CONSTRaining the Nature of DARK MATTER with gravitational LENSING of Fast Radio Bursts

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1. INTRODUCTION

When the “Lorimer” radio burst was discovered in 2007, its unusually high dispersion measure (DM) of 375 pc cm$^{-3}$ and short (12 ms) duration made it a suspicious anomaly (Rane & Lorimer 2017). Since then, about forty fast radio bursts (FRBs) have joined the Lorimer burst in the literature, all with DMs well in excess of the expected contribution from Galactic free electrons and durations on the order of milliseconds, suggesting high-energy, extragalactic sources. Of this small population of detected FRBs, only one has been observed to repeat: FRB 121102. Localized with subarcsecond precision interferometry to a star-forming region in a dwarf galaxy at redshift $z = 0.19$ (Chatterjee et al. 2017; Tendulkar et al. 2017), the repeating FRB challenges the theory that FRBs come from a single source population (Palaniswamy et al. 2018).

The population of FRB models is arguably as diverse as the population of FRBs themselves, ranging from neutron star mergers (Yamasaki et al. 2018) and black hole accretion or merger events (Mingarelli et al. 2015) to flares from magnetars (Popov et al. 2018), and even superconducting strings (Cao & Yu 2018). While the repeating FRB originates from an extreme, dynamic magneto-ionic environment (Michilli et al. 2018), suggestive of a neutron star or magnetar-like source, the population of known FRBs is still too sparse to determine the nature of their sources and differentiate between their (possible) subpopulations. However, with an all-sky rate of $10^4$ FRBs per day per sky
(Rane & Lorimer 2017), new survey efforts with CHIME, HIRAX, and UTMOST are expected to detect hundreds to thousands of FRBs per year.

The expected statistical robustness that will come with so many detected FRBs makes their extragalactic origin worthwhile exploiting for cosmological applications. In particular, FRBs could be lensed by galactic and intergalactic material between the source and observer in three main ways: plasma lensing, strong gravitational lensing, and gravitational microlensing (Li et al. 2017). Lensing might be useful for constraining FRB source models. Plasma lensing, e.g. by filaments in a supernova remnant, could explain the repeating FRB’s aperiodic repetition, amplitude variation, and spectral diversity (Cordes et al. 2017). If a repeating FRB is strongly, gravitationally lensed (e.g. by a galactic halo), there will be multiple, time delayed images of the bursts. The gravitational time delay between different images will produce a detectable, fixed temporal pattern between bursts that, coupled with VLBI resolution of the source, could reveal transverse motion of the source with respect to the observer and lens, thus constraining the source’s physical nature and environment (Dai & Lu 2017).

Strong gravitational lensing of repeating FRBs may also be a measure of cosmological distance. The time delay between FRB images depends on the distances between the source, lens, and observer, and can thus be used to constrain the Hubble parameter ($H_0$) through the distance-redshift relation (Li et al. 2017). Performing such time delay measurements is observationally complex, requiring precise constraints on the mass profiles of the lens and source galaxies, along with mitigation of extraneous effects on the time delays (e.g. transverse motion of the source) and independent distance measurements. However, the short duration of FRBs makes them one of the only source candidates for observing cosmological expansion on the human scale of “real time” (Zitrin & Eichler 2018).

The short duration of FRBs may also make them susceptible to gravitational microlensing by massive astrophysical compact halo objects (MACHOs). MACHOs refer to objects that might account for dark matter; primordial black holes (PBHs), formed from the collapse of overdense, inhomogeneous regions in the early universe, are one example (Carr & Hawking 1974; Alcock et al. 1993). While MACHOs below 20 $M_\odot$ and above 100 $M_\odot$ have essentially been ruled out as dark matter candidates (Muñoz et al. 2016), it remains possible that MACHOs between 20 and 100 $M_\odot$ constitute some or all of the dark matter in the universe, or 27% of the critical density today (Carr
5 concludes the paper. Section 4 briefly discusses theoretical and observational limitations in the lensing analysis, and Section 5 examines the resulting constraints on MACHO parameter space as laid out in Muñoz et al. (2016).

Lensing by a MACHO of mass $\sim 30 \, M_{\odot}$ could produce multiple FRB images with different amplifications and phases, resulting in an interference pattern in the burst intensity (Zheng et al. 2014). Since the lensing behavior is sensitive to the mass profile of the lens, microlensing of FRBs provides a new method of probing the fraction of dark matter in MACHOs, precisely in the mass range that has yet to be constrained (Muñoz et al. 2016).

