Buffered Atmospheres

Several atmospheres in the solar system are in equilibrium with solid or liquid phases at the surface. Examples:

<table>
<thead>
<tr>
<th>Io</th>
<th>SO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pluto</td>
<td>N₂</td>
</tr>
<tr>
<td>Mars</td>
<td>CO₂</td>
</tr>
<tr>
<td>Triton</td>
<td>N₂</td>
</tr>
<tr>
<td>Titan</td>
<td>CH₄ ?</td>
</tr>
</tbody>
</table>

The next few pages explore the thermal effect of latent heat buffering in these situations.

The influence on Mars climate is particularly interesting.
Buffered Atmosphere

Simple idealized model:

\[ m = \text{mass of atm} \]
\[ p = \frac{Mg}{4\pi r^2} \quad \text{surface pressure} \]
\[ T = \text{ice temperature} \]
\[ L = \text{latent heat of vaporization} \]

Neglect atm. heat capacity compared to latent heat.

Then

\[ L \frac{dm}{dt} = \left[ S(t) - cT^4 \right] A \]

\[ S = \text{incoming solar flux} \]

Require that \( p = p_a(T) \) saturation vapor pressure.

Use Clausius-Clapeyron relation \( \frac{dp}{dT} = \frac{L}{T R} \).

\[ \frac{dm}{dt} = \frac{4\pi r^2}{g} \frac{dp}{dT} \frac{dT}{dt} = \frac{4\pi r^2}{g} \frac{L}{R T} \frac{dT}{dt} \]

Call total area \( 4\pi r^2 = A_T \). Simplify

\[ \rightarrow \quad \left( \frac{L}{RT} \right) \frac{R}{g} \frac{dT}{dt} = \left[ S(t) - cT^4 \right] \frac{A}{A_T} \]

analogous to only polar ice heat capacity!

area counts...
Linearized model

How does the system respond to small changes in forcing?

(The linearized equation is easier to solve, easier to interpret.)

Consider can with

\[ S(t) = \bar{S} + S'(t), \quad S' \ll \bar{S} \]

Then can anticipate for \( T \) also

\[ T(t) = \bar{T} + T'(t), \quad T' \ll \bar{T} \]

\[ \bar{S} = \sigma \bar{T}^4 \]

\[ \frac{dT'}{dt} + \frac{T'}{t_B} = \frac{S'}{(\frac{L}{R})^2} \frac{A}{A_T} \]

where

\[ t_B = \frac{c_p p / g}{4 \sigma \bar{T}^3} \quad \frac{R} {c_p} \left( \frac{L}{RT} \right)^2 \frac{A_T}{A} \]

\[ t_R = \frac{1}{3} (20)^2 10 \text{ (for \( \sigma \) seasonal caps)} \]

\( \theta \) seasonal caps, \( t_B \approx 1000 \text{ days} \)
Extremes

a) Imposed $\frac{d}{dt} \gg \frac{1}{t_B}$

$$\Rightarrow \frac{dT'}{dt} \sim \frac{S'}{(\frac{L}{RT})^2 R^2 g} \frac{A}{A_T}$$

Consider harmonic forcing with angular frequency

$$\omega = \frac{2\pi}{P} \quad P = \text{period}.$$ 

Solutions have properties, for $wt_B \gg 1$:

i) Phase lag $\sim \frac{\pi}{2}$

ii) $\frac{T'}{T} \sim \frac{1}{\omega t_B} \frac{1}{4} \frac{S'}{S}$

For $\sigma$, if $t_B \sim 1000$ days,

$$wt_B \sim \frac{2\pi}{P} t_B \sim \frac{2\pi}{600} \cdot 1000 \sim 10$$.
b) Imposed $\frac{d}{dt} << \frac{1}{t_B}$

$$\rightarrow \frac{I'}{t_B} \sim \frac{S'}{(\frac{L}{RT})^2 R F J} \frac{A}{A_t}$$

This is really radiative balance. Solutions have property

i) No phase lag

ii) $\frac{I'}{I} \sim \frac{1}{4} \frac{S'}{S}$

In fully nonlinear formulation, solution is given by

$$\sigma T^4 = S(t)$$

Triton? $p = 1.7 \text{ Pa}$

$T = 38 \text{ K}$

$g = 0.8 \text{ m s}^2$

$t_B = \frac{c_p p / g}{4 \sigma \frac{1}{T^3}} \frac{R}{c_p} (\frac{L}{RT})^2 \frac{A_t}{A} \sim 1000 \text{ d.}$

$t_R = 2 \text{ d.} \frac{1}{3} (20)^2 3$
Neptune's orbital period \( \approx 165 \text{ yr} \), putting Triton in the category

\[
\frac{d}{dt} < < \frac{1}{t_B}
\]

\( \Rightarrow \) radiative equilibrium, no phase lags.
Figure 1: Atmospheric pressure statistics for VL-1 and VL-2 for the complete mission (Tillman, 1985, Zurek et al. 1992). Lower panel: Daily average pressure for Lander 1 (lower traces) and Lander 2 (upper traces) plotted on an annual scale beginning with the landing of VL-1, Ls=97. The difference in pressure between VL-1 and VL-2 is due to altitude. Upper panel: Standard deviation around the daily average pressure for Lander 1. Global (great) dust storms are shown in the upper panel, labeled according to year and sequence, 1977 A, 1977 B, 1982. Transient, hypothesized Kelvin modes, which seem to reappear almost every summer are also shown. Not shown are purported Kelvin modes during dust storm initiation, which are swamped by the storm buildup in this type of plot. Sols with time gaps > 6.0 hours are not plotted.

Figure 2: Spectral analysis of Martian pressure data covering the transient precursor to the 1982 global dust storm, Ls=205. Plot (a) shows the pressure data over an 8 sol segment, while the remaining plots are spectral estimates for these data. The spectral estimates in plot (b) were formed using an autoregressive model with parameters determined by the Yule-Walker method, Tillman 1988. Plot (c) used the same model, but with parameters determined by Burg’s algorithm which provides higher resolution. Plots (b) and (c) show the spectral range between 0.5 and 2.5 cycles/sol on a linear scale. Note the two distinct peaks in (c) near the diurnal and semi-diurnal frequencies (1 and 2 cycles/sol). The two spectral peaks occur immediately prior to and possibly during the buildup of the global dust storms as well as during the summer transient season Ls=150, Tillman and Percival, 1993.