the average number density of galaxies, probability of finding a galaxy in the volume element \( dV \) around \( \mathbf{x} \) is

\[
P_1 = \bar{n} \, dV
\]

In practice, assume \( dV \) is small so that \( P_1 \ll 1 \) and the probability of finding \( >1 \) galaxies in \( dV \) is negligible.

On what scales do galaxies cluster?
Characterizing structure

If one galaxy has comoving coordinate, $\mathbf{x}$, then the probability of finding another galaxy in the vicinity of $\mathbf{x}$ is not random. They are Correlated.

Consider two comoving points, $\mathbf{x}$, and $\mathbf{y}$. If $\bar{n}$ is the average number density of galaxies, probability of finding a galaxy in the volume element $dV$ around $\mathbf{x}$ is

$$P_1 = \bar{n} \, dV$$

In practice, assume $dV$ is small so that $P_1 << 1$ and the probability of finding $>1$ galaxies in $dV$ is negligible.

The Probability of finding a galaxy in $dV$ around $\mathbf{x}$ *and* finding a galaxy in $dV$ around $\mathbf{y}$ is

$$P_2 = (\bar{n} \, dV)^2 \left[ 1 + \xi_g(x, y) \right].$$

If the probabilities were uncorrelated, $P_2 = P_1^2$. Because they are correlated include an extra term, $\xi(x, y)$, which is the correlation function.

On what scales do galaxies cluster?
The spatial correlation function $\xi(r)$

$$P_2 = (\overline{n} \, dV)^2 \left[ 1 + \xi_g(x, y) \right].$$

Correlation function is related to fluctuations in the matter density

$$\langle \rho(x) \, \rho(y) \rangle = \overline{\rho}^2 \, \langle [1 + \delta(x)] \, [1 + \delta(y)] \rangle \quad \delta: \text{density contrast}$$

$$= \overline{\rho}^2 \, (1 + \langle \delta(x) \, \delta(y) \rangle)$$

$$=: \overline{\rho}^2 \, [1 + \xi(x, y)] ,$$

This is because the mean value (the expectation value) of $\delta$ is 0.

$\xi$ can only depend on the difference $x - y$, and because Universe is isotropic, $\xi$ can not depend on direction of separation. Therefore $\xi = \xi(r)$

Measurements of the correlation function show that it follows a powerlaw:

$$\xi_g(r) = \left( \frac{r}{r_0} \right)^{-\gamma} ,$$
Observations of the correlation function

\[ \xi_g(r) = \left( \frac{r}{r_0} \right)^{-\gamma} \]

Typically, \( \gamma \sim 1.8 \).

**strong correlations on <few Mpc scales**

Low-cost: Angular Correlation Function

Correlation properties of galaxies can be determined from their angular positions. The 3D correlation function implies that their angular positions are also correlated. The angular correlation function is the integral of the spatial correlation function along the line of sight.

This does require that one knows (or has an estimate of) the redshift distribution of your sample.

The probability of finding a galaxy at location $\theta_1$ on the sky and a galaxy at location $\theta_2$ is

$$P_2 = (\bar{n} d\omega)^2 \left[ 1 + w(\|\theta_1 - \theta_2\|) \right]$$

The relation between $w(\theta)$ and $\xi(r)$ is given by the Limber equation.

$$w(\theta) = \int dz \, p^2(z) \int d(\Delta z) \times \xi_g \left( \sqrt{[D_A(z)\theta]^2 + \left(\frac{dD}{dz}\right)^2(\Delta z)^2} \right)$$

where $D_A$ is the angular diameter distance, and $dD = -c \, dt$ (physical distance along line of sight).

$p(z)$: redshift distribution of galaxies

dD/dz = c \cdot [(1+z) H(z)]^{-1}
The Power Spectrum

An alternative and equivalent description of the statistical properties of Large Scale Structure is the **Power Spectrum, $P(k)$**. This describes the amplitude of structure on scales of length $L = 2\pi/k$.

Our density fluctuations can be decomposed into the sum of plane waves.

$$\delta(x) = \sum a_k \cos(xk)$$

Power spectrum is the Fourier transform of the correlation function:

$$P(k) = 2\pi \int_0^\infty dr \ r^2 \frac{\sin kr}{kr} \xi(r)$$

⇒ Density perturbation field is a combination of perturbations with different wavelengths (different “modes”)

⇒ Modes with *smaller* wavelengths oscillate *faster*
The Initial Power Spectrum $P_0(k)$

Both $P(k)$ and $\xi(r)$ evolve with time, as dictated by the Growth Factor:

$$\xi(x, t) = D_+^2(t) \xi(x, t_0)$$

$$P(k, t) = D_+^2(t) P(k, t_0) =: D_+^2(t) P_0(k)$$

In the context of linear perturbation theory, knowledge of $P_0(k)$ is sufficient to obtain the power spectrum $P(k)$ at any time.

