Dynamics of Planetary Rings

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Self-Gravity Wakes

• Particles try to accrete, while tides disrupt

\[ Q = \frac{c_r \kappa}{3.36 G \sigma} \approx 2 \]

• Characteristic wavelength:

\[ \lambda_{cr} = \frac{4\pi^2 G \sigma}{\kappa^2} \approx 70 \text{ m in the A ring} \]

Self-Gravity Wakes

- Inputs: Particle size distribution, coeff. of restitution, etc.
- Outputs compared to photometry, varying with viewing geometry

Self-Gravity Wakes

- Density estimation shows that SGWs are an intimate mixture of high- and low-density regions
- Some moderate-density (SGWs recently disrupted)
Self-Gravity Wakes

• Density estimation work predicts that SGWs have a density distribution that is tri-modal \( (\text{Tiscareno et al 2010, AJ}) \)

• Opaque mixed with transparent, complex photometry

• Recent stellar occultation confirms tri-modal \( (\text{Sremčević et al 2009, AGU}) \)

• Can hide a lot of material in those wakes!

• Even at 500 g/cm\(^2\), simulations did not match observed \( \tau \) for B ring \( (\text{Robbins et al 2010, Icarus}) \)
Viscous Overstability

- Unexplained structure abounds in B ring and inner A
- Some structure has these characteristics:
  - Nearly monochromatic wavelength $\lambda \approx 100$ meters
  - Nearly perfect azimuthal alignment
- Basic idea of VO: If viscosity increases sharply with density, overly strong restoring force for small perturbations may overshoot equilibrium
  
Viscous Overstability

- Unexplained structure abounds in B ring and inner A
- Some structure has these characteristics:
  (Thomson et al 2007, GRL)
  - Nearly monochromatic wavelength $\lambda \approx 100$ meters
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- Basic idea of VO: If viscosity increases sharply with density, overly strong restoring force for small perturbations may overshoot equilibrium
Resonances

• When two frequencies are related by a ratio of integers, tugs from one object onto another add up coherently.

• In general, resonances between two frequencies preserve a quantity called the resonant argument.
2:1 Resonance

System: Europa and Io, moons orbiting the planet Jupiter
4:1 Resonance

System: Ganymede and Io, moons orbiting the planet Jupiter
4:3 Resonance

http://www.youtube.com/watch?v=tJXZ05Hphqg

System: Hyperion and Titan, moons orbiting the planet Saturn
Resonances

• Resonances are split by planet’s sizable $J_2$

• Lindblad Resonances pump up the eccentricity of ring particles
  – The $m:(m-1)$ LR has an argument of the form
    \[ \varphi_{LR} = m\lambda' - (m - 1)\lambda - \varpi \]
    where $\lambda$ is longitude and $\varpi$ is direction of pericenter
  – Result in spiral density waves that distribute angular momentum through the disk

• Co-rotation Resonances use perturber’s $e$ to confine the longitude of ring particles
  – The $m:(m-1)$ CER has an argument of the form
    \[ \varphi_{CR} = m\lambda' - (m - 1)\lambda - \varpi' \]
Ring Arcs

- Not just at Neptune anymore!
- Saturn’s G ring, and others
  - Moon(s) embedded in arc
  - Confined by exterior perturber
  - Counter-example: Pallene
- Jupiter’s Main ring
  - Like at Neptune, observed spatial frequencies don’t match CR sites
- Common factors
  - Dust
  - Co-rotation resonances
Ring Arcs and CERs

In the frame of the arc’s mean motion, Mimas moves in and out due to its eccentricity six times during each synodic period with the arc, so Mimas moves in a six-lobed orbit:

\[ 6\lambda_{arc} - 7\lambda_{Mimas} + \varpi_{Mimas} = \pi \]

Arc material librates around one of six stable points

Orbit of Mimas (eccentricity exaggerated)
Neptune’s Ring Arcs

- Porco (1991) proposed that Neptune’s ring arcs are confined within the 42:43 corotation inclination resonance (CIR) with Galatea
- Adjacent stable sites contain material, while others are empty
- Nearby outer Lindblad resonance (OLR) pumps up particle eccentricities to keep them in the resonance despite collisions

Figure from Murray and Dermott (1999)
Neptune’s Ring Arcs

- Refinement of the arcs’ orbits from HST (1999) and Keck (2005) cast doubt on Porco’s original theory.
- Namouni and Porco (2002) used the CER instead, with arc’s mass now a free parameter.

