The Sun - Size and Mass

Masses
\[ M_e = 2.0 \times 10^{30} \text{ kg} \]
\[ M_\oplus = 1.9 \times 10^{27} \text{ kg} \]
\[ M_\odot = 6.0 \times 10^{24} \text{ kg} \]

Note: \[ \frac{M_e}{M_\oplus} : 1000 \quad \frac{M_\odot}{M_\oplus} : 300 \]

Most of solar system mass is in \( \odot \)

Radii
\[ R_e = 696,000 \text{ km} \]
\[ R_\oplus = 71,500 \text{ km} \]
\[ R_\odot = 6,400 \text{ km} \]

\[ \frac{R_e}{R_\oplus} : 10 \]
\[ \frac{R_\odot}{R_e} : 10 \]

Densities
\[ \rho = \frac{M}{\frac{4}{3} \pi R^3} \]
\[ \rho_e = 1.4 \text{ g cm}^{-3} \]
\[ \rho_\oplus = 1.3 \text{ g cm}^{-3} \]
\[ \rho_\odot = 5.5 \text{ g cm}^{-3} \]
Figure 1.1 Solar spectrum and blackbody curve: energy distribution of the Sun and a black body at 5800 K
2.8 Photographic image of the visible solar spectrum obtained with a large grating spectrograph. Wavelengths are given in Angstrom units at the top of the spectrum. The lines labeled B, C, D (shown here as D₁ and D₂), E, F, G, H, K are Fraunhofer’s original description (see text). Also labeled are the hydrogen Balmer lines Hα (Fraunhofer’s C line), Hβ, Hγ, and Hδ. (Courtesy Mt. Wilson Observatory, Carnegie Institution of Washington.)
### TABLE 2.1

Cosmic Abundances of the Elements (in Number of Atoms Relative to Si)

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>Abundance</th>
<th>Element</th>
<th>Z</th>
<th>Abundance</th>
<th>Element</th>
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</table>
Fig. 2 High resolution tracings $\lambda\lambda 4665-4676$ Å of the solar spectrum. The upper tracing refers to disc center; the lower to a point close to the limb at solar radius 98% of $R$. 
Figure 4. Concentrations of 69 chemical elements in Allende, plotted against their concentrations in the solar atmosphere (using logarithmic axes). In both cases, abundances are relative to 1 million silicon atoms.
The missing elements form gases

At inner solar system temperatures:

\[
\begin{align*}
O + 2H & \rightarrow H_2O \\
C + 4H & \rightarrow CH_4 \\
N + 3H & \rightarrow NH_3
\end{align*}
\]

Swept away by solar winds

Remaining: minerals, metals - rocks & grains

Rotation thin disk, accumulation planets

edge-on view of orbiting grains
The Sun as a Star

Is the Sun a typical star? What characterizes “typical” stars?

Possible observations:
  Brightness (energy output rate)
  Mass
  Surface temperature (color)
  Composition (elements in atmosphere)
  Rotation rate

Easiest to measure:
  Brightness (chief difficulty is distance correction)
  Surface temperature

Procedure:
  Measure $L = \text{luminosity} = \text{energy/time}$
  Measure $T_{\text{emission}}$ (from blue/red ratio)
  Make scatter plot $L$ vs. $T_e$, for lots of stars
  Look for patterns in the scatter plot

Such a scatter plot is called a *Hertzsprung-Russell diagram*.

Comment: It is best to do this for star clusters because the stars are then all at the same distance.
FIG. 6-7 (Above) A Hertzsprung–Russell diagram compiled in 1936, containing all stars for which spectral class and luminosity were known at the time. The striped appearance of the plot is a consequence of the finite number of spectral classes and subclasses that have been defined; actually there is a smooth distribution of stars with surface temperature. Figure by W. Gyllenberg, Lund Observatory, Sweden.

FIG. 6-8 (Right) Names applied to principal groupings of stars in Fig. 6-7. Numbers refer to the masses of individual stars in the main sequence relative to the mass of the sun.
H-R diagram, showing the locations of the brightest stars.

Figure 8.15 Luminosity classes on the H-R diagram. (Figure from Kaler, *Stars and Stellar Spectra*, © Cambridge University Press 1989. Reprinted with the permission of Cambridge University Press.)
H-R diagram for a typical Open Cluster.

= Sun

H-R diagram for a typical Globular Cluster.
Hertzsprung-Russell Diagram

Stars are strung out along the main sequence according to their masses. Small mass \( \text{→} \) low luminosity, low surface temperature

Sun is typical mid-life MS star, but a bit smaller than average.

Principal other groupings in the H. R. diagram:

- Red giants: large stars, recently left MS, He cores
- Horizontal branch: post-MS stars burning helium.
- White dwarfs: small old stars, depleted in hydrogen

Summary: Mass and age are principal factors determining the properties of a star. Composition, rotation, or other factors are secondary.
Solar Internal Conditions

Few direct observations
- need reasoning based on physical principles
- but conditions are extreme….risky

Strategy:
• Know density is low & temperature high \[ \frac{\text{bar}}{\text{g}} \] assume ideal gas state
• Then pressure \[ p = \frac{\text{bar}}{\text{g}} \] R T /m, where \( m \) is mean molecular weight
• Know total mass \[ \frac{\text{bar}}{\text{g}} \] pressure can be estimated from gravity
• Then knowing \( p \), \[ \frac{\text{bar}}{\text{g}} \] \[ \frac{\text{bar}}{\text{g}} \] internal temperature if \( m \) is known.
Solar Internal Pressure

