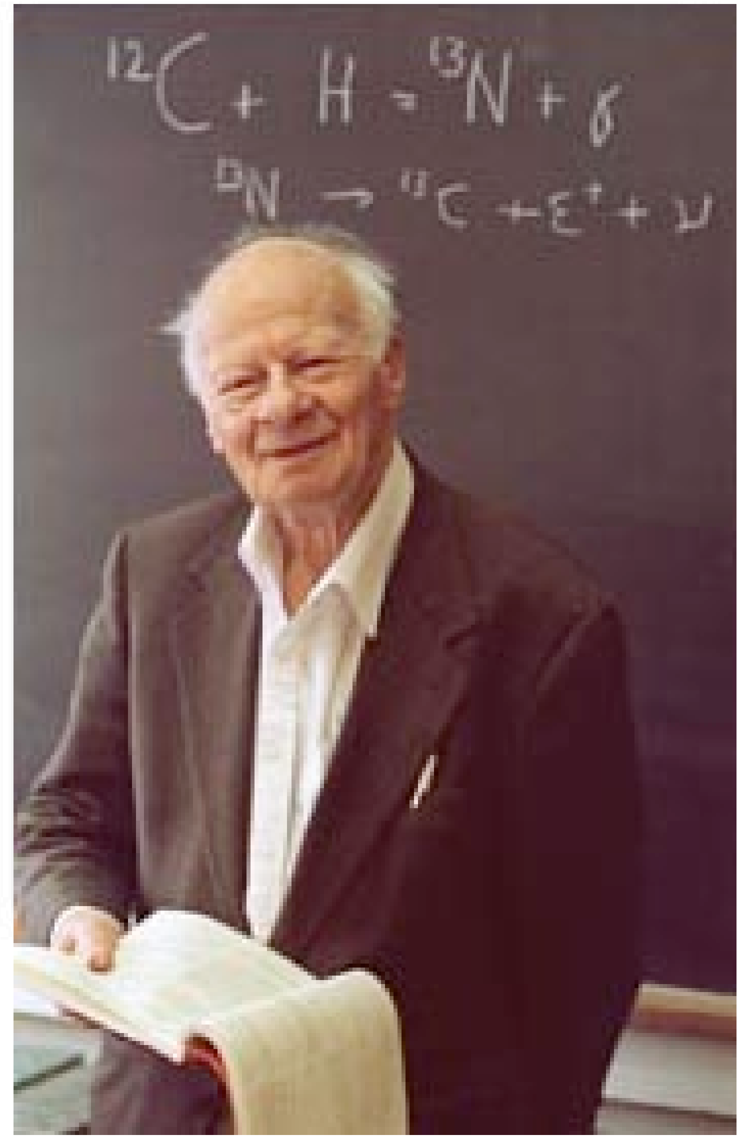
Solar Neutrinos: Theory and Connections to other Neutrino Physics

This began an effort to combine precise weak-interactions/nuclear microphysics with careful modeling of solar evolution. Ultimately the goal would be 1% predictions of quantities like the core temperature that govern pp-cycle branching ratios. Solar neutrino detection would be the experimental test of the model and its microphysics.

Based on Bahcall and Davis History of the Solar Neutrino Problem



Hertzsprung-Russell diagram

- T_s, L, R simplest stellar properties

- Stefan-Boltzmann black-body:

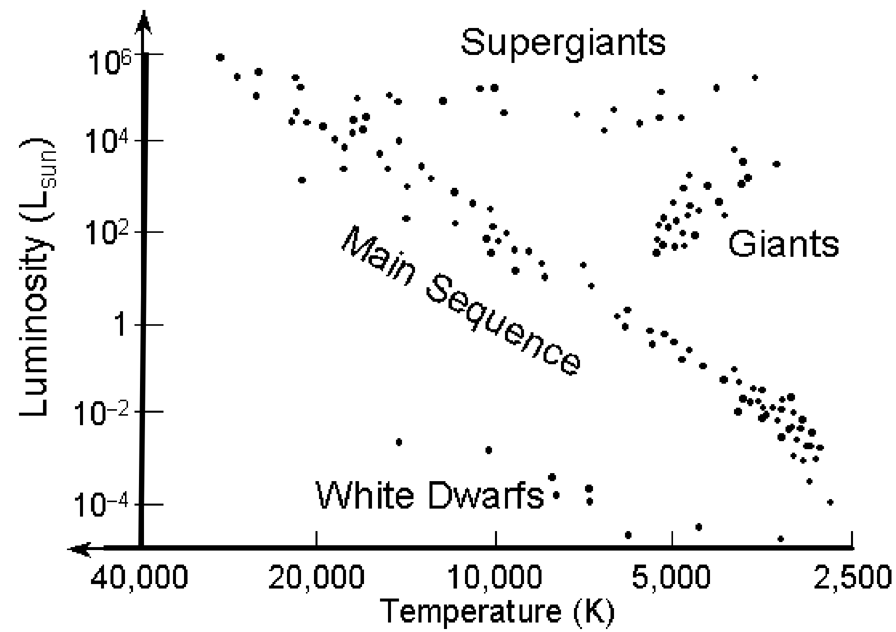
$$L = 4\pi R^2 \sigma T_s^4$$

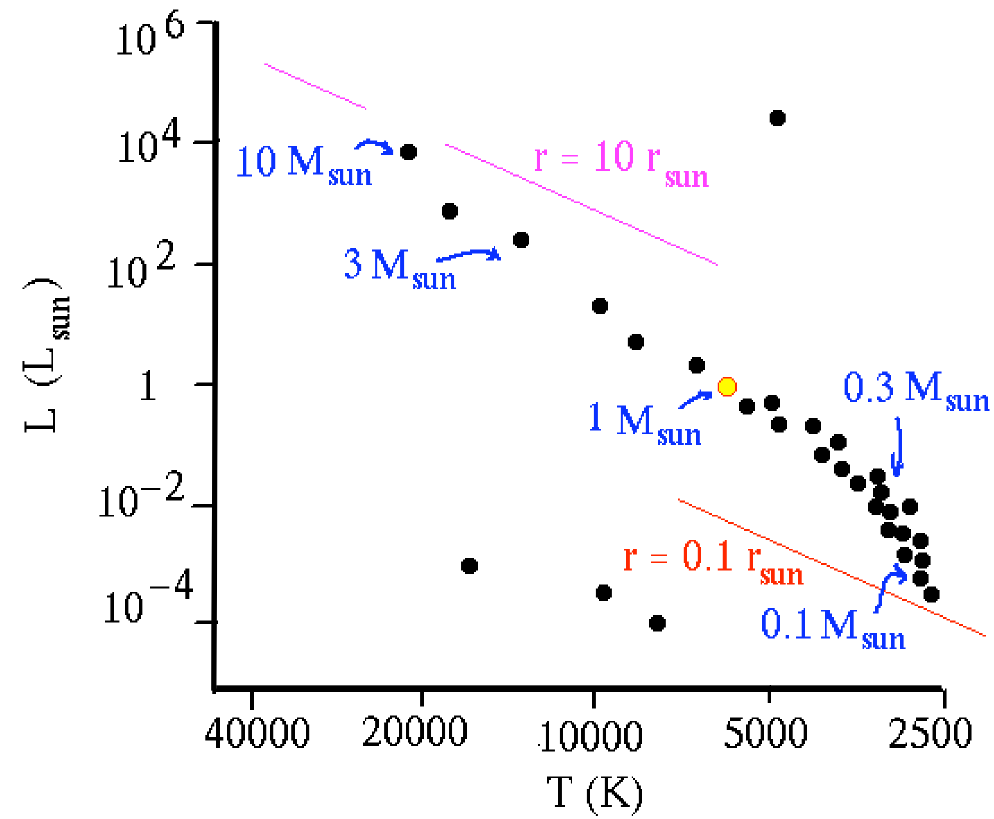
$$\left[\frac{L}{L_\odot}\right] = \left[\frac{R}{R_\odot}\right]^2 \left[\frac{T}{T_\odot}\right]^4$$

- ⇒ one-parameter HR trajectory

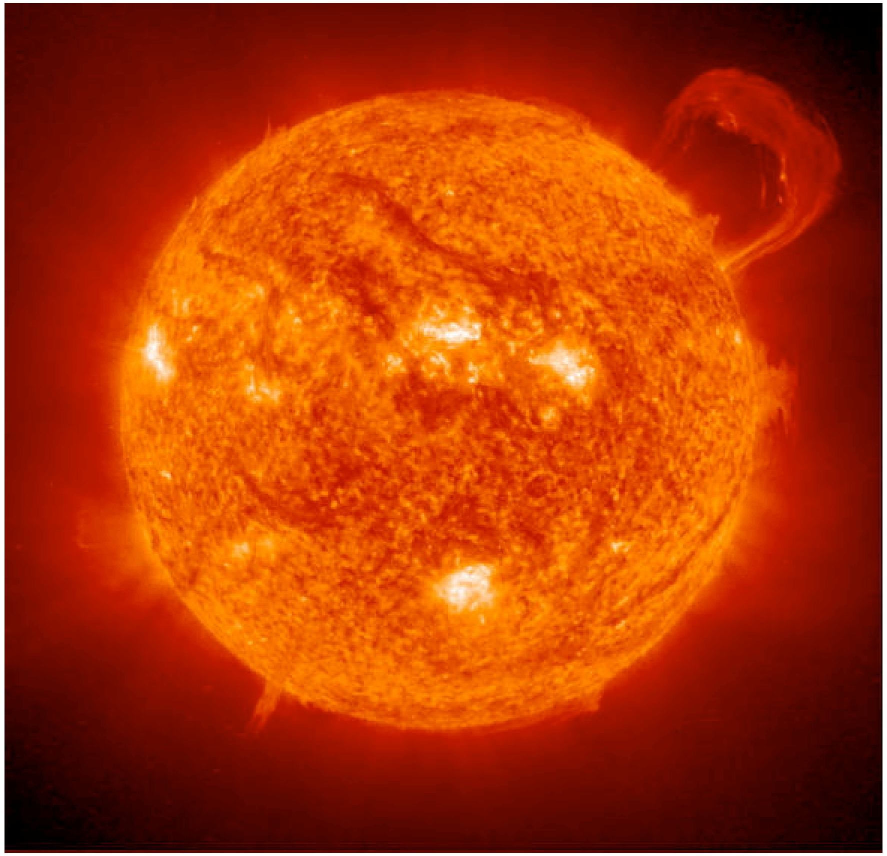
- "main sequence" of H-burning stars to which sun belongs

- sun a stellar-evolution test case: M, L, R, T_s, age, surface composition





The Standard Solar Model assumptions



- sun burns in hydrostatic equilibrium: gravity balanced by gas pressure gradient (need EOS)
- energy transported: radiation, convection (convective envelope, radiative core)
- solar energy generated by H fusion