**The Initial Power Spectrum**

At early times, the scale factor grew at $a(t) \sim t^{1/2}$. At that time there should be no natural length-scale. The only mathematical function that depends on a length with no characteristic scale is a power law, therefore we expect that

$$P(k) \propto k^{n_s}$$

Harrison, Zeldovich (and Peebles and others) have argued that $n_s=1$ for based on scaling relations. A power spectrum with $n_s=1$ is a *Harrison-Zeldovich spectrum.*
The Initial Power Spectrum

The Initial Power Spectrum

\[ P(k) \propto k^{n_s} \]

We therefore have an equation for the evolution of the Power Spectrum:

\[ P(k, t_1) = D_+^2(t_1) A k^{n_s} \]

where the constant A is the overall normalization -- it can *not* be determined from theory but must be fixed by measurements of the power spectrum.

The initial power spectrum is:

\[ P_0(k) = A k^{n_s} \]

Overlay of different “modes” on different scales
Cosmic Microwave Background Radiation

After subtraction of the Earth’s motion and contribution of the Galaxy, the CMB is remarkably isotropic.

It reflects the baryonic density inhomogeneities ~380,000 yrs after the Big Bang

\[ \frac{\delta T}{T} \approx 0.00001 \]
CMB as the surface of last scattering

At redshift $z>1100$, hydrogen is fully ionized.
(Helium is fully ionized at $z>6000$)

$\Rightarrow$ The Universe is optically-thick against Thomson scattering at $z>1100$.

$\Rightarrow$ The universe becomes unobservable at redshifts $\sim 1000$, which represents the last-scattering plane for photons

If fluctuations in the photon temperature are present, they will be preserved at decoupling and should be visible as “frozen into” the CMB (except if the universe would have been re-ionized at time when it was still mostly homogenous)
Cosmic Microwave Background Radiation

- The CMB fluctuations, recently observed by WMAP at a high angular resolution, show a characteristic size of $\sim 1^\circ$
- How do we mathematically describe this behavior? How do we compare models to these observations?
CMB: The Fluctuation Power Spectrum

\[ P(k) \propto k^{n_s} \quad \text{Harrison-Zeldovich spectrum} \]

We therefore have an equation for the evolution of the Power Spectrum:

\[ P(k, t_i) = D_{+}^2(t_i) A k^{n_s} \]

where the constant \( A \) is the overall normalization -- it can \textbf{not} be determined from theory but must be fixed by measurements of the power spectrum.

The initial power spectrum is:

\[ P_0(k) = A k^{n_s} \]

Overlay of different “modes” on different scales
WMAP: measuring the power spectrum

different “modes” on different scales,
(“frozen in” at $t_{\text{recomb}}$)
Cosmic Microwave Background Radiation

- Historical note: The accuracy of CMB measurements has been improving fast!

- Any scalar field on a sphere can be expressed as a series of spherical harmonic multipoles:

\[ T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{l,m} Y_{l,m}(\theta, \phi) \]

⇒ Measure the amplitude of the structure on different scales, construct power spectrum.

⇒ CMB sky should show fluctuations that can be described by a predictable function of \( l \)

- Monopole: \( l=0, m=0 \)
- Dipole: \( l=1, m=0,1 \)
- Quadrupole: \( l=2, m=0,1,2 \)
- etc.

To 3\(^{rd}\) peak

To 7\(^{th}\) peak?
Angular spectrum of the CMB

Origin of acoustic peaks in CMB multipole moment $l$-spectrum:

Pressure of photon-baryon fluid resists gravity, giving rise to oscillations $\Rightarrow$ sound waves

Compressing a gas heats it up, expanding a gas cools it down
$\Rightarrow$ Temperature fluctuations

Modes stop oscillating at recombination (phases are “frozen in”)

different “modes” on different scales
WMAP CMB Ripples vs. Density

Click animation to start
Angular spectrum of the CMB

For a Harrison-Zel’dovich spectrum, the angular spectrum of the CMB should behave as:

\[ \ell(\ell + 1)C_\ell \approx \text{const} \quad \text{for} \quad \ell \ll \frac{180^\circ}{\theta_{\text{H,rec}}} \approx 100, \]

Basic flat WMAP parameters: \( \omega_L = 0.71, \omega_m = 0.29 \) (\( \omega_c = 0.24, \omega_b = 0.047 \)), \( n = 0.93, h = 0.71 \).