Gravitational force

\[ F = \frac{GM_1 G_2}{r^2} \]

Apply to two halves of Sun to make rough estimate:

Each side has 1/2 solar mass

Use 1/2 solar radius to estimate distance

pressure = force/area

\[ F = \frac{F \frac{M}{2} \frac{M}{2}}{\left(\frac{r}{2}\right)^2} \]

\[ p = \frac{F}{\text{area}} = \frac{F}{\pi r^2} \]

\[ \Rightarrow \rho = \frac{GM^2}{\pi r^4} = \frac{(6.7 \times 10^{-11} \text{ MKS})(2 \times 10^{30} \text{ kg})^2}{\pi \left(7 \times 10^8 \text{ m}^4\right)} = 3.5 \times 10^{14} \text{ MKS} = 3.5 \times 10^9 \text{ atmospheres!} \]
Solar Internal Temperature

Use ideal gas assumption, \( p = \rho RT \)

Where

\[
T = \frac{p}{\rho R}
\]

\[
p = 3.5 \times 10^{14} \text{ MKS}
\]

\[
\rho = 1.4 \frac{gm}{cm^3} = 1.4 \times 10^3 \frac{kg}{m^3}
\]

\[
R = \frac{8.3 \times 10^3}{0.5} \frac{J}{g\text{mole}^{-1} \cdot g^0 K^{-1}}
\]

This assumes atomic weight = 0.5 \( \frac{g}{\text{mole}} \), half protons, half electrons.

\[
\Rightarrow T = 15 \times 10^6 \text{ K}
\]

Agrees with theoretical estimates of the requirement for nuclear fusion.

**Note:** We use here the meteorological convention of writing \( R = \frac{\mathcal{R}}{m} \), where \( \mathcal{R} \) is the universal gas constant and \( m \) is the mean atomic weight.
What Determines Brightness?

- On main sequence, temperature drops from $15 \times 10^6$ K at core of star to much smaller values near edge.
- Temperature gradient $\Delta$ diffusion of radiative energy out. This radiative diffusion rate determines the luminosity $L$.
- Surface temperature adjusts so that $L$ radiates to space.

$$L = 4\pi r^2 \sigma T^4$$

where $\sigma$ = Stefan-Boltzmann constant = $5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$.
- It turns out that $L$ is proportional (approximately) to $M^4$.

(Which stars have longest main sequence lifetimes, high mass or low mass?)
Figure 1. A cross-section of the Sun's interior. Energy produced through the fusion of hydrogen in the core is transported outward, first by countless absorptions and emissions within the radiative zone and then by convection. The wholesale motion of ionized matter within the interior generates magnetic fields that express themselves at the surface as sunspots, prominences, and active regions.
*This concept is just as applicable to solid planets as to the sun. Heat is transferred in planets by conduction unless the adiabatic gradient of the rock is exceeded, whereupon solid-state convection can occur if the rock is hot and "soft" enough. Depending upon the effective viscosity of the rock, the convection may be so sluggish as to have a negligible effect on heat transport; or it may comprise an important second mode of heat transport, as in the sun.*
Over a star’s lifetime the mass usually stays constant but the radius will change. A star starts its
life as a diffuse cloud, and then shrinks. An estimate of internal $T$ gives

$$ T = \frac{p}{\rho R} = \frac{GM^2}{M} / (4\pi r^3 / 3)R = \frac{3}{4} \frac{GM}{R} \frac{1}{r} $$

Temperature varies inversely with radius. In the diagram, central temperature increases toward
the right.

Gravity intensifies for a compact object $\checkmark$ pressure increase $\times$ temperature increase

Gravitational energy release $\checkmark$ (a) higher central $T$ and (b) loss of heat to space by radiation.

When nuclear burning ignition temperature (about $15 \times 10^6$ K) is reached, shrinking stops and
main sequence is reached.

From equation, main sequence large mass $\checkmark$ large radius.
Comments

- Fusion of H $\rightarrow$ He stage (main sequence) for 1M star lasts ~10 billion yrs; Sun is ~ half way through.
- Sun appears to be a stable hydrogen burner.
- After H $\rightarrow$ He, we have He $\rightarrow$ C plus burning in shells. Complicated structure.
  
  $\rightarrow$ red giants, nove & supernovae (more massive stars only).
  $\rightarrow$ return of material to interstellar medium
- Neutrino counters can test nucleosynthesis reaction theories
  
  $RT \approx \frac{GM}{r}$ says thermal energy ; gravitational

  Example of Virial Theorem
Billions of years from now the Sun will grow enormously in size and luminosity. Note the different time scales, expanded near the end of the Sun’s life to show relatively rapid changes. The orbits of the planets enlarge due to mass loss from the Sun. By the time our star becomes a white dwarf, it will have only 0.51 to 0.58 of its present mass.
1. The Sun is an ordinary main sequence star, slightly smaller than average.
2. Its age is about 4.5 billion years, about half of its main sequence lifetime.
3. Its luminosity is $3.9 \times 10^{26}$ J s$^{-1}$ and the solar flux at the Earth’s mean distance is $1368 \, \text{W} \, \text{m}^{-2}$.
4. The solar radius is $7 \times 10^8$ m. Its emission temperature is about 5780 K.
5. The surface composition of the Sun is very similar to that of primitive meteorites, except for the most volatile species (H, He, Ne, Ar, C, N, O).
6. Over its lifetime on the main sequence, the solar luminosity is predicted by stellar structure theory to increase by 15 or 20%.
7. In another 7-8 Gyr, the Sun will expand into a red giant, first roasting and then perhaps engulfing the terrestrial planets.