This paper examines the promise of FRB microlensing as a probe of dark matter in MACHOs. Section 2 provides a theoretical introduction to gravitational microlensing of FRBs, while Section 3 examines the resulting constraints on MACHO parameter space as laid out in Muñoz et al. (2016). Section 4 briefly discusses theoretical and observational limitations in the lensing analysis, and Section 5 concludes the paper.

2. GEOMETRY OF GRavitational MICrolensing

Gravitational microlensing refers to the deflection of light from a background source by an intervening lens. While the deflection produces two images of the source, their angular separation is often too small to resolve—hence the name microlensing. The following geometrical setup follows Perryman (2011), adapted from his emphasis on exoplanets to FRBs.

**Figure 1.** A background source lensed by a point mass will result in two images. Here $\alpha_{GR}$ is the angle at which light is deflected by the lens; $\theta_S$ is the offset between the source and observer; $\theta_I$ is the angular position of one image; $b$ is the impact parameter; and $D_L$, $D_S$, and $D_{LS}$ are the angular diameter distances between the observer and lens, observer and source, and lens and source, respectively (Perryman 2011).

et al. 2016).
Consider a FRB source lensed by a MACHO of mass $M_L$, which can be treated as a point mass. The lensing geometry is shown in Fig. 1. The angular diameter distances between the observer and lens, observer and source, and lens and source are $D_L$, $D_S$, and $D_{LS}$, respectively. It follows from general relativity (cf. Narayan & Bartelmann (1996)) that a FRB propagating near the lens will be deflected by an angle $\alpha_{GR}$, where

$$\alpha_{GR} = \frac{4GM_L}{bc^2},$$

(1)

$b$ is the impact parameter (the amount of transverse deflection—see Fig. 1), $G$ is the gravitational constant, and $c$ is the speed of light. In order for the FRB to reach the observer, the angle between the lens and deflected FRB signal, $\theta_I$, must be related to the angle between the observer and source, $\theta_S$, by

$$\theta_I^2 - \theta_S \theta_I - \frac{4GM_L}{c^2} \frac{D_{LS}}{D_L D_S} = 0$$

(2)

or, letting $\sqrt{4GM_L D_{LS}/c^2 D_L D_S} = \theta_E$, the angular Einstein radius,

$$\theta_I^2 - \theta_S \theta_I - \theta_E^2 = 0.$$  

(3)

This simple quadratic equation demonstrates that this lensing geometry results in two possible images of the source, located at $\theta_I = (1/2)(\theta_S \pm \sqrt{\theta_S^2 + 4\theta_E^2})$. The two images will have amplifications of the form

$$A_{\pm} = \frac{1}{2} \left( \frac{u^2 + 2}{u \sqrt{u^2 + 4}} \pm 1 \right),$$

(4)

where $u \equiv \theta_S/\theta_E$ (Perryman 2011). The ratio of these amplifications is

$$R_f = \frac{u^2 + 2 + u \sqrt{u^2 + 4}}{u^2 + 2 - u \sqrt{u^2 + 4}}.$$  

(5)

The time delay between the images will be

$$\Delta t = (1 + z_L) \frac{4GM_L}{c^3} \left( \frac{1}{2} u \sqrt{u^2 + 4} + \ln \left( \frac{\sqrt{u^2 + 4} + u}{\sqrt{u^2 + 4} - u} \right) \right)$$

(Zheng et al. 2014). Note that $\Delta t$ will be longer for higher redshift, heavier MACHOs.

A typical angular image separation will be of order $\theta_E$, which can be estimated for microlensed FRBs by re-expressing $\theta_E$ as

$$\theta_E \sim (7 \times 10^{-9}) \circ \left( \frac{M_L}{30 M_\odot} \right)^{1/2} \left( \frac{D_L}{340 \text{ Mpc}} \right)^{-1/2} \left( \frac{D_{LS}}{D_S} \right)^{1/2}.$$  

(7)
Taking the FRB source to be at a distance of 680 Mpc ($z \approx 0.2$) and the MACHO lens to be 30 $M_\odot$ and halfway (340 Mpc) between the source and observer, the angular image separation is about $7 \times 10^{-9}$ degrees, too small to resolve. Such an angular separation corresponds to the two images taking paths that differ in length by tens of meters, making them indistinguishable at the detector. However, if the images are out of phase with each other, they will constructively and destructively interfere, producing a total image amplification of the form

$$A(\omega) = \frac{u^2}{u\sqrt{u^2 + 4}} + \frac{2}{u\sqrt{u^2 + 4}} \cos(\omega t),$$

where $\omega$ is the angular frequency measured by the observer (Zheng et al. 2014).