WMAP + other: \( \omega_L = 0.71, \omega_m = 0.27 \) (\( \omega_c = 0.23, \omega_b = 0.044 \)), \( n = 0.93, h = 0.71 \).

Characteristic size scale of fluctuations at recombination

Observation: \( \theta_{\text{peak}} = 1^\circ \)

The universe is flat:

\[ \Omega_{\text{Tot}} = [\theta_{\text{peak}}(\text{deg})]^{-1/2}. \]

\[ \Omega_{\text{Tot}} = 1 \]
Cosmic Microwave Background Radiation

The shape of the fluctuation spectrum changes with the density parameters. This is how we can measure them based on the CMB!

Fig. 8.24. Dependence of the CMB fluctuation spectrum on cosmological parameters. Plotted is the square root of the power per logarithmic interval in $\ell$, $\Delta_T = \sqrt{\ell(\ell+1)C_\ell/(2\pi)T_0}$. These power spectra were obtained from an accurate calculation, taking into account all the processes previously discussed in the framework of perturbation theory in General Relativity. In all cases, the reference model is defined by $\Omega_m + \Omega_\Lambda = 1$, $\Omega_\Lambda = 0.65$, $\Omega_b h^2 = 0.02$, $\Omega_m h^2 = 0.147$, and a slope in the primordial density fluctuation spectrum of $n_s = 1$, corresponding to the Harrison–Zeldovich spectrum. In each of the four panels, one of these parameters is varied, and the other three remain fixed. The various dependences are discussed in detail in the main text.
Large Scales: Sachs-Wolfe Effect (SWE)

The SWE arises as CMB photons travel through the evolving gravitational potential of the expanding universe.

1. Photon entering the potential well of a supercluster of galaxies
   - The photon starts at a certain energy.

2. Photon nearing the minimum point of the potential well of a supercluster of galaxies
   - As the photon travels through the potential well, it gains a little energy.
   - At the same time, dark energy causes the supercluster to expand, and the potential well loses some depth.

3. Photon exiting the potential well of a supercluster of galaxies
   - The supercluster continues to expand.
   - When the photon climbs out of the potential well, it loses some energy; however, the well is less deep, so the photon exits with more energy than it entered.
Large Scales: Sachs-Wolfe Effect (SWE)

The SWE dominates the power spectrum on very large scales.

First peak due to mode that just reaches max compression in valley/rarefaction on hill top for first time at $t_{\text{recomb}}$.

Second peak due to mode that just reaches maximal rarefaction in valley/compression on hill top for first time.

At troughs, temperature fluctuations are not zero due to motions of photon-baryon fluid $\Rightarrow$ Doppler effect.

Density perturbations due to gravity.

ISW: Integrated SWE

- Early ISW
- Primordial Doppler Effect
- Late ISW
(Photon) Diffusion Damping

Recombination is not instantaneous; rather the last scattering surface has finite thickness $d$.

Consequently, fluctuations on scales smaller than $d$ are “washed out” (as photons on those scales can travel between hot and cold regions, reducing their $\Delta T$ during the recombination process), causing damping on small angular scales.
Sunyaev-Zel’dovich Effect (SZE)

The photons of the CBR suffer Inverse Compton scattering against the hot electrons of the intracluster medium, preferentially gaining energy. The CMB spectrum gets shifted to higher frequencies: at wavelengths $<1.4$ mm the clusters appear as bright patches in the CMB.

To first order, the CMB distortion is proportional to the integral along the line of sight of the electron density times its thermal energy:

$$\frac{\Delta I}{I} = 2y, \quad y = \int n_e \sigma_T \frac{kT}{m_e c^2} dl$$

Carlstrom, Holder & Reese, 2002, ARAA, 40, 643
Sunyaev-Zel’dovich Effect (SZE)

SZE depends on \textit{cluster mass}, not \textit{redshift}

\(\Rightarrow\) Useful to find high-z galaxy clusters, where X-ray measurements of the cluster ICM are challenging due to surface brightness dimming \(\propto (1+z)^4\)

\(\Rightarrow\) Lack of \(z \gg 1\) detections due to lack of mature clusters with massive, hot X-ray ICM

For unresolved clusters, the strength of the \(SZ\) signal measures the thermal energy of the electrons:

\[ Y = \int ydA \approx \int n_e T \, dV \propto M_{\text{gas}} T \]
Sunyaev-Zel’dovich Effect (SZE)

Spectrum of SZE in RX J1347.5-1145

- SZE expected to cross “null” at \( \sim 217 \) GHz going from emission into absorption

