Tertiary-Induced BH Binary Mergers

Dong Lai
Cornell University

Caltech Tapir Seminar 3/6/2020
Gravitational waveform gives $M_1, M_2, \chi_{\text{eff}}$

\[ \chi_{\text{eff}} \equiv \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2} \cdot \hat{L} \]
Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation: several flavors (star clusters, triples...)

How to distinguish different channels?
Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation: several flavors (star clusters, triples…)

How to distinguish different channels?

Rates (uncertain)?
Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation: several flavors (star clusters, triples…)

How to distinguish different channels?

Rates (uncertain)?
Residual eccentricity when enter LIGO band (10Hz)
Formation Channels of Merging BH Binaries

-- Isolated Binary Evolution

-- Dynamical Formation: several flavors (star clusters, triples…)

How to distinguish different channels?

Rates (uncertain)?
Residual eccentricity when enter LIGO band (10Hz)
Spin-orbit misalignmement
Standard Binary Evolution Channel:

many papers, uncertain physical ingredients

Produce circular orbit at 10 Hz aligned spin-orbit

Belczynski +16
Dynamical Formation Channels
several flavors...

1. Dense clusters: binary-single scatterings $\rightarrow$ tight binary

Produce mostly circular orbit when enter LIGO band (10 Hz)??
Expect random spin-orbit orientations

Portegies Zwart & McMillan 2000; Rodriguez et al.2015; Chatterjee et al.2017; Samsing et al. 2018; ...
Dynamical Formation Channels
several flavors...

1. Dense clusters: binary-single scatterings ➔ tight binary

2. Tertiary-Induced Mergers:
   Mergers induced by (gentle) perturbations from tertiary companion
   stellar triples in galactic field, binary around SMBH
Tertiary-Induced Binary Mergers

merger window, residual eccentricity, spin-orbit misalignments

Liu & DL 2017, 2018, 2019
Liu, DL & Wang 2019a, b

Previous/related works (in various contexts):
e.g. Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Thompson 2011; Antonini et al. 2012, 2014, 2017, Silsbee & Tremaine 2017; Petrovich & Antonini 2017...
Lidov-Kozai oscillations

- **Inner binary + tertiary companion**
  - Eccentricity and inclination oscillations induced if \( i > 40^\circ \)
  - If \( i \) large (85-90 degrees), get extremely large eccentricities (\( e > 0.99 \))

- **Orbital (secular) evolution**

![Graphs showing orbital evolution over time](image)
LK oscillation + Gravitation Radiation

\[ a_1 = 30 \, M_{\odot}, \quad a_2 = 20 \, M_{\odot}, \quad a_3 = 30 \, M_{\odot} \]
\[ a_{\text{in}} = 100 \, \text{AU}, \quad a_{\text{out}} = 6000 \, \text{AU} \]
\[ e_{\text{in}} = 0.001 \]
\[ e_{\text{out}} = 0.6 \]
Inclination window for merger

**Fixed inner binary:** \( m_1 = 30M_\odot, m_2 = 20M_\odot, a_{in,0} = 100\text{AU} \)

**Fixed** \( m_3/a_{out}^3 \) **value**

Quadrupole LK: \( e_{max} \) vs \( I_0 \) analytic: LK driving compete with GR apsidal precession

Merger window (almost) analytic
Inclination window for merger

Fixed inner binary: $m_1 = 30M_\odot$, $m_2 = 20M_\odot$, $a_{in,0} = 100$ AU

Fixed $m_3/a_{out}^3$ value

- $e_{out} = 0$
- $e_{out} = 0.3$
- $e_{out} = 0.9$
- $e_{out} = 0.6$
Inclination window for merger

Fixed inner binary: $m_1=30M_\odot$, $m_2=20M_\odot$, $a_{in,0}=100$AU

Fixed $m_3/a_{out}^3$ value

With octupole, $e_{lim}$ is still analytic

Octupole broadens the LK merger window
What about the BH Spin?
Spin-Orbit Coupling