Note that $\theta_E$ will be larger if the MACHO is closer and more massive, though this correlation does not necessarily imply that detection of FRB microlensing will be easier or more likely for closer, larger MACHO lenses. In particular, the correlation between the time delay $\Delta t$ between images and the lens mass and redshift (Eq. 6) suggests that higher redshift, more massive MACHOs will correspond to a higher likelihood of microlensing detection, because the observer will need some minimum time delay in order to distinguish the images.

3. CONSTRaining DARK MATTER WITH FRB MICROLENSING

3.1. Probability of Microlensing

Muñoz et al. (2016) show that the probability of FRB microlensing can be calculated in terms of the fraction of dark matter existing in MACHOs. This probability can be found using the lensing optical depth to a FRB, which at a redshift $z_S$ is

$$\tau(M_L, z_S) = \int_0^{z_S} d\chi(z_L) (1 + z_L)^2 n_L \sigma(M_L, z_L),$$

where $\chi$ is the comoving distance, $n_L$ is the comoving number density of lenses, and $\sigma$ is the lensing cross section. The lensing optical depth can be re-written in terms of cosmological parameters as

$$\tau(M_L, z_S) = \frac{3}{2} f_{DM} \Omega_c \int_0^{z_S} \frac{d z_L}{c H(z_L)} \frac{H_0^2}{D_L D_{LS}} \frac{D_L D_{LS}}{D_S} (1 + z_L)^2 (u_{\text{max}}^2 - u_{\text{min}}^2),$$

where $f_{DM}$ is the fraction of dark matter in MACHOs, $\Omega_c$ is the current cold dark matter density, and $H$ is the Hubble parameter. The lensing cross section $\sigma$ has been re-written in terms of the minimum and maximum normalized impact parameters ($u_{\text{min}}, u_{\text{max}}$). Requiring a minimum time
delay to detect the microlensed images sets $u_{\text{min}}$ (see Eq. 6), while requiring a maximum flux ratio for detection sets $u_{\text{max}}$ (see Eq. 5; cf. Muñoz et al. (2016) for more detail). Note that $\tau(M_L, z_S)$ will be lower for lower redshift, lower mass MACHOs, reflecting the direct correlation between time delay and lens mass and redshift.

The lensing probability must also take into account the redshift distribution of FRBs propagating to the observer. Muñoz et al. (2016) and Wang & Wang (2018) consider two possible FRB redshift distributions:

- FRBs in a given volume have a constant comoving number density. The corresponding redshift distribution $N_{\text{const}}$ is related to the size of the comoving volume in which the FRBs are propagating (cf. Equation 7 in Muñoz et al. (2016)).

- The density of FRBs evolves with redshift like the cosmological star formation rate density (cf. Caleb et al. (2016)). The corresponding redshift distribution $N_{\text{SFH}}$ is related to the size of the comoving volume in which the FRBs are propagating, along with the star formation rate density (cf. Equations 8 and 9 in Muñoz et al. (2016)).

In both cases, the redshift distribution $N(z)$ includes a Gaussian term $\sim e^{-1/z_c}$ where $z_c$ is the cutoff redshift above which the instrumental signal-to-noise ratio is too low to detect FRBs. For the rest of this paper $z_c = 0.5$, following Muñoz et al. (2016).

In the optically thin regime, the lensing probability $\bar{\tau}$ is just the lensing optical depth $\tau(M_L, z_S)$ integrated with the redshift distribution function $N(z)$ of FRBs:

$$\bar{\tau} = \int \tau(M_L, z_S) N(z) dz.$$ \hspace{1cm} (11)

The lensing probability for different MACHO masses is shown in Fig. 2 for both $N(z) = N_{\text{const}}$ and $N(z) = N_{\text{SFH}}$. Overall the lensing probability is lower for $N(z) = N_{\text{const}}$. The dependency in Eq. 11 on $\tau(M_L, z_S)$ means that $\bar{\tau}$ is sensitive to the image time delay and amplification ratio thresholds set by $(u_{\text{min}}, u_{\text{max}})$. The lensing probabilities for three different minimum time delays (0.3, 1, and 3 ms) are shown in Fig. 2. Increasing the minimum time delay between images necessary for detection shifts the probability distribution to higher MACHO masses. Assuming $N(z) = N_{\text{const}}$ and about $10^4$
FRBs are detected per year in new survey efforts, the maximum number of detectable microlensed FRBs will be \(0.013 \times 10^4 = 130\) FRBs (Muñoz et al. 2016).