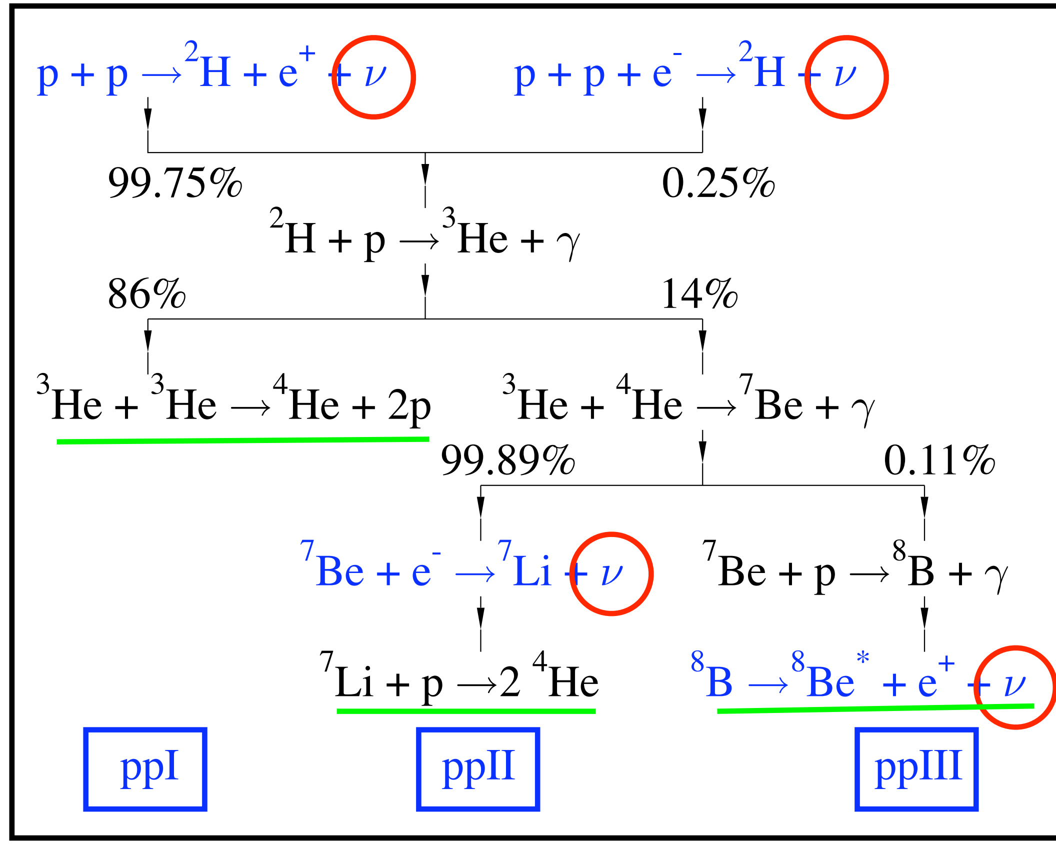
$$4\mathrm{p} \quad \rightarrow \quad {}^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_\mathrm{e}$$
$$T_c \quad \sim \quad 1.5 \cdot 10^7\mathrm{K} \leftrightarrow \mathrm{E}_\mathrm{cm} \sim 2\mathrm{keV}$$

pp chain, CNO cycle

- boundary conditions: solar age, *L*, *M*, *R*, today's surface composition
  H : He : Z needed, where Z denotes metals (A ≥ 5)
  assume t=0 sun homogenized ⇒ Z from today's surface abundances
  H+He+Z = 1, He/H adjusted to produce today's *L*

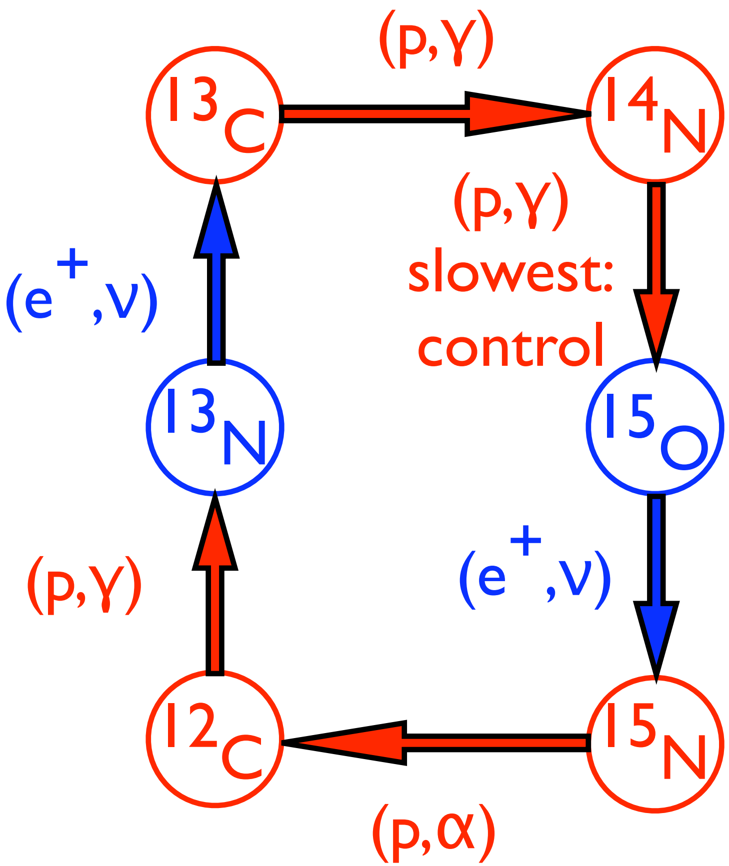
Solar pp chain

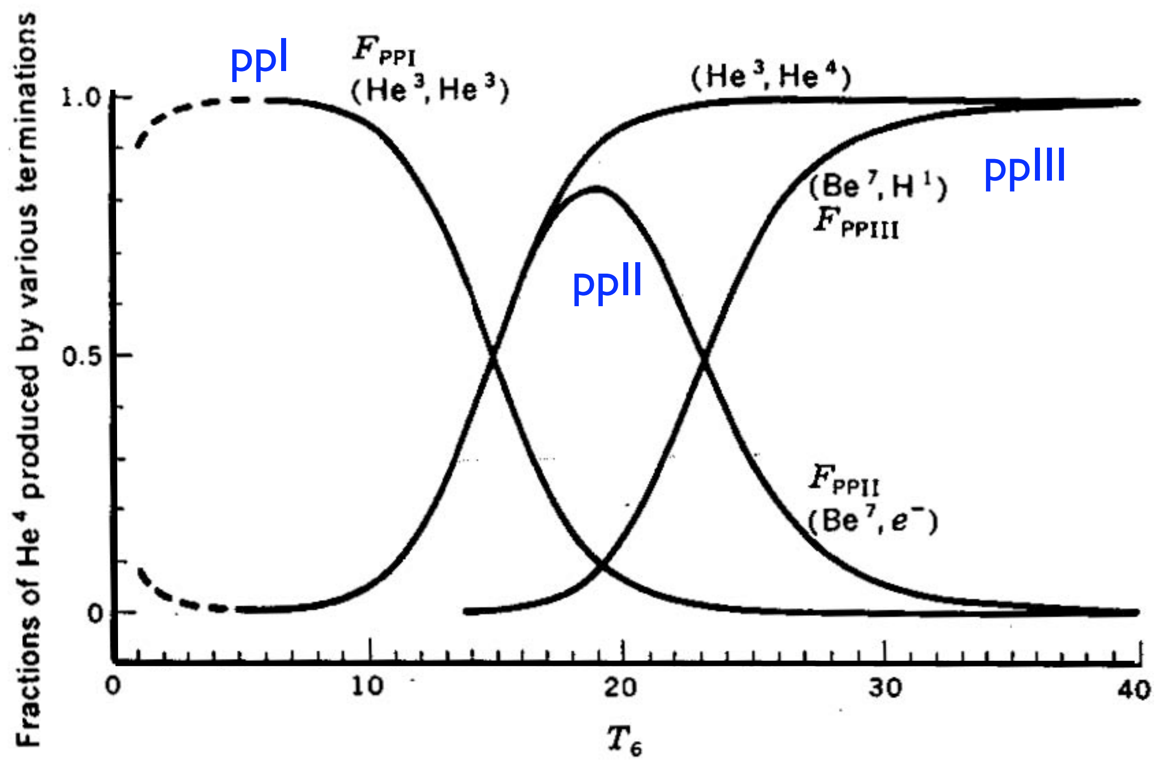
- 1920s: Eddington - recognized solar energy is nuclear
- 1928: Gamow - QM tunneling would allow stellar reactions
- 1930s: Bethe - describes nuclear reactions, effective range theory

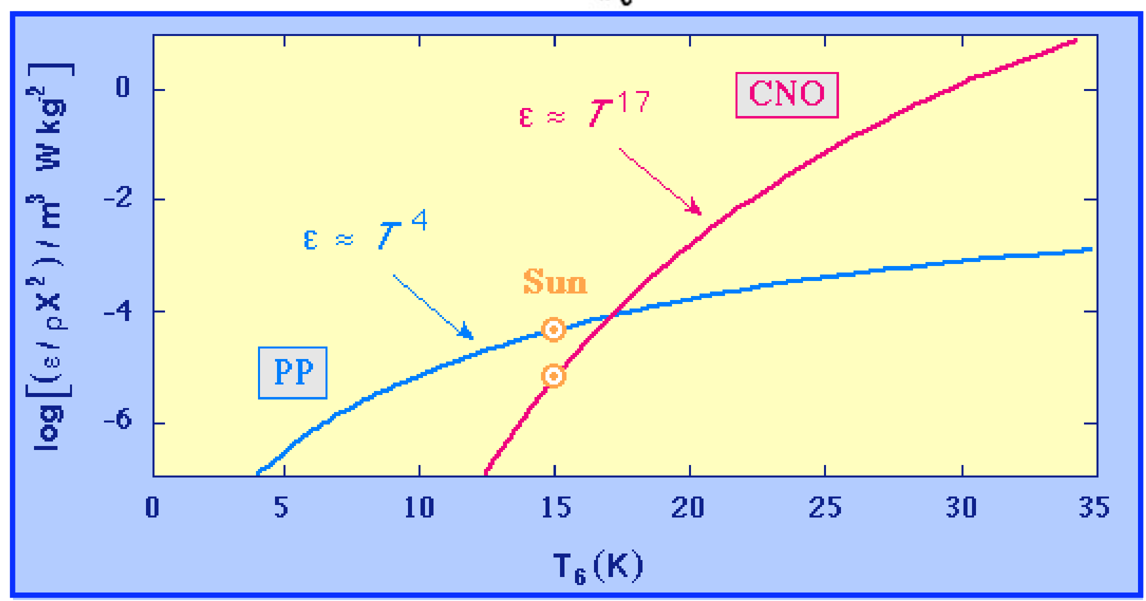


- Weizacker, Bethe, and Critchfield explored 4p → He; Bethe and Critchfield - pp chain in 1938; Bethe - CNO cycle in 1939
- pp-chain governs energy production in lower-mass main-sequence stars: three competing cycles -- ppI, ppII, ppIII -- each synthesizing He
- tagged by three distinctive neutrinos ⇒ probe of core T

CNO cycle







- Hans identified this as the cycle relevant for hotter MS stars
- C,N,O catalysts for 4p → He
- competes well with pp chain only at the sun's center: $T_6 \sim 1.55$