Measurement:

\[ \nu_0 = 225.8 \pm 2.5 \text{ (stat.)} \pm 1.2 \text{ (sys.)} \text{ GHz} \]

⇒ Differs from “canonical” value due to ICM structure

“hole in the sky”
In addition, to the temperature fluctuations, we can learn about the polarization.
WMAP Polarization Maps

K band

Ka band

V band

Q band

W band

T(µK)

0

50

0

35
CMB polarization: Inflation

The explosive expansion during inflation created ripples for which should be evidence in the polarization of the last-scattered photons, i.e. the CMB.
Inflation and Gravity Waves - I

- Inflation predicts two forms of fluctuations:
  - Scalar modes (*density* perturbations) with slope $n_s$:
    - generate CMB anisotropy and lead to structure formation
  - Tensor modes (*gravity waves*) with slope $n_t$:
    - generate CMB anisotropy but do not contribute to structure formation
- Gravity wave amplitude, $r$, proportional to energy scale of inflation:

\[
r^{1/4} \propto \frac{V^{1/4}}{m_{pl}} = \frac{E_{\text{infl}}}{3.3 \times 10^{16} \text{ GeV}} \quad \text{with} \quad r = \frac{P(k_0)_{\text{tensor}}}{P(k_0)_{\text{scalar}}}
\]

- Both types of fluctuations contribute to *CMB temperature anisotropy*:

![Graphs showing TT anisotropy for different values of r, with observed and predicted signals for scalar and tensor modes.](image)
• Both types of fluctuations contribute to CMB polarization anisotropy:
  – Scalar modes produce only “E-mode” polarization patterns, by symmetry
  – Tensor modes produce both “E-mode” and “B-mode” polarization patterns (see below)
• The observation of B-mode polarization uniquely separates scalar and tensor modes from inflation and measures the energy scale of inflation.
• Only known probe of physics at $E \sim 10^{16}$ GeV… 12 orders of magnitude higher than planned accelerators.
WMAP has produced a visual demonstration that the *polarization pattern* around hot and cold spots follows the pattern expected in the *standard model*.

The standard model predicts a specific linked *pattern of temperature and polarization around hot and cold spots* in the map. WMAP now sees the predicted pattern in the map, as shown in the figure.

The hunt is on for *B-modes* from inflation… (e.g., BICEP-2, Planck)
CMB Lensing: Different B-modes

- Gravitational lensing distorts and amplifies CMB fluctuations
  - Lensing acts to smooth the temperature and E-polarization peaks
  - Subtle effect, only reaches 10% deep in damping tail
  - Statistically detectable, but measurement difficult due to secondary effects and foregrounds that dilute the signal
- Warping of the polarization field generates B-modes from E-modes at recombination (100 sq deg.)
- Provides integrated line-of-sight mass measure back to recombination
- Lensing signal detected at >25 sigma with Planck (2013 results, paper XVII)

*Hu & Okamoto (2001)*

*Seljak (1996)*
Observational constraints on dark energy

1. Four observational techniques dominate the White Papers received by the task force. In alphabetical order:
   a. **Baryon Acoustic Oscillations (BAO)** are observed in large-scale surveys of the spatial distribution of galaxies. The BAO technique is sensitive to dark energy through its effect on the **angular-diameter distance vs. redshift** relation and through its effect on the **time evolution of the expansion rate**.
   b. **Galaxy Cluster (CL)** surveys measure the spatial density and distribution of galaxy clusters. The CL technique is sensitive to dark energy through its effect on a combination of the **angular-diameter distance vs. redshift** relation, the **time evolution of the expansion rate**, and the growth rate of structure.
   c. **Supernova (SN)** surveys use Type Ia supernovae as standard candles to determine the **luminosity distance vs. redshift** relation. The SN technique is sensitive to dark energy through its effect on this relation.
   d. **Weak Lensing (WL)** surveys measure the distortion of background images due to the bending of light as it passes by galaxies or clusters of galaxies. The WL technique is sensitive to dark energy through its effect on the **angular-diameter distance vs. redshift** relation and the **growth rate of structure**.

Other techniques discussed in White Papers, such as using $\gamma$-ray bursts or gravitational waves from coalescing binaries as standard candles, merit further investigation. At this time, they have not yet been practically implemented, so it is difficult to predict how they might be part of a dark energy program. We do note that if dark energy dominance is a recent cosmological phenomenon, very high-redshift ($z \gg 1$) probes will be of limited utility.
Observational constraints on dark energy

If the dark energy density evolves as

$$\rho_{\text{DE}} \propto a^{-n}$$

then a DE-dominated universe obeys

$$a \propto t^{2/n}$$

which implies acceleration for

$$n < 2$$

(becomes super-linear)

But people usually use the “equation-of-state parameter”

$$w = \frac{\rho_{\text{DE}}}{\rho_{\text{DE}}} = \frac{1}{3}(n - 3)$$

so that acceleration happens for

$$w < \frac{1}{3}$$

The objective is to try to constrain $w(z)$ via a variety of independent techniques.
Acoustic Oscillations in the CMB

- Although there are fluctuations on all scales, there is a characteristic angular scale.
CMB temperature anisotropies provide a standard ruler.

