Thermal emission from surfaces

To estimate the thermal emission from a planetary surface, we must consider the sub-surface temperature profile $T(z)$.

Below the surface, heat transport occurs primarily by conduction:

$$\rho c \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{F} = -\frac{\partial F}{\partial z} \tag{87}$$

and:

$$F = -k \frac{\partial T}{\partial z} \tag{88}$$

with:

- $\rho$ the density
- $c$ the heat capacity
- $F$ the thermal flux
- $k$ the conductivity

This gives the diffusion equation:

$$\frac{\rho c \partial T}{k \partial t} = \frac{\partial^2 T}{\partial z^2} \tag{89}$$
Describe the temperature of the surface as:

\[ T(0, t) \equiv T_s(t) = A' + \sum_{n=1}^{\infty} T_n Re \left[ e^{i\omega t + i\phi_n} \right] \quad (90) \]

\( \omega \) is e.g. 2\( \pi \)/day or 2\( \pi \)/year
\( \phi_n \) is a phase shift

A solution of Equation 89 can then be written as a Fourier series:

\[ T(z, t) = A + B(z) + \sum_{n=1}^{\infty} T_n Re \left[ e^{\lambda_n z} e^{i\omega t + i\phi_n} \right] \quad (91) \]
\( \lambda_n \):
Solve the diffusion equation per Fourier term:
\[
\frac{\rho c \partial T_n(z, t)}{k} \frac{\partial T_n(z, t)}{\partial t} = \frac{\partial^2 T_n(z, t)}{\partial z^2}
\]  
(92)
This leads to:
\[
\lambda_n = \pm \left( \frac{n \omega \rho c}{2k} \right)^{\frac{1}{2}} (i + 1) = \pm \gamma_n (i + 1)
\]  
(93)
The physical solution is: \( \lambda_n = + \gamma_n (i + 1) \)

\( B(z) \):
Use Equation 88 and the boundary condition:
as \( z \to -\infty \), \( F \to F_{int} \)
\[
F_z = -k \frac{\partial T}{\partial z}
\]
\( F_{int} = \) internal flux
This leads to:
\[
B(z) = -\frac{F_{int}}{k} z
\]  
(94)
$A + B(z)$ and $T_1$ graphically:

\[ \phi_n = \frac{H}{z} \]

The characteristic damping length or the thermal skin depth is:

\[ l_n = \gamma_n^{-1} = \left( \frac{2k}{n\omega \rho c} \right)^\frac{1}{2} \]  

(95)

The wavelength is:

\[ \lambda_n = 2\pi \left( \frac{2k}{n\omega \rho c} \right)^\frac{1}{2} = 2\pi l_n \]  

(96)

The phase-lag between the temperature at the surface and that at a depth $z$ is:

\[ \Delta t(z) = \left( \frac{\rho c}{2kn\omega} \right)^\frac{1}{2} z \]  

(97)
The flux at the surface is:  \[ F = -k \frac{dT}{dz} \]

\[ F(0, t) = F_{\text{int}} - \sum_{n=1}^{\infty} (n\omega k \rho c)^{\frac{1}{2}} T_n \cos \left( n\omega t + \phi_n + \frac{\pi}{4} \right) \]

\[ \equiv \sigma T_s^4(t) - F_{\text{inc}}(t) \quad \text{(98)} \]

with \( F_{\text{inc}} \) the flux that is incident on the surface.

The surface temperature variation depends on the *thermal inertia*: \( (k \rho c)^{\frac{1}{2}} \).
### THERMAL PARAMETERS

<table>
<thead>
<tr>
<th>Substance</th>
<th>$\rho$ (g cm$^{-3}$)</th>
<th>$\alpha$ (cm$^2$ s$^{-1}$ K$^{-1}$)</th>
<th>$\kappa$ (cm$^2$ s$^{-1}$ K$^{-1}$)</th>
<th>$(\kappa \rho \alpha)^{1/2}$ (cm$^2$ s$^{-1/2}$ K$^{-1}$)</th>
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<tbody>
<tr>
<td>copper</td>
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<td>0.0914</td>
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<td>granite</td>
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<td>0.006</td>
<td>0.06</td>
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<tr>
<td>concrete</td>
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<td>0.23</td>
<td>0.0022</td>
<td>0.034</td>
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<tr>
<td>ice</td>
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<td>0.50</td>
<td>0.0053</td>
<td>0.049</td>
</tr>
<tr>
<td>dry soil</td>
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<td>0.00063</td>
<td>0.014</td>
</tr>
<tr>
<td>water</td>
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<td>1.0</td>
<td>0.00144</td>
<td>0.038</td>
</tr>
<tr>
<td>snow (fresh)</td>
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<td>0.5</td>
<td>0.00025</td>
<td>0.0035</td>
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<tr>
<td>lunar soil</td>
<td>* 1.0</td>
<td>0.15</td>
<td>$7 \times 10^{-6}$</td>
<td>0.0010</td>
</tr>
</tbody>
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### Skin Depth

$$\text{skin depth} = \left( \frac{2\kappa}{\rho \alpha} \right)^{1/2}$$

<table>
<thead>
<tr>
<th></th>
<th>granite</th>
<th>dry soil</th>
<th>lunar soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24$ hrs</td>
<td>0.17 m</td>
<td>0.07 m</td>
<td>0.012 m</td>
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<tr>
<td>$29$ days</td>
<td>0.94 m</td>
<td>0.40 m</td>
<td>0.06 m</td>
</tr>
<tr>
<td>$365$ days</td>
<td>3.3 m</td>
<td>1.4 m</td>
<td>0.22 m</td>
</tr>
</tbody>
</table>

**Note:** 1 cal = 4.184 joules.
Fig. 4. The dependence of diurnal temperatures on thermal inertia ($I$), calculated for the equator of the time of Viking landing. The albedo for this set is 0.30. The average value of $I$ based on Mariner 9 is 0.006, and that for unweathered granite is about 0.055. Hour angle is measured from local midnight.

Fig. 2. Diurnal temperature curves at the Viking landing season for a nominal homogeneous model of the Martian surface (bolometric albedo = 0.25, effective emissivity = 0.85, and thermal inertia = 0.006 cal cm$^{-2}$ sec$^{-1/2}$ deg$^{-1/2}$). Each curve is labeled by latitude. Hour angle is measured from noon.