Standard Solar Model properties

- over 98% of the sun's energy generated through pp cycle -- ppI dominance

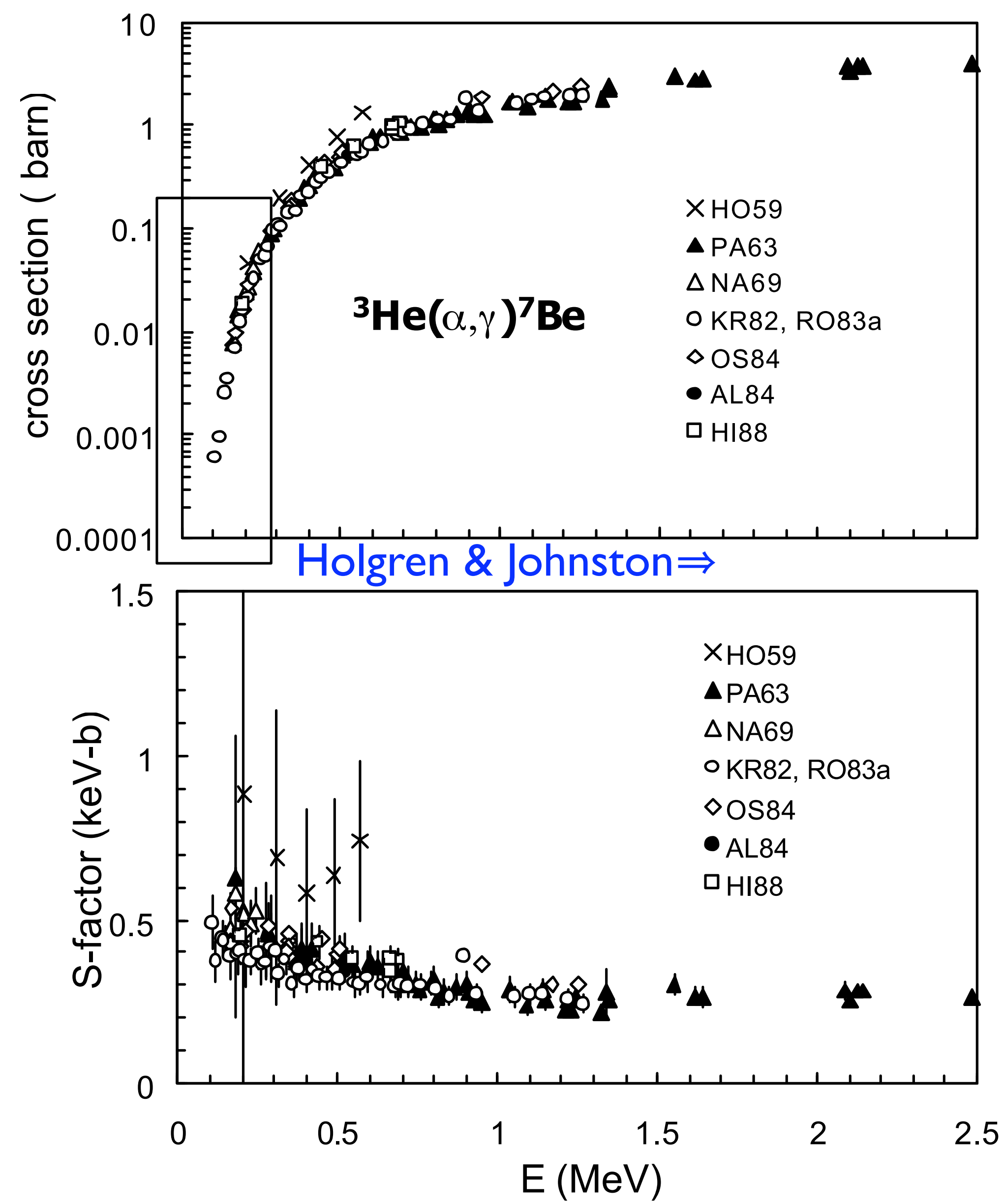
- dynamic sun
  - 44% luminosity rise over past 4.7 Gy
  - ^{8}B (ppIII) ν flux doubled over past 0.9 Gy (T^{22})
  - large composition gradient in ^{3}He established

- significant successes
  - correct depth of convective zone ⇔ solar acoustic oscillations
  - good agreement with helioseismology probing to depth (more later)

- as implemented by Bahcall and others, the SSM is 1D and static
  - sun's Li depletion large -- no dynamical convection
  - complicated magnetic phenomena of convective zone beyond SSM

Solar neutrinos

- Hans and others were limited by SSM and cross section uncertainties
- SSM nuclear cross sections had to be measured at much higher energies, extrapolated to give S(0)
- in 1959 became apparent solar νs might be measurable: a strong path to the ppII/III cycles
- Fowler brought John Bahcall to Caltech to work with Iben and Sears

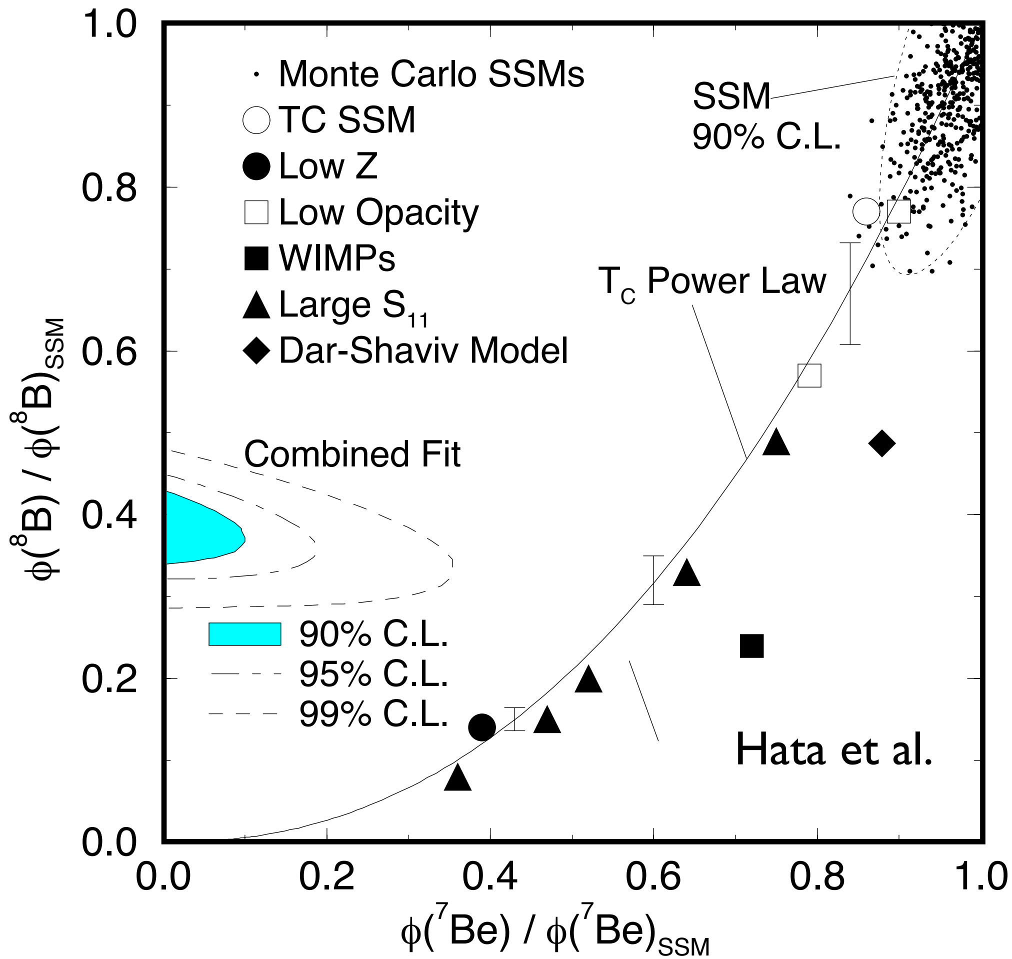


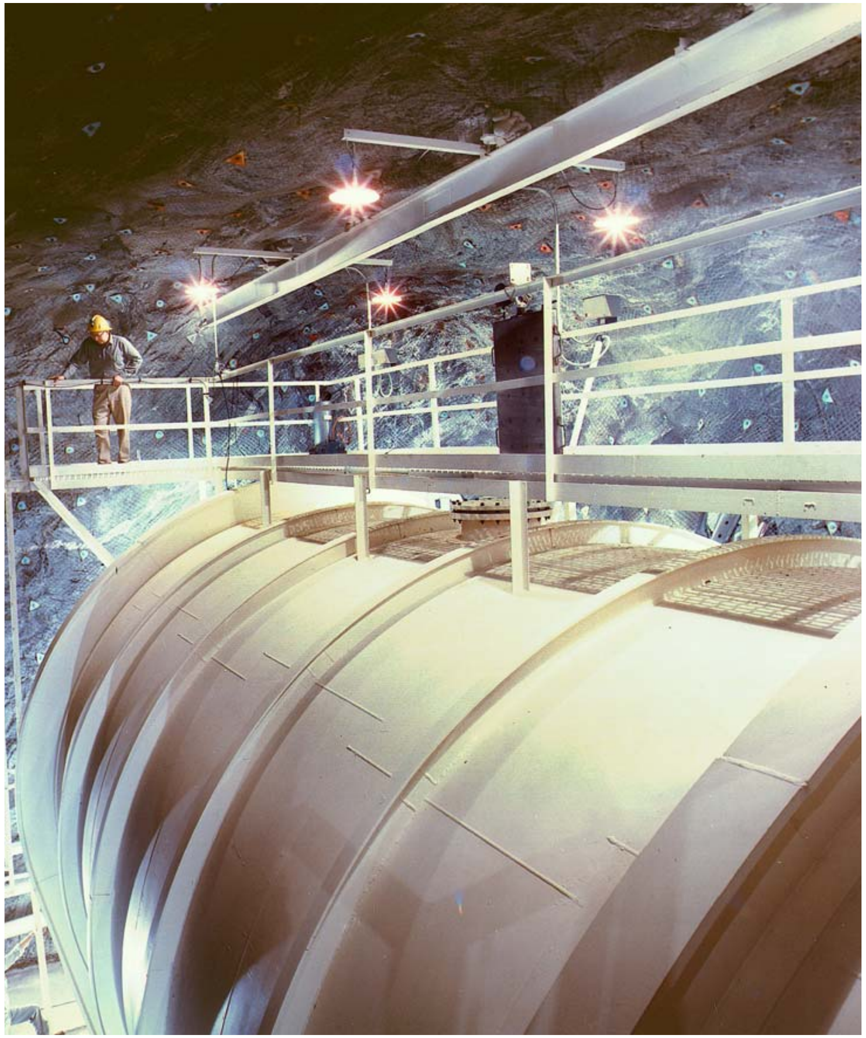
- with new SSM results and Cl capture cross sections in hand, Bahcall and Davis proposed Cl in 1964; excavation in 1965; first results in 1968

Cl, Ga, Kamioka experiments:

$$\phi(pp) \sim \phi^{SSM}(pp)$$
$$\phi(^7Be) \sim 0$$
$$\phi(^8B) \sim 0.43\ \phi^{SSM}(^8B)$$