![Figure 2](image.png)

**Figure 2.** The lensing probability \(\bar{\tau}\) (Eq. 11) as a function of MACHO lens mass \(M_L\). The red curve corresponds to \(\bar{\tau}\) calculated with \(N(z) = N_{\text{const}}\) and the blue curve to \(N(z) = N_{\text{SFH}}\). Three minimum time delays between burst images are considered: 0.3 ms (dashed), 1 ms (solid), and 3 ms (dotted) (Muñoz et al. 2016).

![Figure 3](image.png)

**Figure 3.** Fraction of dark matter allowed for different MACHO masses if no microlensed FRBs are detected out of \(10^4\) FRBs (shaded regions are forbidden), assuming \(N(z) = N_{\text{const}}\). Results are shown for three different minimum time delays between images: 0.3 ms (dashed), 1 ms (solid), and 3 ms (dotted). Constraints from the MACHO and EROS collaborations are shown in red and green, and from wide binary disruption in blue (Muñoz et al. 2016).
3.2. Constraints on MACHO Parameter Space

Since the lensing optical depth (and the lensing probability) depend on the fraction $f_{DM}$ of dark matter in MACHOs (see Eq. 10), detection or nondetection of microlensed FRBs would place constraints on $f_{DM}$. Fig. 3 shows the $f_{DM}$ allowed for different MACHO masses if no microlensed FRBs are detected out of $10^4$ FRBs, assuming the FRB redshift distribution is $N(z) = N_{\text{const}}$. The constraints from FRB microlensing nondetection are much broader than those from the MACHO and EROS collaborations and from wide binary disruptions. If no microlensed FRBs are detected in this setup, $f_{DM}$ for MACHOs with $M_L > 100 \, M_\odot$ is constrained to less than 0.8%, while $f_{DM}$ for MACHOs with $20 < M_L < 100 \, M_\odot$ could be up to $\sim 20\%$. If MACHOs in the 20 to 100 $M_\odot$ range make up most of dark matter, then there should be roughly 10 or more detectable, microlensed FRBs out of every $10^4$ FRBs observed (Muñoz et al. 2016).

4. LIMITATIONS TO FRB MICROLENSING

Thus far two main constraints on the detectability of FRB microlensing have been mentioned: the time delay between burst images and their amplification ratios. The principle behind the former is that the time delay must be of order or larger than the pulse width, otherwise only a single burst will be detected. However, the pulse width is affected by scattering off of material in the interstellar medium (ISM). Scattering off of an interstellar screen could destroy the coherence of the rays, decreasing the likelihood of detection, although observing at higher frequencies would mitigate this effect (Zheng et al. 2014). Interstellar scattering of FRBs is not fully understood; in particular, the difference between the scattering contributions of the sources’ host galaxies, the intergalactic medium (IGM), and the Galaxy remains an open question.

One overarching complication that has not yet been discussed in this paper is the distinction between intrinsic and extrinsic burst repetition. How does the observer distinguish between a source that intrinsically repeats and a source that appears to repeat because of lensing? In the latter case, the amount of apparent repetition will be limited by the number of images that can arise from the lensing geometry. Even with a source that does intrinsically repeat, like FRB 121102, it is not clear whether that repetition is intrinsically aperiodic, or whether its aperiodicity arises from lensing
Muñoz et al. (2016) argue that the correlation between image magnification and time delay can be used to distinguish between intrinsic burst repetition and lensed bursts. Convolving the ratio of the image amplifications $R_f$ (Eq. 5) with the time delay between images $\Delta t$ (Eq. 6) results in a joint probability distribution (PDF) for $\Delta t$ and $R_f$, assuming certain parameters, such as the lens mass $M_L$ and the redshift distribution of FRBs $N(z)$, are fixed. The PDF calculated by Muñoz et al. (2016) for a 30 $M_\odot$ MACHO is shown in Fig. 4. Higher $R_f$ correspond to higher $\Delta t$. The brighter burst image should typically arrive first.