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Dumas et al (1999)
Narrow Ring Dynamics

• A unique assemblage of matter, behaving as a single self-contained object yet ephemerally thin
• How and why they form is not known
• Theory: Eccentricity gradient compensates for differential precession, allows ring to precess as if rigid (Borderies, Goldreich, and Tremaine 1983, AJ)
  – Confirmation and detailed understanding still needed
• Shepherd moons, when present, may be more perturbers than shepherds
Narrow Ring Dynamics

- Normal modes detected
  - Occultations precisely measure edge radii
  - Azimuthal variations in radial width are seen in images as variations in brightness
- Several modes were detected by occultations and Voyager
- Azimuthal structure could be due to
  - Shepherding and/or perturbing moons
  - Self-sustaining eccentricity gradients
- Some narrow rings and edges at Saturn have been found to exhibit a handful of modes (Spitale et al)
Gap Edges

- Moon gives passing ring particles an eccentricity, resulting in wavy gap edges.
- It follows from Kepler’s 3rd Law that $\lambda = 3\pi \Delta a$.
- $\lambda$ increases with $\Delta a$, forming “moonlet wakes” that penetrate into the ring (Showalter et al. 1986, Icarus).
- Expect smooth sinusoidal edges, amplitude proportional to the mass of the moon, then decays as streamlines cross.
• Wavy edges persist until next Pan encounter (~1000 orbits)
• Amplitude and wavelength are well-behaved ($3\pi \Delta a$) immediately after encounter, but then complex
Spiral Density Waves

- Dominate fabric of the outer part of the ring system
- Excited by LRs and VRs
- Ring properties (density, viscosity) can be derived from these waves
Spiral Density Waves

- At an $m:(m-1)$ Lindblad resonance, perturbing moon raises an $m$-lobed pattern, which is stationary in the moon’s reference frame.
- Compression and rarefaction at resonance site excites a density wave that propagates outward.
- The wave carries angular momentum away from the resonance site, facilitating a disk-mass exchange.

\[
m = 1 \quad m = 2 \quad m = 3 \quad m = 4
\]
• In standard linear theory, five variables parameterize the wave \( (Goldreich \& Tremaine 1982, Shu 1984)\)
  
  – \( \sigma_0 \): Surface density, governs wavelength dispersion
  
  – \( r_L \): Resonance location, translation in the radial direction
  
  – \( \xi_{SD} \): Damping parameter, related to ring viscosity
  
  – \( A_L \): Amplitude, proportional to moon’s mass
  
  – \( \phi_0 \): Initial phase, set by local longitude relative to moon
Spiral Waves as Scientific Instruments

- Wavelet analysis (spatially-resolved power spectrum) helps to extract wave parameters from radial profile
- Wavenumber $k \approx (r-r_{res})/\sigma$ (may decrease)

*Tiscareno et al (2007, Icarus)*
Spiral Waves as Scientific Instruments

- Sensitivity of the wavelet allows easy detection of features not apparent to the unaided eye
- Waves superposed upon one another are picked out and separated in frequency space

*Tiscareno et al 2007, Icarus*
Spiral Density Waves

- **Non-linearity**: In strong waves, density perturbations saturate the background (Longaretti and Borderies 1986, Icarus)
  - Peaks sharpen, troughs flatten
  - Wavelength dispersion deviates from linear theory
- Nearly all *Voyager* waves are strong enough to be non-linear
- Weak waves, seen by Cassini, give disk parameters with better fidelity
Spiral Density Waves

- Surface density $\sigma$ peaks in mid-A Ring
- Dividing optical depth by $\sigma$ gives mass extinction
  - Implies smaller particles in Cassini Division
- Viscosity places upper limit on vertical thickness
  - Meaningful in Cassini Division (few m) and inner A Ring (10-15 m)

Tiscareno et al. (2007, Icarus)
Colwell et al. 2009, Icarus
Tiscareno et al. (2013, Icarus)
The Curious Iapetus -1:0 Bending Wave