Using reaction T-dependences ⇒





$$\phi(^8B) \sim T_c^{22} \Rightarrow \text{colder sun}$$
$$\frac{\phi(^7Be)}{\phi(^8B)} \sim T_c^{-10} \Rightarrow \text{warmer sun}$$

so a contradiction

Neutrino oscillations (vacuum)

flavor states $|\nu_e>$ $\leftrightarrow$ $|\nu_L>$ m_L mass states

$|\nu_\mu>$ $|\nu_H>$ m_H

assume that these bases are not coincident, do an experiment:

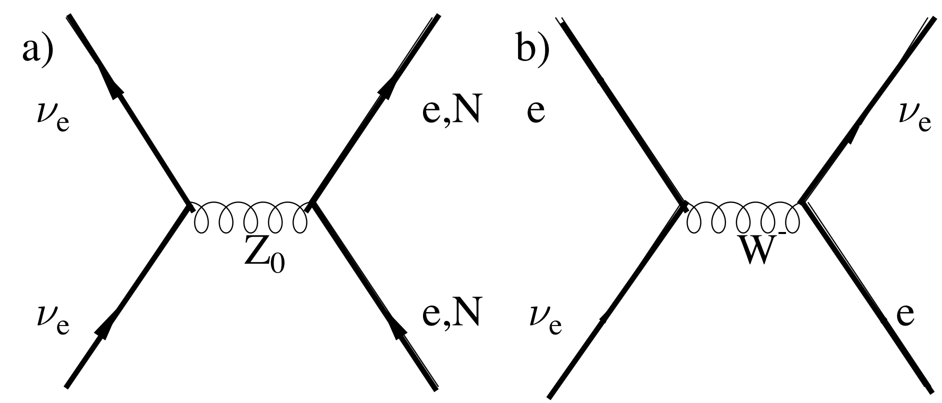
$$\begin{aligned}|v_e> &= \cos\theta|\nu_L> + \sin\theta|\nu_H> \\ |v_\mu> &= -\sin\theta|\nu_L> + \cos\theta|\nu_H>\end{aligned}$$

$$\begin{aligned}|\nu_e^k> &= |\nu^k(x=0,t=0)> \qquad E^2 = k^2 + m_i^2 \\ |\nu^k(x\sim ct,t)> &= e^{ikx}\left[e^{-iE_Lt}\cos\theta|\nu_L> + e^{-iE_Ht}\sin\theta|\nu_H>\right] \\ |<\nu_\mu|\nu^k(t)>|^2 &= \sin^2 2\theta\sin^2\left(\frac{\delta m^2}{4E}t\right), \qquad \delta m^2 = m_H^2 - m_L^2\end{aligned}$$

ν_μ appearance downstream $\Leftrightarrow$ vacuum oscillations

Bethe and the MSW mechanism (1987)

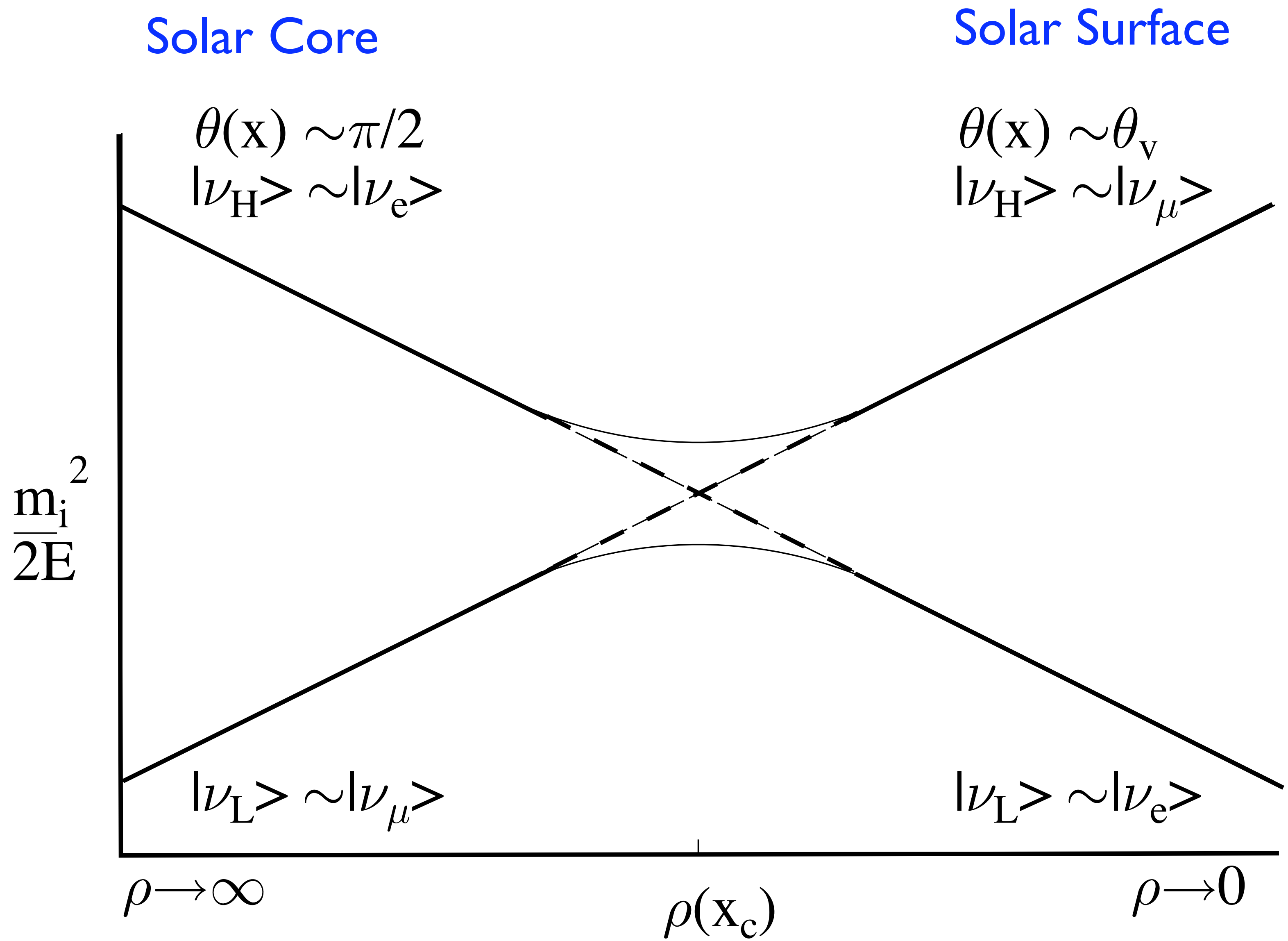
$$|\nu^k(0)> = a_e(0)|\nu_e> + a_\mu(0)|\nu_\mu>$$



$$i\frac{d}{dx}\begin{bmatrix} a_e(x) \\ a_\mu(x) \end{bmatrix} = \frac{1}{4E} \times$$

$$\begin{bmatrix} -\delta m^2 \cos 2\theta \boxed{+ 2E\sqrt{2}G_F\rho_e(x)} & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \boxed{-2E\sqrt{2}G_F\rho_e(x)} + \delta m^2 \cos 2\theta \end{bmatrix} \begin{bmatrix} a_e(x) \\ a_\mu(x) \end{bmatrix}$$

- Matter effects on oscillations first discussed by Wolfenstein
- In 1987 Mikheyev and Smirnov numerically integrated equation ⇒ large regions of $\sin^2 2\theta$ - δm^2 plane yielded large suppressions of solar ν flux
- Bethe: pointed out level crossing at $\rho_e(x) = \rho_c = \delta m^2 \cos 2\theta / 2E\sqrt{2}G_F$



Solar Core
Solar Surface
$\theta(x) \sim \pi/2$
$|\nu_H> \sim |\nu_e>$
$\theta(x) \sim \theta_v$
$|\nu_H> \sim |\nu_\mu>$
$\frac{m_i^2}{2E}$
$|\nu_L> \sim |\nu_\mu>$
$|\nu_L> \sim |\nu_e>$
$\rho \rightarrow \infty$
$\rho(x_c)$
$\rho \rightarrow 0$

Think in terms of local mass (instantaneous) eigenstates:

$$|\nu(x)>= a_H(x)|\nu_H(x)> + a_L(x)|\nu_L(x)>$$

$$i\frac{d}{dx}\left[\begin{array}{c} a_H(x) \\ a_L(x) \end{array}\right] = \frac{1}{4E}\left[\begin{array}{cc} m_H^2(x) & i\alpha(x) \\ -i\alpha(x) & m_L^2(x) \end{array}\right]$$

Observe:

- mass splitting at ρ_c small: avoided level crossing
- $\nu_H \sim \nu_e$ in large density limit
- if vacuum angle is small, $\nu_H \sim \nu_\mu$ in vaccum

Thus there is a local matter angle $\theta(x)$ which rotates from $\pi/2$ to the vacuum value θ as $\rho_e(x)$ goes from infinity $\rightarrow$ 0

- α(x) must be ~ dρ(x)/dx: so small if density change gentle

- if small (everywhere), ignore relative to diagonal elements ⇒ integrate

Bethe: $P^{adiab}_{\nu_e} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i$ path independent

  - this led Hans to predict a solar neutrino solution with $\delta m^2 \sim 10^{-5}$ eV^2

- can generalize: most nonadiabatic region is very near crossing point

beat frequency lowest ⇔ period largest ⇔ best chance to "see" dρ/dx

- derivative at ρ_c governs nonadiabatic behavior: Landau-Zener

$$P^{LZ}_{\nu_e} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

$$P_{hop} = e^{-\pi\gamma_c/2} \quad \gamma_c = \frac{\sin^2 2\theta}{\cos 2\theta}\frac{\delta m^2}{2E}\frac{1}{|\frac{1}{\rho_c}\frac{d\rho}{dx}|}$$

