Formation of Eccentric Black Hole Binaries

Éamonn O’Shea
I. INTRODUCTION

LIGO has so far detected $O(10)$ binary black hole (BBH) mergers through gravitational waves. An important assumption in the current detection pipeline is that BBH are expected to merge with negligible eccentricity. The justification for this is that eccentricity is radiated away by gravitational waves quite quickly, and it is expected that the majority of BBH mergers have effectively circularised when they reach the LIGO band. However as has been shown by Samsing [1], interactions of binaries in globular clusters lead to BBH that have appreciable eccentricity within the LIGO band.

Such environments have stellar densities a million times that of our solar neighborhood. This significantly increases the probability of interaction between binaries and single stars over their lifetime. As such binary populations are characterised by their past interactions more than the properties of the primordial stars.

Dynamical friction in these clusters tend to cause heavier stars to tend towards the core of the cluster, and binaries are effective at ”soaking up” heavier stars as lighter stars are kicked out. Since the binaries thus tend to increase in mass, and also separation, the cross section for another exchange interaction increases, and binaries are likely to undergo many encounters.

A large fraction of these interactions form temporary resonances, where the three stars follow chaotic orbits for a time. During these chaotic interactions, close encounters become more likely, and during such encounters, energy and angular momentum loss due to gravitational radiation becomes noticeable, and the objects can be driven together, and even merge, emitting GW’s which may be detectable by LIGO.

II. TYPES OF INTERACTIONS

Figure 1. summarises the different types of interactions that can occur. The case we are most interested in is the intermediate binary-single states (IMS) states which occur as a consequence of a close interaction. A close interaction is defined as a third body passing within $r_{CI}$ of the binary center of mass, with this defined:

$$r_{CI} = \frac{m_2}{m_1 + m_2} a_0.$$  

The cross section for such an interaction can be calculated analytically:

$$\sigma_{CI} = \pi r_{CI}^2 \left( 1 + \frac{2Gm_{\text{tot}}}{r_{CI} v_\infty^2} \right).$$
Whether the first (geometric) term, or the second (gravitational) term dominates depends on the relative energy of the incoming object to that of the binary.

We can thus classify binaries as either hard or soft depending on their energy relative to the interacting third binary. When the characteristic speed of the binary $v_c$, is less than $v_\infty$, the incoming object has sufficient energy to split the binary, whereas the hard binary case $v_\infty < v_c$ tend to survive the interaction. As such hard binaries tend to become harder after each interaction, whereas soft binaries tend to evaporate.

During a CI, the system is in a chaotic three body state which is unstable, and will inevitably
evolve to an outcome from Figure 1. We are interested in the cross sections for each of these outcomes relative to the CI cross section itself, as a function of the initial velocity of the third object. Samsing [1] proceeds by studying such binary-single interactions numerically. \( N_{\text{tot}} \) binary-single interactions were performed by sampling a disc at infinity with radius \( b \), such that the cross section for outcome of type \( O_i \) (fly-by, exchange, inspiral, etc.) is

\[
\sigma_i = \frac{N_i}{N_{\text{tot}}} \pi b^2
\]

A. Results: Newtonian and Post-Newtonian Results

Samsing [1] performs simulations to find the cross sections in both the Newtonian case and the effects of including Post-Newtonian (PN), of course PN effects are necessary for the binaries to coalesce and merge at all. Figure 2 shows the outcomes for these simulations for the newtonian case and the case of including PN terms.

Let us focus our attention to the resonant interactions only. For the newtonian case, resonant interactions are just as likely to either be fly-by’s or exchanges. This is a consequence of that fact that resonant interactions tend to erase the initial information present in the binary. We can see that as \( v_\infty/v_c \to 1 \), in each case the probability for a resonant interaction drops. This is intuitive: Intermediate states don’t form if the energy of the incident object is too high. There is a small probability of exchange or ionisation for \( v_\infty/v_c \gtrsim 1 \) in both cases.

Most importantly, by including PN effects we get two more possible outcomes than in the newtonian case; inspirals and collisions. Collisions only occur in the Neutron star case, and inspirals can possibly cause GW emission in the LIGO band.

III. FORMATION OF INSPIRALS

It is shown by Samsing [1] that the cross section for the formation of inspirals is:

\[
\sigma_{\text{insp}} \propto a_0^{2/7} \frac{m_{12}^{12/7}}{v_\infty^2}.
\]

Interestingly, the cross section increases with the size of the target. The \( v^{-2} \) dependence can be seen in Figure 2, as a dashed line. Furthermore, the \( v_\infty \) dependence of the formation of ionised states, (in which )
IV. ECCENTRIC INSPIRALS IN THE LIGO BAND

LIGO’s current template bank searches are optimized for circular binaries. Huerta and Brown have shown that for $e = 0.2$, the signal-to-noise ratio (SNR) for current searches degrades by 50%. An example of such a waveform with $e = 0.1$ is shown in figure 3.

The eccentricity and semi-major axis of binaries as they evolved in Samsing’s simulation is shown in figure 4. An estimation of the peak gravitational wave frequency of the binary as it mergers is:

$$f_{\text{GW}}^{\text{peak}} \propto \left( a(1 - e^2) \right)^{1.5}$$

so forms (approximate) straight lines on the figure. Also, it can be seen that the binaries evolve along straight lines with the same slope in this figure. This leads us to the conclusion, the only
FIG. 3: Eccentricity $e = 0.1$ waveform. It can be seen to be qualitatively different from the usual non-eccentric waveform which is instantaneously monochromatic which leads to a loss in SNR. Eccentric binaries in the LIGO band are those that have formed in the LIGO band. However we do see that an appreciable number of binaries do have non-negligible eccentricity.

FIG. 4: Semi-major axis vs. eccentricity for simulations of $M = M_{\text{sun}}$. Squares represent inspirals and