Tiscareno et al. (2013, Icarus)

Tiscareno et al. 2013, Icarus
• SDWs are generally thought of as a static phenomenon
• But group velocity $v_g = \frac{\pi G \sigma_0}{\kappa}$ is of order km/yr
• Spiral waves propagate for tens of km, so any information would take many years to travel through

• *Density waves as dynamic records of history*
Janus and Epimetheus

- Co-orbital moons “swap” orbits every 4 years
- How do rings respond to change in forcing?
J/E 2:1 Density Wave

- Very strong wave
- Clearly non-linear, makes detailed analysis difficult
- Unmistakable in the middle of the wave is a “stutter step”
- Record of a previous J/E reversal?
J/E 4:3 Density Wave

- Two “stutter steps”, separated by 65 km
- If we assume 4-yr libration interval,
  - group velocity
    ~ 16 km/yr
  - surface density
    ~ 34 g/cm²
- Agrees with nearby surface density values from wavelet analysis
J/E 9:7 Density Wave

- Best-resolved example of a second-order Janus/Epimetheus density wave
- Because second-order waves are weaker, dispersion remains linear, so can linearly superpose wave segments
- We will use it as an archetype as we construct our model
J/E Wave Model

- **Step 1:** J and E waves propagate in one configuration for four years

- **Step 2:** J/E reversal
  - Old waves continue to propagate, but are “headless”
  - New waves are generated in new locations

- **Step 3:** Another reversal
  - Multiple “headless” waves
  - New waves now generated in original locations

*Tiscareno et al 2006, ApJL*
J/E 9:7 Density Wave

• The unusual morphology is largely explained!
• Peak locations generally fit better than amplitude
• Main section is due to interference between old and new Janus waves
• Out front is wave currently generated by Epimetheus
• Downstream, older Epimetheus wave appears and interferes

J/E 9:7 Density Wave

- May 2005 data, nearly a year after SOI
- Much of the pre-reversal wave has damped away
- But remnants can still be seen downstream, both in model and in data
- This model is clearly a first-order solution, but quantitative fits may require departure from linear theory

*Tiscareno et al 2006, ApJL*
“Propellers”

• Moonlets embedded in the disk
• We see “propeller-shaped” disturbance around moonlet
• First time ever tracking the orbit of an object orbiting within a disk, not in empty space
• Best-observed example (“Bleriot”), seen for 5 yr, shows subtle changes in its orbit
  – Is it interacting with the disk? Perhaps implications for Type I?
Non-Keplerian Orbital Motion

- What is the nature of the changes in Blériot’s orbit?
- **Resonant Libration?**
  - \( \lambda(t) \) would be sinusoidal
  - Corotation resonance? (M. Sremčević, pers. comm., 2011)
- **Episodic Constant Drift?**
  - \( \lambda(t) \) would be piecewise quadratic
  - Plausible (Kirsh et al. 2009, Icarus), needs more study
- **Random walk?**
- **Modified “Type I” Migration?**
  - Powered by radial surface density variation
  - \( \lambda(t) \) would be exponential

*Tiscareno et al. 2010, ApJL
Tiscareno 2013, P&SS*
“Type I” Migration

- Must re-derive for particle disk by tracing streamlines
- Torque is still asymmetric, due to orbit curvature
- However, the resulting migration rate is too small (≈0.2 m/yr), and is only in the inward direction
Random Walk

- Torque fluctuations due to self-gravity wakes and/or collisions could lead to a continual random walk in $a$
- Variation in longitude could be surprisingly large
- No preferred period, thus any period is possible
- Prediction: Current periodic trends will not hold

*Crida et al. 2010, AJ
Rein and Papaloizou 2010, A&A
H. Rein, personal communication, March 2011*
Modified “Type I” Migration

- Moonlet encounter with a particularly large ring particle or massive clump can “kick” semimajor axis suddenly (Lewis and Stewart 2009, Icarus)
- Radial variations in ring surface density, so must convolve torque
- Locations of stable equilibrium are possible ($\Delta \lambda \propto e^{At}$)
- Prediction: Episodic profile, with longitude trending exponentially back towards equilibrium value