- so γ_c >> 1 ⇔ slowly varying density ⇒ P_{hop} ~ 0 ⇔ adiabatic crossing

  allows strong $\nu_e \rightarrow \nu_\mu$ conversion

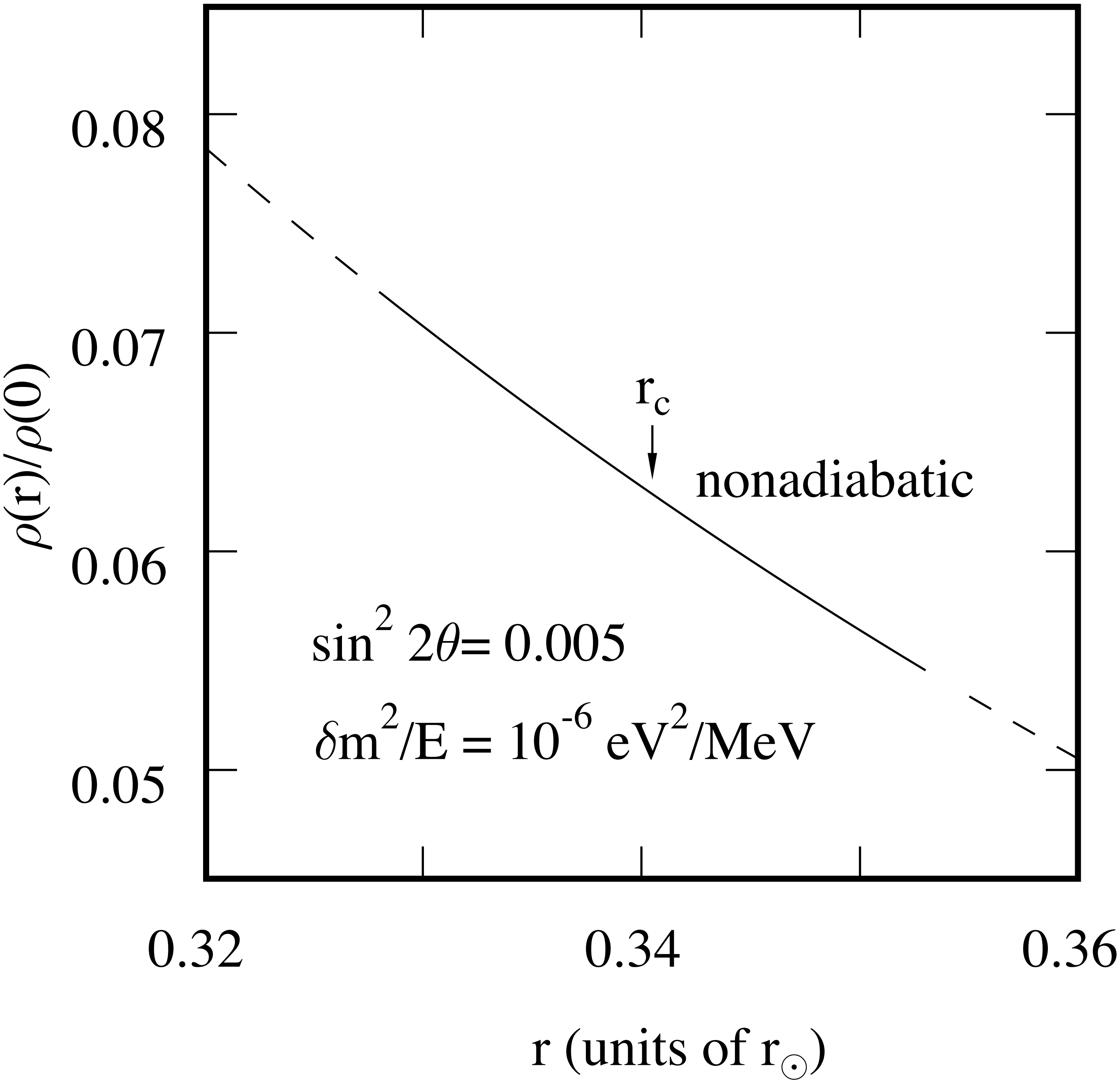
- and γ_c << 1 ⇔ sharply varying density ⇒ P_{hop} ~ 1 ⇔ nonadiabatic crossing

  hops to lower level: no flavor conversion

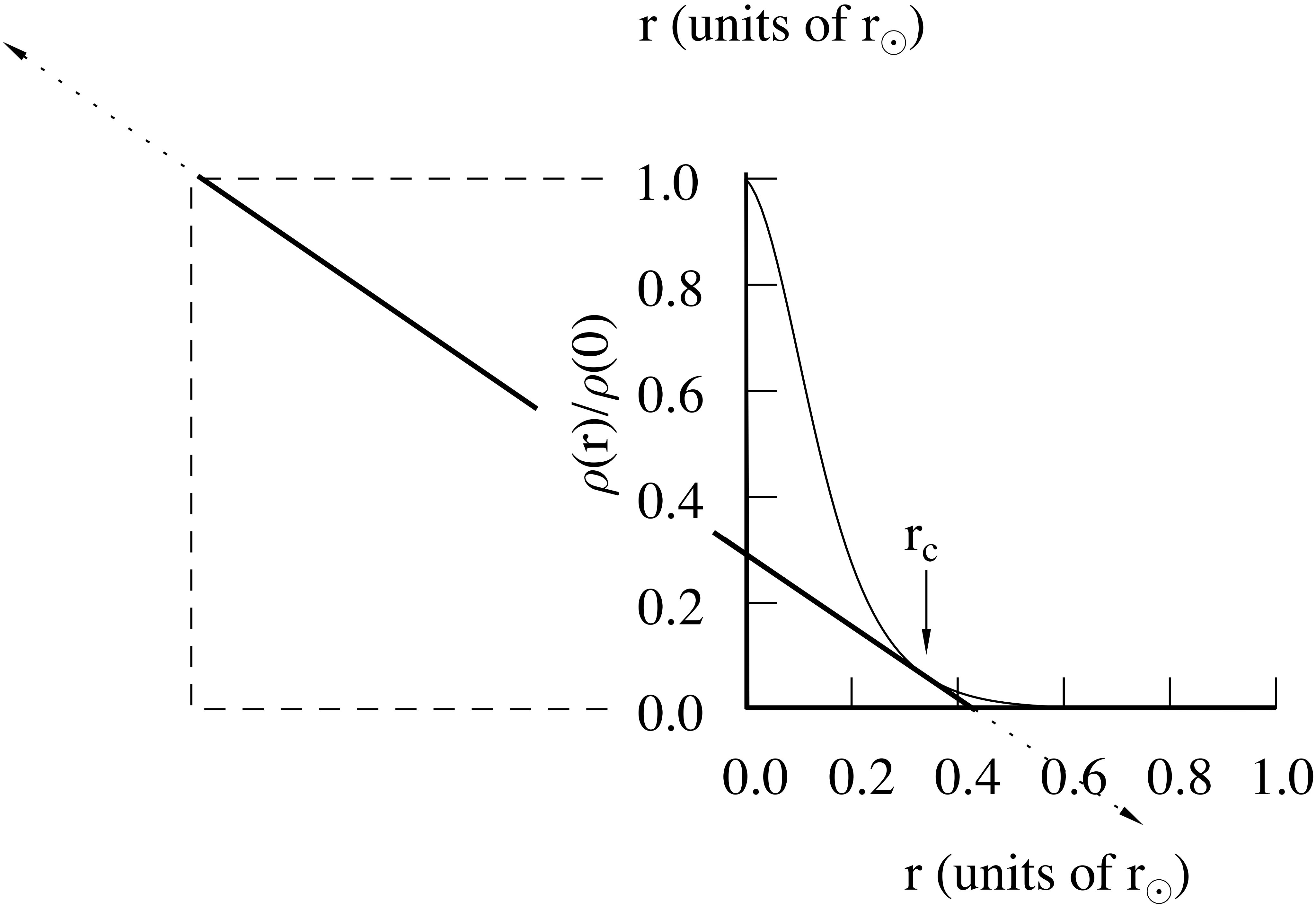
so two conditions necessary for strong flavor conversion

a level crossing must occur ($\theta_i \sim \pi/2$)