They were produced about 380,000 years after the Big Bang, and should be most prominent at a physical size of 380,000 light years across.

\[ \Omega_{\text{Tot}} = [\theta_{\text{peak (deg)}}]^{-1/2}. \]

Observation: \( \theta_{\text{peak}} = 1^\circ \)

The universe is flat:

\[ \Omega_{\text{Tot}} = 1 \]
Sound Waves in the Early Universe

Before recombination:
- Universe is *ionized*.
- Photons provide enormous *pressure* and restoring force.
- Perturbations oscillate as *acoustic waves*.

After recombination:
- Universe is *neutral*.
- Photons can travel *freely* past the baryons.
- Phase of oscillation at $t_{\text{rec}}$ affects *late-time amplitude*.

Big Bang

<table>
<thead>
<tr>
<th>Ionized</th>
<th>Recombination $z \sim 1100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 380,000$ years</td>
<td></td>
</tr>
</tbody>
</table>

Today
Prediction of Baryon Acoustic Oscillations

• Acoustic signatures are formed by sound waves excited by perturbations in the early Universe
• At recombination, the sound speed decreases dramatically, ending the wave propagation
• The modes of oscillation are “frozen in”

\[ c_s = \frac{c}{\sqrt{3}} \sim 0.57c \]

Unlike the CMB acoustic oscillations, baryon oscillations reflect the velocity of the fluid at recombination rather than the density. The velocity at release from photon drag kinematically forms baryon over-densities.
Sound Waves

• Each initial overdensity (in DM & gas) is an overpressure that launches a spherical sound wave.

• This wave travels outwards at 57% of the speed of light.

• Pressure-providing photons decouple at recombination. CMB travels to us from these spheres.

• Sound speed plummets. Wave stalls at a radius of 150 Mpc.

• Overdensity in shell (gas) and in the original center (DM) both seed the formation of galaxies. Preferred separation of 150 Mpc.
Prediction of Baryon Acoustic Oscillations

Overdensities in dark matter and in the baryon shell both seed future galaxy formation.
Response of a point perturbation

Remember: this is a tiny ripple on a big background

Based on CMBfast outputs (Seljak & Zaldarriaga). Green’s function view from Bashinsky & Bertschinger 2001.
A Statistical Signal

• The Universe is a superposition of these shells.
• The shell is weaker than displayed.
• Hence, you do not expect to see “bulls-eyes” in the galaxy distribution.
• Instead, we get a 1% bump in the correlation function.
A Standard Ruler

- The acoustic oscillation scale depends on the sound speed and the propagation time.
  - These depend on the matter-to-radiation ratio ($\Omega_m h^2$) and the baryon-to-photon ratio ($\Omega_b h^2$).

- The CMB anisotropies measure these and fix the oscillation scale.

- In a redshift survey, we can measure this along and across the line of sight.

- Yields $H(z)$ and $D_A(z)$!
Galaxy Redshift Surveys

- Redshift surveys are a popular way to measure the 3-dimensional clustering of matter.
- But there are complications from:
  - Non-linear structure formation
  - Bias (light ≠ mass)
  - Redshift distortions
- Do these affect the acoustic signatures?
Introduction to SDSS LRGs

- SDSS uses color to target luminous, early-type galaxies at 0.2<z<0.5.
  - Fainter than MAIN (r<19.5)
  - About 15/sq deg
  - Excellent redshift success rate

- The sample is close to mass-limited at z<0.38.
  Number density ~ $10^{-4} \, h^3 \, \text{Mpc}^{-3}$.

Science Goals:
- Clustering on largest scales
- Galaxy clusters to z~0.5
- Evolution of massive galaxies
Large-Scale Correlations: A Prediction Confirmed!

- Standard inflationary CDM model requires acoustic peaks.
  - Important confirmation of basic prediction of the model.
- This demonstrates that structure grows from $z=1100$ to $z=0$ by linear theory.
  - Survival of narrow feature means no mode coupling.

Warning: Correlated Error Bars

Acoustic series in $P(k)$ becomes a single peak in $\xi(r)$!

Pure CDM model has no peak.