Astronomy 6570

Physics of the Planets

Outer Planet Interiors
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>15 – 317 ( M_{\oplus} )</td>
</tr>
<tr>
<td>Radius</td>
<td>3.9 – 11.2 ( R_{\oplus} )</td>
</tr>
<tr>
<td>Density</td>
<td>0.69 – 1.67 ( \text{gcm}^{-3} )</td>
</tr>
<tr>
<td>Rotation period</td>
<td>9.9 – 18 hours</td>
</tr>
<tr>
<td>Obliq.</td>
<td>3° – 98°</td>
</tr>
<tr>
<td>Vis. Surf.</td>
<td>clouds; zonally banded (N?)</td>
</tr>
<tr>
<td></td>
<td>decreasing contrast: ( J \rightarrow S \rightarrow U )</td>
</tr>
<tr>
<td>Atmos. Comp.</td>
<td>( H_2 + He ) (roughly solar)</td>
</tr>
<tr>
<td></td>
<td>+ ( CH_4, NH_3, H_2O ) ... (enhanced)</td>
</tr>
<tr>
<td>Atmos. Struct.</td>
<td>adiabatic below ( \sim 1 ) bar</td>
</tr>
<tr>
<td></td>
<td>warm stratospheres</td>
</tr>
<tr>
<td>Energy output</td>
<td>( \sim 2 \times ) solar input (exc. U)</td>
</tr>
<tr>
<td>Atmos. Circ’n.</td>
<td>Zonal winds of 100 – 400 ms</td>
</tr>
<tr>
<td>Mag. Field</td>
<td>0.14 – 4.2 Gauss</td>
</tr>
<tr>
<td></td>
<td>tilt = 0 – 59°</td>
</tr>
<tr>
<td>Satellites</td>
<td>inner, regular sats (( e \sim i \sim 0 ))</td>
</tr>
<tr>
<td></td>
<td>outer, irreg. sats</td>
</tr>
<tr>
<td>Ring system</td>
<td>increasing mass: ( J \rightarrow N \rightarrow U \rightarrow S )</td>
</tr>
<tr>
<td></td>
<td>assoc. with small satellites</td>
</tr>
<tr>
<td></td>
<td>ephemeral structures?</td>
</tr>
</tbody>
</table>
Temperature profiles are assumed to be convective (i.e., adiabatic) below the visible clouds, as indicated by the dashed curves.

Warm stratospheres are due to near-IR absorption by CH$_4$. 
Planetary insolation patterns

- small obliquity (J)
- large obliquity (U)
Emitted infrared flux and equivalent brightness temperatures versus latitude for the four outer planets. The radiation is emitted, on average, from the 0.3 to 0.5 bar pressure levels. The equator-to-pole temperature differences are small. The largest temperature gradients occur at the extrema of the zonal velocity profile. (Ingersoll, 1990)
Zonal wind profiles (Voyager data)
Fig. 1. Saturn's rotation periods. Cloud tracking of atmospheric features, as measured during the Voyager 1 and 2 encounters in 1980 and 1981, are represented by a dark continuous line (15). Violet crosses mark points measured from 1994 to 2004 with the Hubble Space Telescope (9). Yellow triangles are the equatorial data measured by Cassini (10). Outside the equator, the Voyager, Hubble, and Cassini data essentially coincide. The dots indicate short radio emission bursts that have been assigned to lightning produced by storms [yellow dot, Voyager (16); red dot, Cassini (17)].

Internal circulation models: Saturn

Fig. 12. Columnar convection cells (a) and cyclindrical zonal flow (b). As shown by Busse (1976), the columnar mode is the preferred form of convective instability in a uniformly rotating, viscous, conducting fluid. The cylindrical mode is the most general form of steady zonal motion in an inviscid adiabatic fluid. The interaction of these two modes is analogous to the behavior of transverse convective disturbances in a sheared horizontal layer, according to Ingersoll and Pollard (1982).
Magnetic field comparison:

**Obliquity**
\[
\cos^{-1}(\Omega \cdot \mathbf{N}) = \begin{array}{cc}
23.5^\circ & 3.1^\circ \\
26.7^\circ & 97.9^\circ \\
29.6^\circ & 
\end{array}
\]

**Dipole Tilt**
\[
\cos^{-1}(\Omega \cdot \mathbf{M}) = \begin{array}{cc}
+11.7^\circ & -9.6^\circ \\
-0.0^\circ & -59^\circ \\
-47^\circ & 
\end{array}
\]