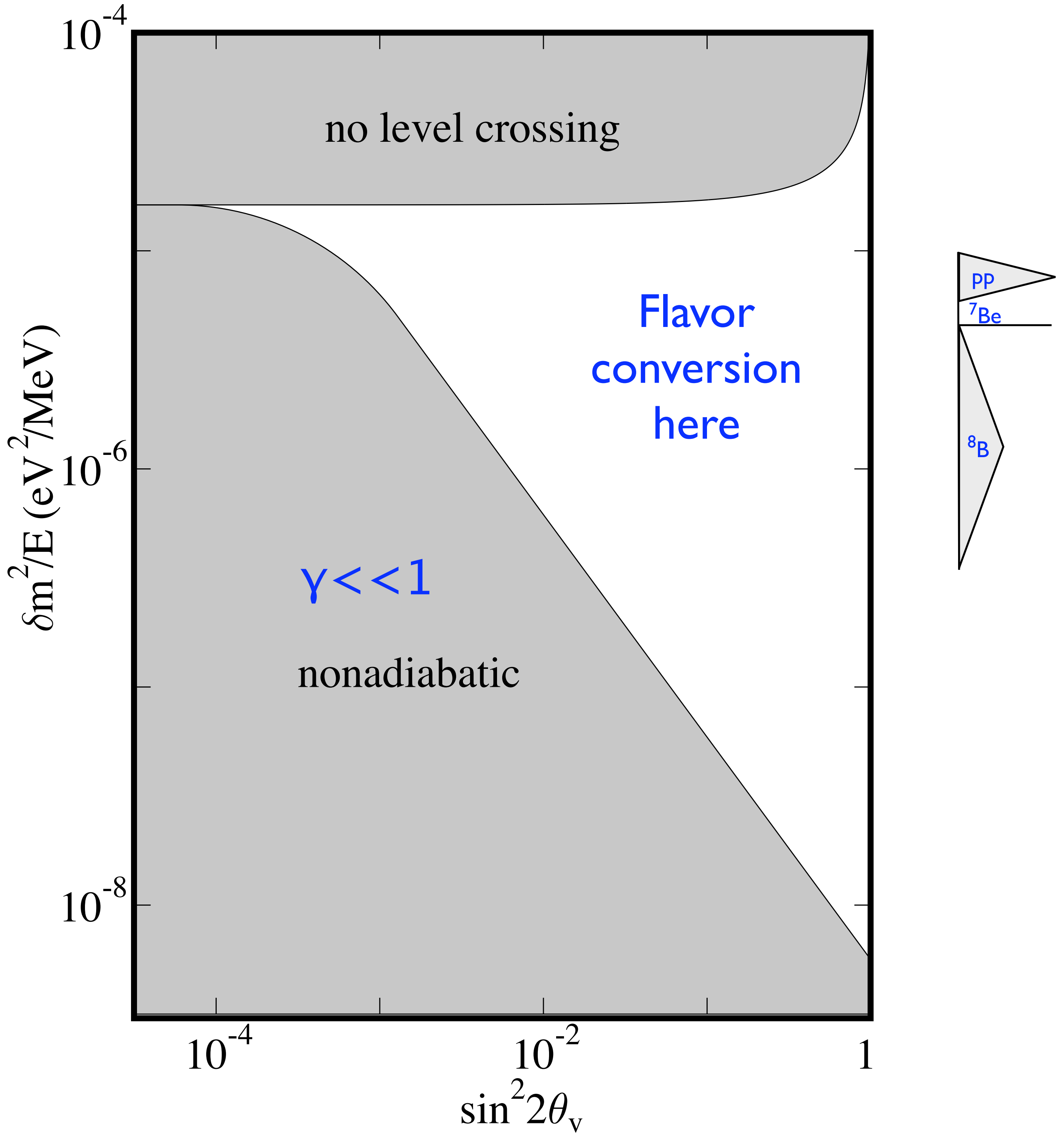
the crossing must be adiabatic



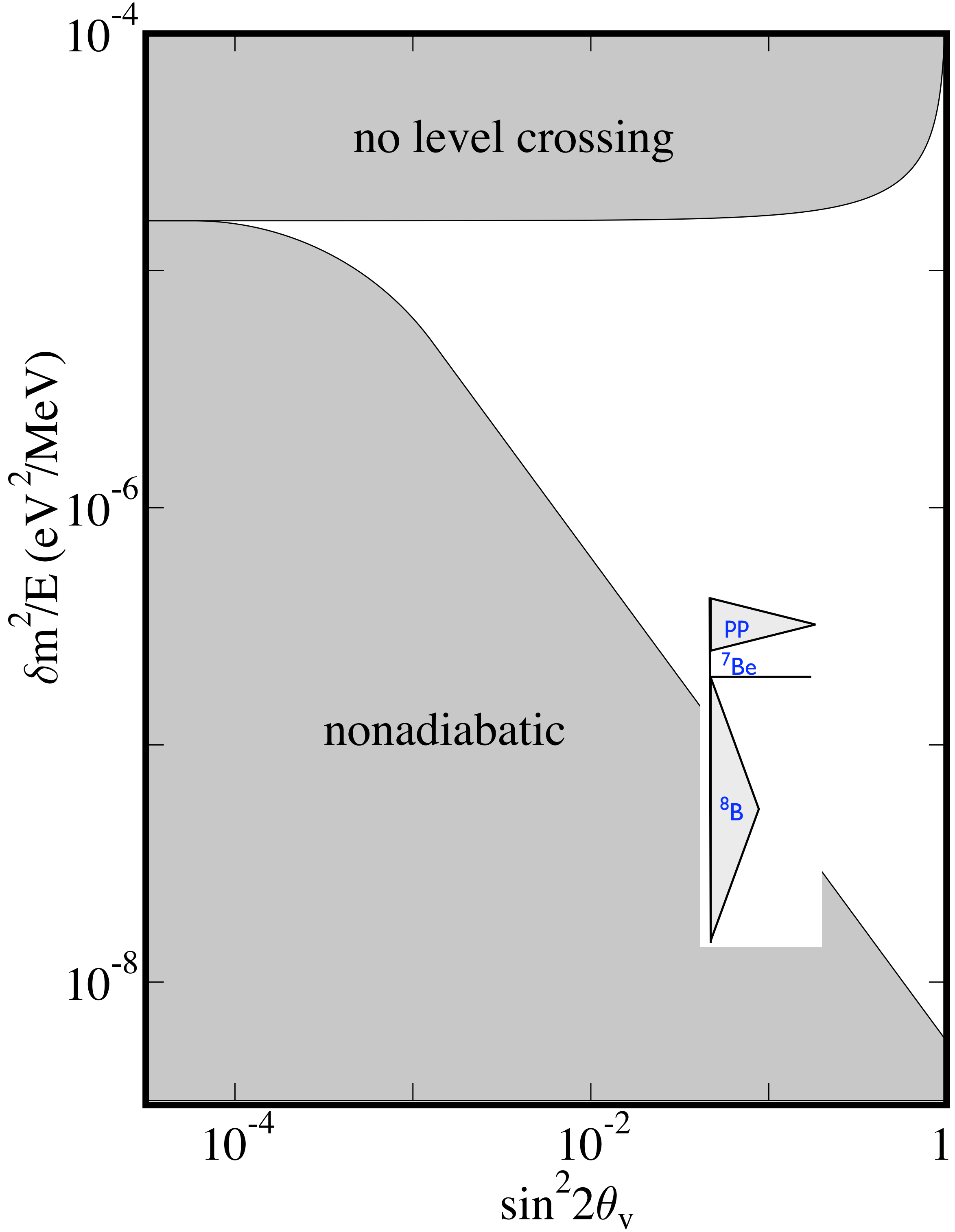
0.08
0.07
0.06
0.05
$\rho(r)/\rho(0)$
r_c
nonadiabatic
$\sin^2 2\theta = 0.005$
$\delta m^2/E = 10^{-6}\ eV^2/MeV$
0.32
0.34
0.36
r (units of $r_\odot$)



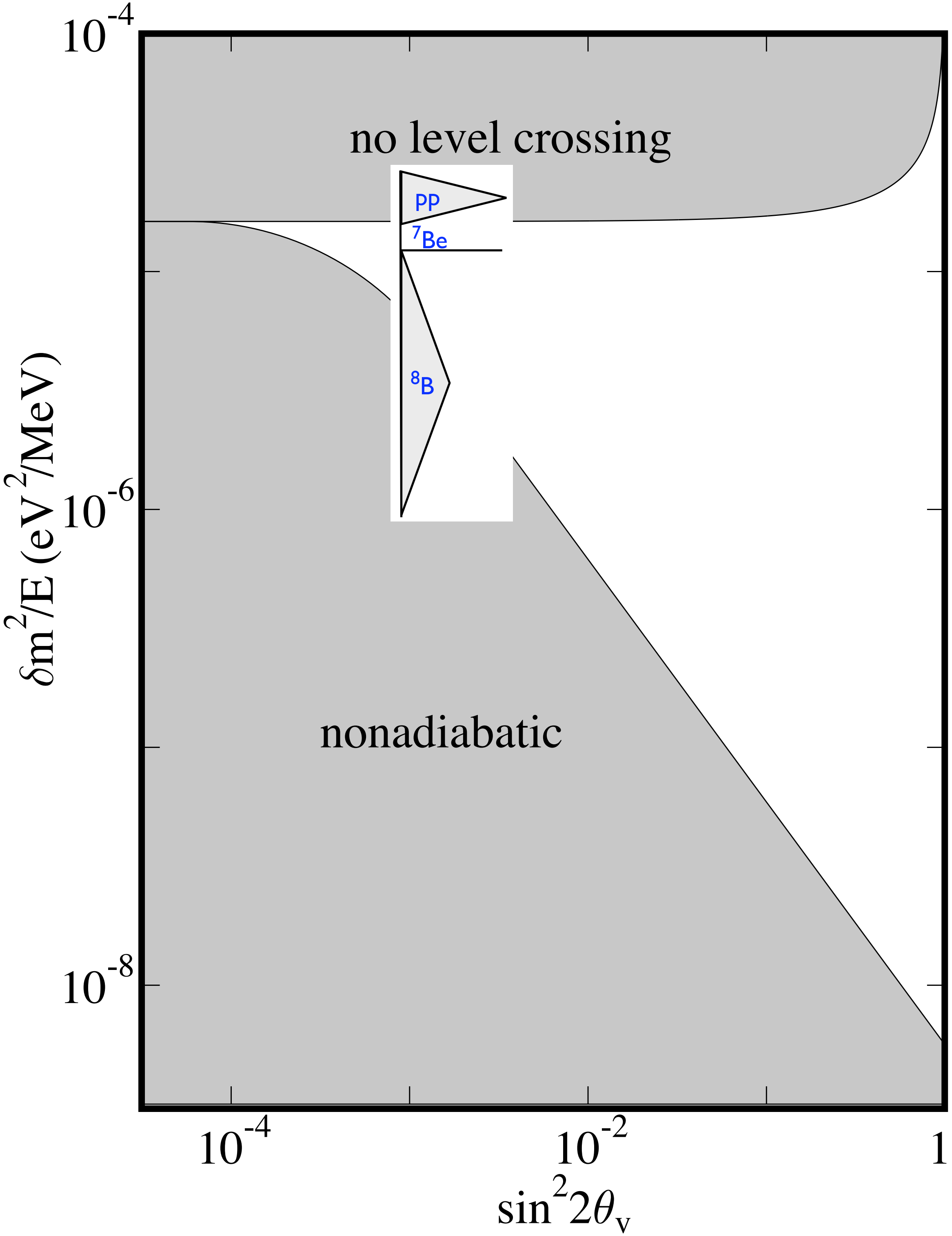
r (units of $r_\odot$)
$\rho(r)/\rho(0)$
1.0
0.8
0.6
0.4
0.2
0.0
r_c
0.0
0.2
0.4
0.6
0.8
1.0
r (units of $r_\odot$)



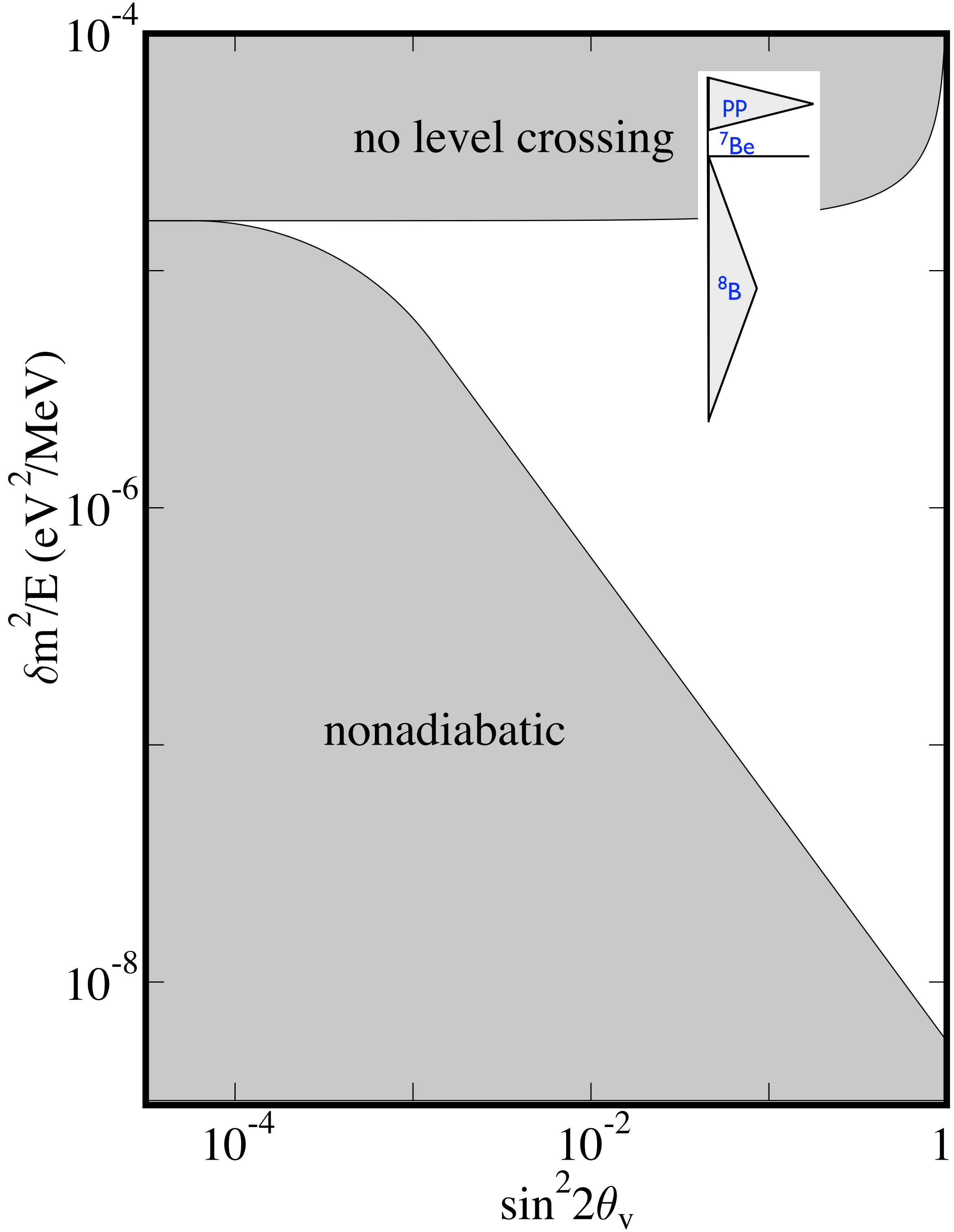
no level crossing
Flavor conversion here
γ<<1
nonadiabatic
pp
7Be
8B
10-4
10-6
10-8
10-4
10-2
1
δm2/E (eV2/MeV)
sin22θv