Moreover, a lensed burst will have fixed time delays between its images, set by the difference between the paths the burst images take from source to observer. If the source exhibits aperiodic repetition and is lensed multiple times by the same MACHO, then the lensed bursts will be distinguished by the equal time delays between each image.

Ultimately, a statistically robust detection of FRB microlensing and resultant constraint on the fraction of dark matter in MACHOs will require an extremely large sample (tens of thousands) of FRBs, largely because such fortuitous alignment is needed between a FRB source, a compact lens,
FIG. 4: Four windows in which PBHs could conceivably provide the dark-matter density. Upper left panel: (A) Intermediate-mass black holes. The constraints in this mass range are EROS and MACHO microlensing bounds [27] (in blue), dynamical constraints (in red) from the life-time of the central star cluster in the Eridanus II dwarf galaxy [192], as well as dynamical constraints (in green) from the existence of wide-binary star systems [37].

Upper right panel: (B) Sublunar black holes; in this case the constraints (in blue) are again the femtolensing of GRBs from [187], while the limits from neutron-star capture (in green) are taken from [36]. The red-shaded region to the right-hand side of the plot denotes microlensing constraints from the Kepler survey [189], while the red-shaded region to the plot’s left-hand side shows constraints from white-dwarf explosions [188].

Lower left panel: (C) Subatomic black holes. The constraints here (red-shaded region) stem from non-detections of extragalactic $\gamma$-rays that would be observable from the evaporation of PBHs of these masses [11, 35], and (in blue) femtolensing of $\gamma$-ray bursts (GRBs) taken from Fermi data [187].

Lower right panel: (D) Planck-mass relics from PBH evaporations. This shows the mass range of the initial PBHs if they derive from inflation [62] but there are no observational constraints on such relics. Details on all these regimes and the meaning of the constraints can be found in the subsections on the respective scenarios.

gravitationally. It has been suggested that PBHs in window (A) could naturally arise in various inflationary scenarios [39, 110–112] but this applies equally for the other windows since the mass-scale is essentially arbitrary.

We now discuss each of the mass windows in turn. For the largest one (A), we will present our analysis in some detail in order to demonstrate the methodology. For the next two mass windows (B and C), we have performed a similar analysis but just state the main results. Finally, the Planck-mass relic scenario (D) is discussed, although there is only the trivial constraint $f<1$ in this mass range. We stress that we are not making definite conclusions about the viability of PBH dark matter in any particular range. We are merely considering how conclusions can be drawn from certain observational claims in the literature, which may or may not be justified.

There is an independent constraint on $f_{DM}$ in the 20 to 100 $M_\odot$ MACHO mass range from the ultra faint dwarf galaxy Eridanus II, shown in Fig. 5 (Brandt 2016; Carr et al. 2016). The constraints from Eridanus II lie in the same region of parameter space as constraints from FRB lensing, and will thus provide a useful check on future FRB lensing constraints.

5. CONCLUSIONS

While dozens of FRBs, including one repeater, have been observed, their physical cause(s) remain(s) a mystery. Regardless of the physical source, their extragalactic origin, coupled with
the expected statistical robustness of upcoming FRB surveys, make them attractive candidates for cosmological applications. In particular, the detection of gravitationally lensed FRBs could be used to constrain the nature of dark matter, as discussed in this paper. Gravitational microlensing of FRBs by dark matter MACHOs would produce multiple images of the FRB, with amplifications and time delays set by the MACHO’s mass and distance. Detection or nondetection of microlensed FRBs in a large scale survey would constrain the fraction of dark matter existing in MACHOs between 20 and 100 $M_\odot$, a mass range of MACHO that has yet to be ruled out. While an extremely large sample of FRBs will be necessary to perform a microlensing detection or conclude a nondetection and extrapolate the resultant constraint on dark matter MACHOs, CHIME, HIRAX, and UTMOST should make such a constraint possible.

REFERENCES

Cao, X.-F., & Yu, Y.-W. 2018, PhRvD, 97, 023022
Carr, B., Kühnel, F., & Sandstad, M. 2016, PhRvD, 94, 083504
Rane, A., & Lorimer, D. 2017, Journal of Astrophysics and Astronomy, 38, 55
Yamasaki, S., Totani, T., & Kiuchi, K. 2018, PASJ, 70, 39