**Solar Wind Angle**
\[
\cos^{-1}(\Omega \cdot \mathbf{R}) = \begin{array}{cc}
66.5 - 113.5^\circ & 86.9 - 93.1^\circ \\
63.7 - 116.5^\circ & 7.9 - 172.1^\circ \\
60.4 - 119.6^\circ & 
\end{array}
\]

*Figure 2* Orientations of the planets and their magnetic fields.
# Comparison of planetary magnetic fields

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Jupiter(^a)</th>
<th>Saturn(^a)</th>
<th>Uranus(^a)</th>
<th>Neptune(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius, (R_{\text{planet}}) (km)</td>
<td>6,373</td>
<td>71,398</td>
<td>60,330</td>
<td>25,559</td>
<td>24,764</td>
</tr>
<tr>
<td>Spin Period (Hours)</td>
<td>24</td>
<td>9.9</td>
<td>10.7</td>
<td>17.2</td>
<td>16.1</td>
</tr>
<tr>
<td>Magnetic Moment/(M_{\text{Earth}})</td>
<td>1(^b)</td>
<td>20,000</td>
<td>600</td>
<td>50</td>
<td>25</td>
</tr>
</tbody>
</table>

## Surface Magnetic Field (Gauss)

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Dipole Equator, (B_0)</td>
<td>0.31</td>
<td>4.2</td>
<td>0.22</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.24</td>
<td>3.2</td>
<td>0.18</td>
<td>0.08</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.68</td>
<td>14.3</td>
<td>0.84</td>
<td>0.96</td>
<td>0.9</td>
</tr>
<tr>
<td>Dipole Tilt and Sense(^c)</td>
<td>+11.3°</td>
<td>-9.6°</td>
<td>-0.0°</td>
<td>-59°</td>
<td>-47°</td>
</tr>
</tbody>
</table>

<p>| | | | | | |</p>
<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>Distance (A.U.)</td>
<td>1(^d)</td>
<td>5.2</td>
<td>9.5</td>
<td>19</td>
<td>30</td>
</tr>
<tr>
<td>Solar Wind Density (cm(^{-3}))</td>
<td>10</td>
<td>0.4</td>
<td>0.1</td>
<td>0.03</td>
<td>0.005</td>
</tr>
<tr>
<td>(R_{\text{CF}})</td>
<td>8 (R_E)</td>
<td>30 (R_J)</td>
<td>14 (R_S)</td>
<td>18 (R_U)</td>
<td>18 (R_N)</td>
</tr>
<tr>
<td>Size of Magnetosphere</td>
<td>11 (R_E)</td>
<td>50-100 (R_J)</td>
<td>16-22 (R_S)</td>
<td>18 (R_U)</td>
<td>23-26 (R_N)</td>
</tr>
</tbody>
</table>


\(^b\) \(M_{\text{Earth}} = 7.96 \times 10^{25}\) Gauss cm\(^3\) = 7.906 \times 10^{15}\) Tesla m\(^3\).

\(^c\) Note: Earth has a magnetic field of opposite polarity to those of the giant planets.

\(^d\) 1 A.U. = 1.5 \times 10^8\) km.
Jupiter’s UV aurora from HST.
(Note offset from N pole & Io “hot spot”.)
Saturn’s UV aurora from HST. (Note alignment with N pole.)
Planetary Interior Models (general considerations)

Assume spherical symmetry (for simplicity only!)

1. Hydrostatic equilib.: \[ \frac{dP}{dr} = -\rho g = -\frac{Gm(r)\rho}{r^2} \]

2. Mass conservation: \[ \frac{dm}{dr} = 4\pi r^2 \rho \]

3. Equation of state: \[ P = f(\rho, T; \text{composition}) \]

4. Heat transfer:
   \[ \begin{cases} k \frac{dT}{dr} = F \ldots \text{conduction} \\ \frac{dT}{dr} = \left( \frac{\partial T}{\partial P} \right)_S \frac{dP}{dr} \ldots \text{convection} \end{cases} \]

- in approximate treatments (4) may be dropped and (3) replaced by \[ P = f(\rho) \]

3 first-order D.E.'s \( \Rightarrow \) **3 boundary conditions**

1. \( m(0) = 0 \)

2. \( P(R) = 0 \)

3. \( T(R) = \begin{cases} T_{\text{surf}} \text{ (solid surface)} \\ T_{\text{eff}} \text{ (jovian planets)} \end{cases} \)

Adjust model parameters (e.g., composition, \( M_{\text{core}} \), etc.) to fit observables:

\( M, R, J_2 \left( \sim \frac{c}{MR^2} \right), J_4, \text{ etc.} \)

Model \( \Rightarrow \rho(r), P(r), g(r), T(r) \)
Central pressures:

\[ \frac{dP}{dr} = - \frac{Gm\rho}{r^2} \]

\[ \Rightarrow \text{roughly} \quad \frac{P_c - P_0}{R} \approx \frac{4\pi}{3} \frac{GM^2}{R^5} \]

\[ \therefore \quad P_c \approx \frac{GM^2}{R^4} \]

Case I: a uniform-density planet:

\[ m(r) = \frac{4\pi}{3} \rho r^3 \Rightarrow \]

\[ g = \frac{4\pi}{3} G\rho r \Rightarrow \]

\[ \frac{dP}{dr} = - \left( \frac{4\pi}{3} \right) G\rho^2 r \]

\[ \therefore \quad P(r) = P_c - \frac{2\pi}{3} G\rho^2 r^2 \]

Boundary condition \[ \Rightarrow P_c = \frac{2\pi}{3} G\rho^2 R^2 = \frac{3}{8\pi} \frac{GM^2}{R^4} \]
Adiabatic equations of state for principal constituents:

HYDROGEN

H-He

P $\propto \rho^2$

PRESSURE (Pa)

DENSITY (g/cm$^3$)

10^13

10^12

10^11

10^10

10^9

0.1

1

10

1 Mb
Case II:

\[ P = \mathbb{C} \rho^2 \quad (n = 1 \text{ polystrope} \sim \text{Jupiter}) \]

\[ \Rightarrow P_c = \frac{2\pi^3}{9} G \rho^2 R^2 \quad \text{(see Polytrope notes)} \]

### Examples

<table>
<thead>
<tr>
<th>Body</th>
<th>( \rho ) (g cm(^{-3}))</th>
<th>R (km)</th>
<th>Case I (Mb = 10(^{11}) Nm(^{-2}))</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>3.34</td>
<td>1738</td>
<td>0.047</td>
<td>0.155</td>
</tr>
<tr>
<td>Earth</td>
<td>5.52</td>
<td>6371</td>
<td>1.73</td>
<td>5.68</td>
</tr>
<tr>
<td>Uranus</td>
<td>1.32</td>
<td>25,600</td>
<td>1.60</td>
<td>5.25</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1.33</td>
<td>71,400</td>
<td>12.6</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Detailed models:  Earth = 3.5 Mb
Jupiter = 70 Mb
Uranus = 8 Mb
“Cold” & adiabatic planet models with pure compositions.

- Jupiter & Saturn are near the curve for 75% H, 25% He (by mass).
- Uranus & Neptune fall between pure H/He models and a mixture of heavier elements denoted as “ice” (CH$_4$, NH$_3$ & H$_2$O).

Stevenson (c 1977).
Brown Dwarf Gliese 229B has a mass ~ 40M\(_J\) but about the same radius as Saturn!

He, C & Mg/Fe white dwarfs have radii ~0.015 \(R_{\odot}\), or ~2 Earth radii!
Figure 6.26  (a) Models of the interior structure of Jupiter and Saturn, assuming a fully convective hydrogen–helium envelope (adiabatic models). (Adapted from Guillot et al. 1995) (b) Models of the density as a function of normalized radius for Jupiter and Saturn. A helium deficit in the molecular hydrogen regions, together with enrichment in the metallic hydrogen region, causes the small density change in Saturn at 0.55 \( R_h \). For Jupiter, models with and without a core are shown. (Adapted from Marley and Fortney 2007)
Constraints on heavy element content in Jupiter & Saturn.

(Guillot)

Note that these models predate the acquisition of gravity data by Juno & Cassini!
H/He immiscibility region (SS physics calculation, Morales et al 2013) + Saturn profile from Cassini.
Saturn interior models

less, et al. (2019)
Jupiter & Saturn gravity fields: Iess et al. (2017, 2018)
Uranus & Neptune models

Figure 6.27 (a) Schematic representation of the interiors of Uranus and Neptune. (After Stevenson 1982) (b) Models of the density as a function of normalized radius for three Neptune and one Uranus interior models. The solid, dashed and dot-dashed curves represent the range in possible Neptune models, where the core could be absent or extend out to a fractional radius of 20%. The dotted curve represents a single Uranus model. Because of Neptune’s larger mass, it is denser than Uranus at each fractional radius. The inset shows the region of transition from a hydrogen-rich atmosphere to the icy mantle in more detail. (Marley and Fortney 2007)