Low
solution



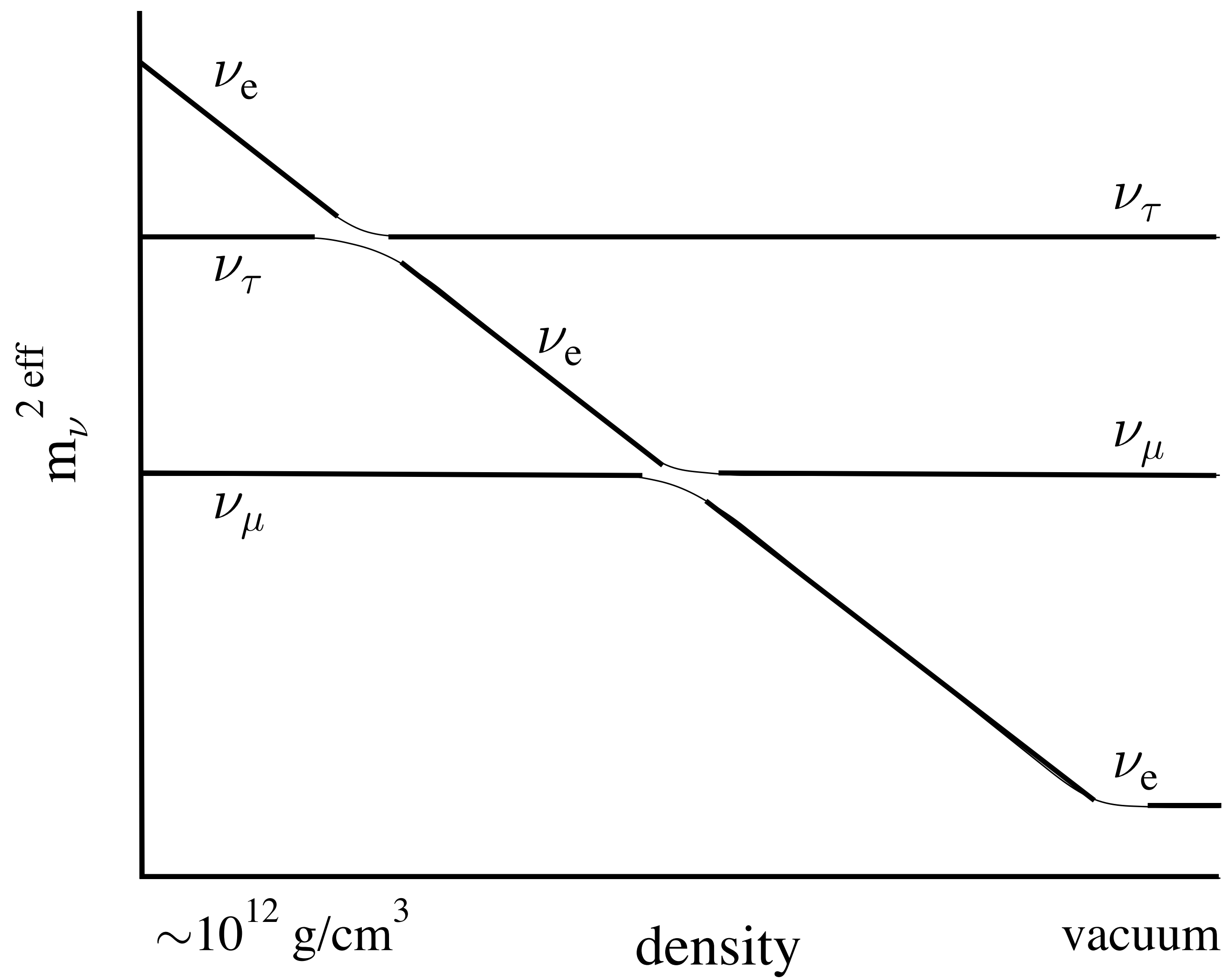
Small angle solution



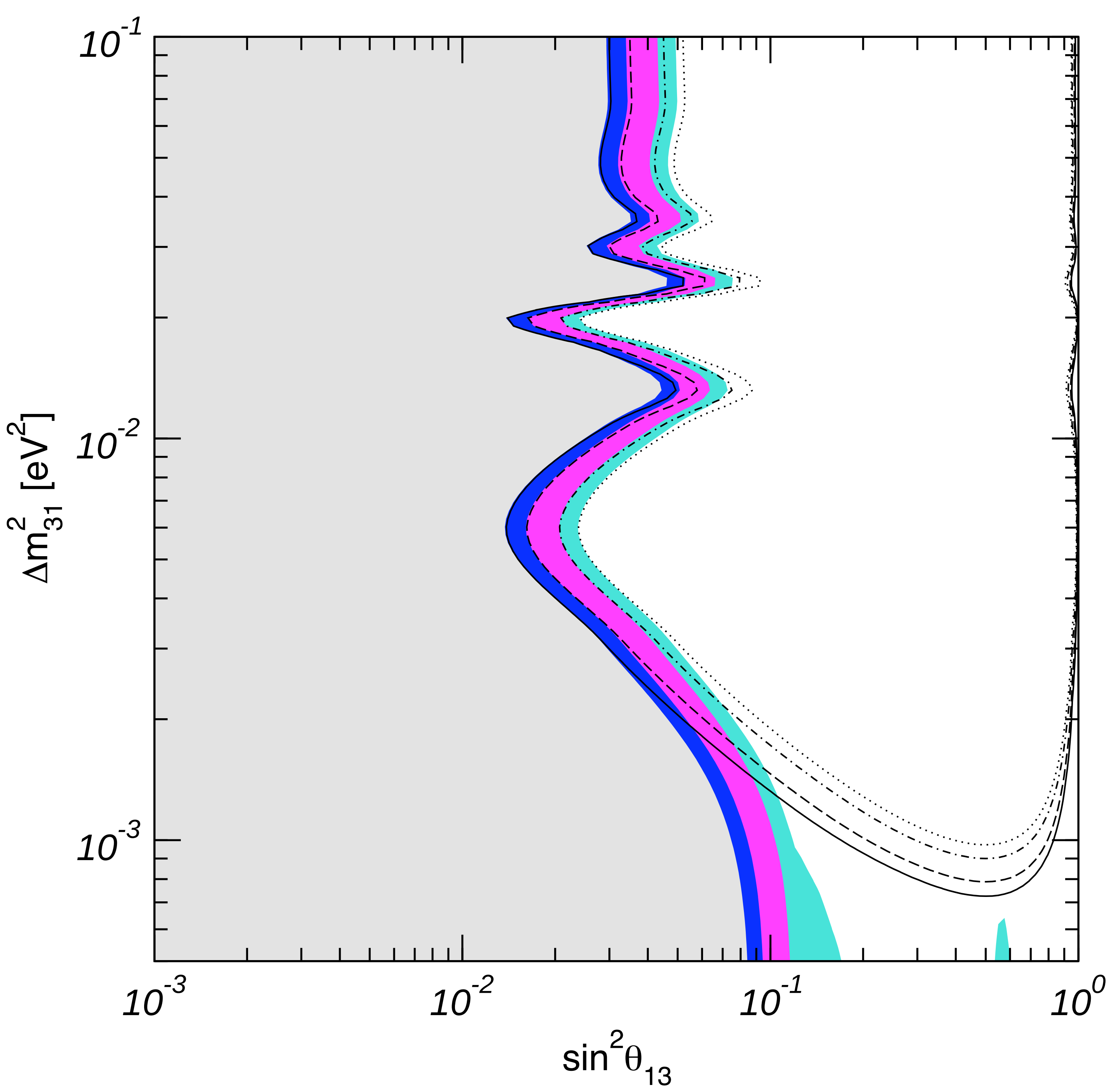
Large angle solution

this is the solution matching SNO and SuperK results + Ga/Cl/KII

$\tan^2\theta_v \sim 0.40$



ν_e
ν_τ
ν_τ
ν_e
ν_μ
ν_μ
ν_e
$m_\nu^{2\ \mathrm{eff}}$
$\sim 10^{12}$ g/cm^3
density
vacuum



Maltoni et al.

Art's talk: SNO and SuperK, KamLAND, K2K, MINOS, ...

What do we know about masses?

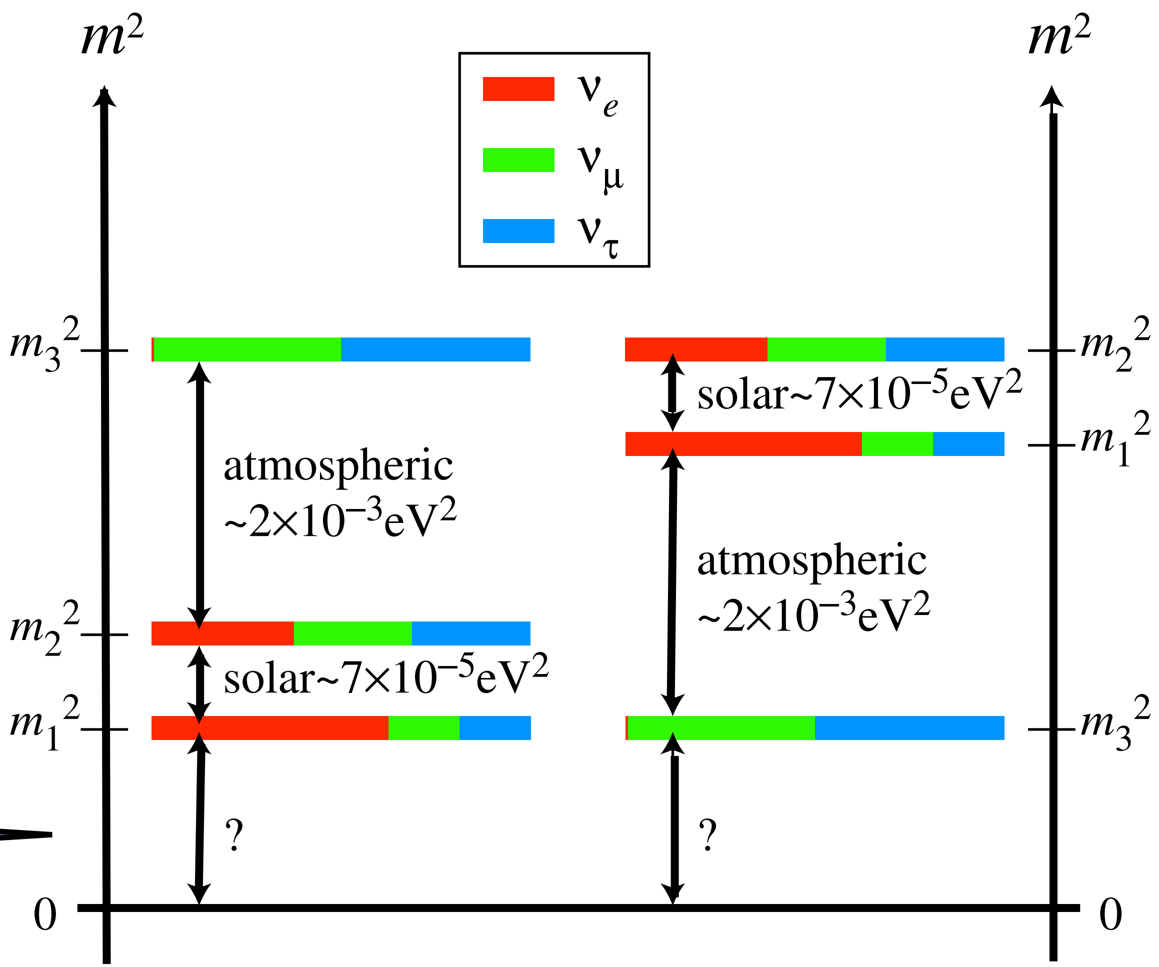
$m_{12}{}^2 \sim (8 \pm 1)\times10^{-5}\ eV^2$

$|m_{23}{}^2| \sim (2.2 \pm 0.8)\times10^{-3}\ eV^2$

WMAP + LSSS

$\sum m_i < 1\ eV$

Degenerate or hierarchical schemes allowed within this constraint



Mohapatra et al., APS study

Neutrino mixing status: θ_{12}, θ_{23}

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

	atmospheric	ν_e disappearance	solar
results:	$\theta_{23} \sim 45^\circ$	$\sin\theta_{13} \leq 0.17$	$\theta_{12} \sim 30^\circ$

Neutrino mass may be the first signature of physics at the GUT scale

$$\bar{\psi}_R m_D \psi_L + h.c. \qquad \bar{\psi}^c_L m_L \psi_L + \bar{\psi}^c_R m_R \psi_R$$

neither allowed in the minimal standard model

$$\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \Rightarrow m_\nu^{light} = m_D (\frac{m_D}{m_R})$$

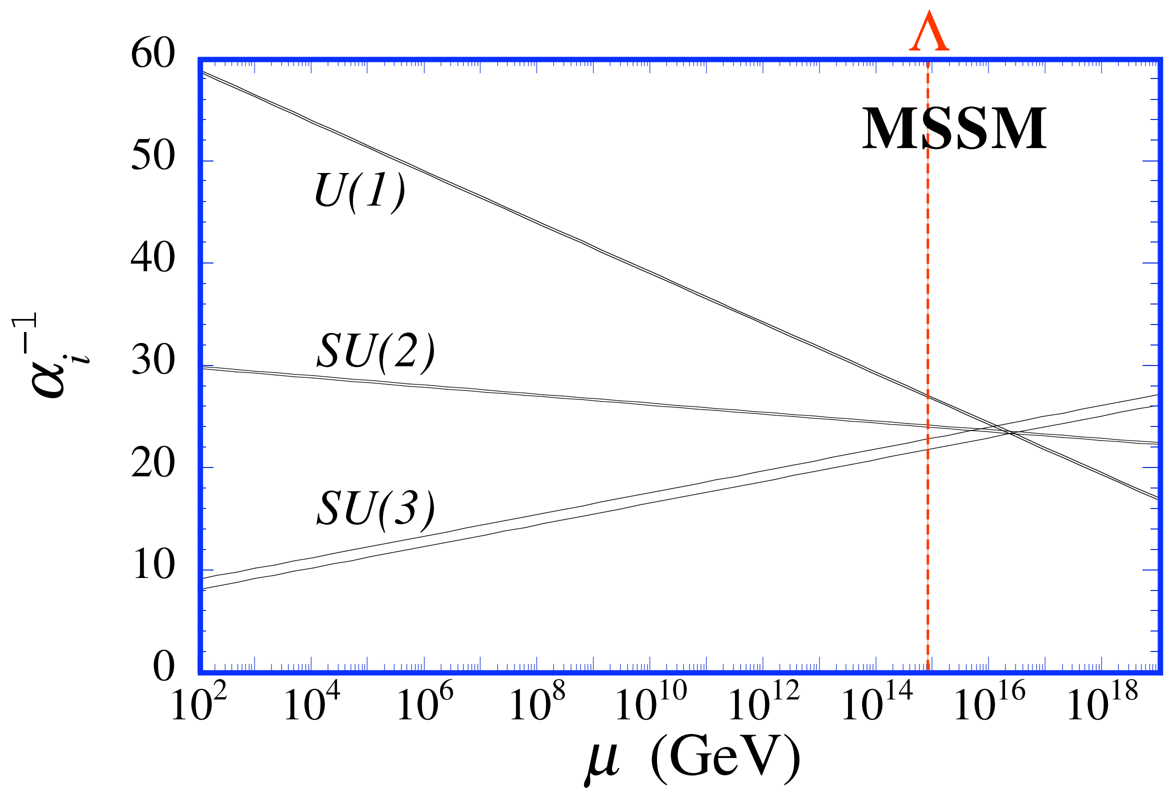
a natural explanation of the suppression (m_D/m_R) of light ν masses relative to the Dirac scale (m_D) of other SM fermions

using $m_3 \sim 0.05$ eV

$m_D \sim m_{top} \sim 180$ GeV

$\Rightarrow m_R \sim 0.3 \times 10^{15}$ GeV

Mohaptra et al.
APS study



Great program of future physics has been mapped out

- determining the absolute scale of neutrino mass: near-term $\beta\beta$ exps. and cosmological tests should reach 50 meV; future efforts to 10 meV
- measuring the unknown mixing angle θ_{13} in reactor or LB off-axis exps.
- demonstrating that Majorana masses exist in $\beta\beta$ decay
- distinguishing between the inverted and normal hierarchies in LB or next-generation atmospheric ν studies of subdominant oscillations
- seeing the effects of the Dirac CP phase in LB exps.
- once the masses and mixing angles are known, do the nuclear physics to high precision to constrain the Majorana phases in $\beta\beta$ decay

- includes future solar, supernova experiments to do astro-ν physics

Hans's CNO cycle and solar/stellar evolution

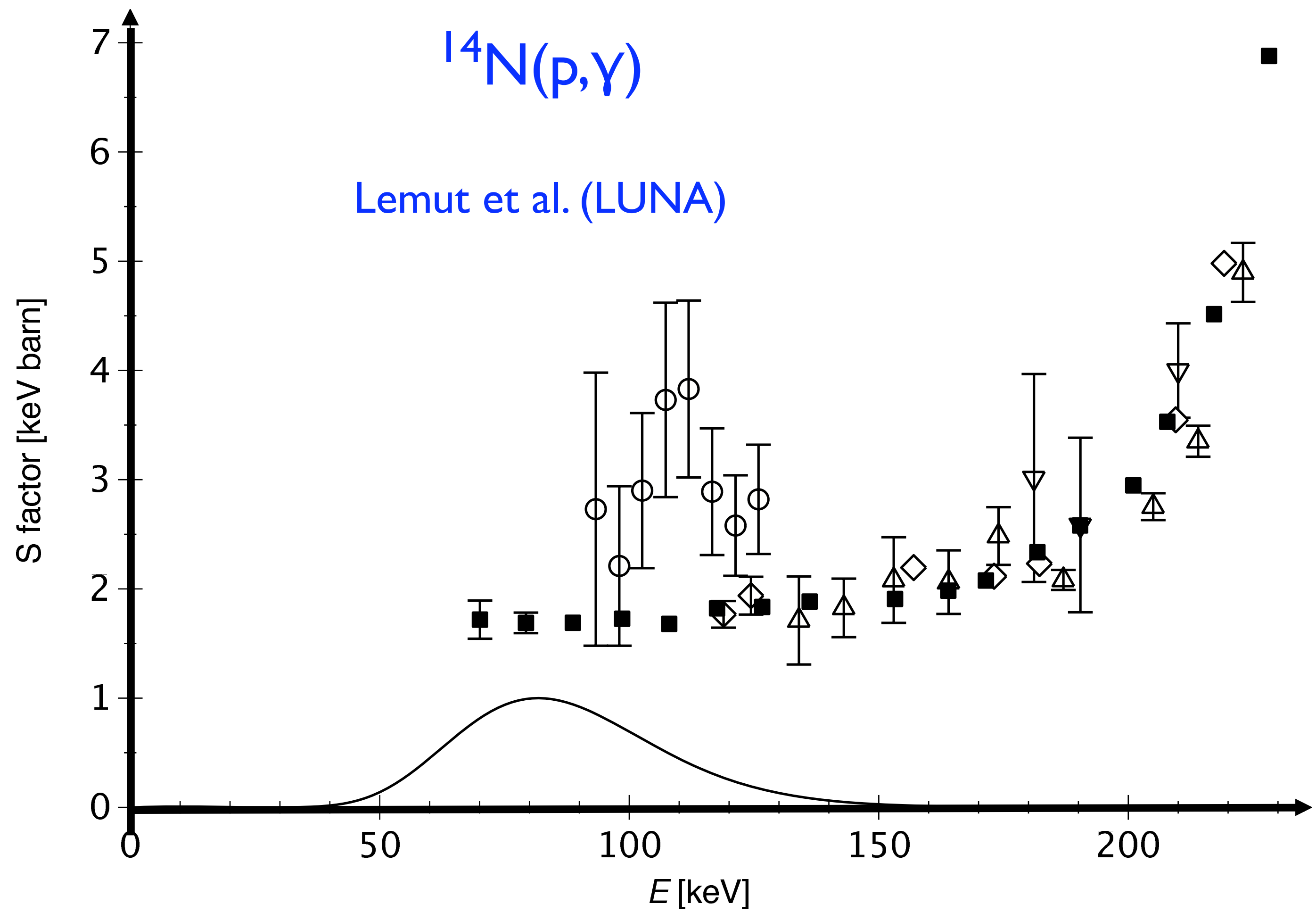
- while CNO cycle is a minor contributor to solar energy, CNO νs are measurable; test core metalicity crucial to solar opacity and SSM
- key SSM assumption: homogeneity on entering the main sequence
- current zero-age SSM metals ⇔ today's solar surface abundances
- tested in helioseismology: depth of the convective zone sensitive to Z
- CNO elements also control very early evolution of core: out-of-equilibrium burning keeps core convective for almost 100 My
- recent improvements in modeling solar atmosphere (3D) have revised surface Z downward by 30% ! ⇒ destroys previous SSM ⇔ helioseismology concordance and alters other SSM properties

Recent precise measurement of the controlling CNO cycle reaction

- S(0) measured in the Gamow peak, lower by 50%: 1.61± 0.08 keV-b

- delays point at which CNO cycle will take over from pp chain in hydrogen-burning stars

- lowers CNO ν flux



- solar-like stars within globular clusters important to formation history of the host galaxy

- GCs are a standard ruler for the age of the universe post "first light"

- new S(0) delays onset of CNO cycle and its steep T^{17} dependence: slows evolution along main sequence and onto subsequent RG and He-burning (AGB) tracks, with variable star "clocks"

- effect is estimated to be an increase in the ages of the oldest such stars of 0.8 Gy

Hans would be delighted by LUNA's exquisite measurement and by its effects on his 67-year-old theory of MS stellar evolution. He would be intrigued by the consequences.