The de Sitter precession of spin around the angular momentum axis of the binary

\[ \frac{\dot{S}_1}{dt} = \Omega_{ds} \hat{L} \times \hat{S}_1 \]

\[ \Omega_{ds} = \frac{3Gn(m_2 + \mu/3)}{2c^2a(1-e^2)} \]
BH spin dynamics during LK oscillations

\[ \frac{d\mathbf{S}_1}{dt} = \Omega_{dS} \mathbf{L} \times \mathbf{S}_1 \]  
(de Sitter Precession)

But \( \mathbf{L} \) precesses and nutates during LK oscillations

\[ \Omega_{pl} \approx \frac{3(1 + e^2)}{t_{LK} \sqrt{1 - e^2}} |\sin 2I| \]
BH spin dynamics during LK oscillations

\[ \frac{d\hat{S}_1}{dt} = \Omega_{dS} \hat{L} \times \hat{S}_1 \]  
(de Sitter Precession)

But \( \mathbf{L} \) precesses and nutates during LK oscillations:

\[ \Omega_{pl} \approx \frac{3(1 + e^2)}{t_{LK} \sqrt{1 - e^2}} |\sin 2I| \]

Spin dynamics depends on

\[ \Omega_{dS} \text{ vs } \Omega_{pl} \]
BH spin evolution in LK-induced orbital decay

\[ \frac{\Omega_{dS}}{\Omega_{pl}} \]

changes from \( << 1 \) (non-adiabatic) to \( >> 1 \) (adiabatic) as the orbit decays

\( \Rightarrow \) Final spin-orbit misalignment angle
Merger Window and Final Spin-Orbit Misalignments

Fixed inner binary: $m_1 = 30 M_\odot$, $m_2 = 20 M_\odot$, $a_{in,0} = 100$ AU

Fixed $m_3/a_{out}^3$ value
Merger Window and Final Spin-Orbit Misalignments

Fixed inner binary: $m_1 = 30M_\odot$, $m_2 = 20M_\odot$, $a_{in,0} = 100$ AU

Fixed $m_3/a_{3\text{out}}^3$ value

90° “attractor” (Adiabatic Invariance)
Effective Spin Distribution

A unique signature of LK-induced mergers
Effective Spin Distribution

For “reasonable” initial binary/triple parameters ($e_0=0$, distant companions)
Effective Spin Distribution

For “reasonable” initial binary/triple parameters ($e_0=0$, distant companions)

Consider ALL possible parameters
Effective Spin Distribution

For “reasonable” initial binary/triple parameters (e\(_0\)=0, distant companions)

Consider ALL possible parameters

Observed so far
Residual Eccentricity (at 10 Hz)

BH-BH mergers

10% have $e_m > 0.1$

1% have $e_m > 0.9$
Residual eccentricity vs Spin-orbit Misalignment

Circular Mergers \( (e_m < 10^{-3}) \) prefer \( \theta_{sl}^f \sim 90^\circ \)

More eccentric Mergers has random \( \theta_{sl}^f \)
What happens if the tertiary is a Supermassive BH?

--- Relativistic Effects induced by the SMBH
1. **Lense-Thirring Precession of $L_{\text{out}}$ around $S_3$**

--- introduced by the spin of the SMBH

\[
\left. \frac{dL_{\text{out}}}{dt} \right|_{L_{\text{out}}S_3} = \Omega_{L_{\text{out}}S_3} \hat{S}_3 \times L_{\text{out}},
\]

\[
\left. \frac{de_{\text{out}}}{dt} \right|_{L_{\text{out}}S_3} = \Omega_{L_{\text{out}}S_3} \hat{S}_3 \times e_{\text{out}}
- 3\Omega_{L_{\text{out}}S_3} (\hat{L}_{\text{out}} \cdot \hat{S}_3) \hat{L}_{\text{out}} \times e_{\text{out}}
\]

\[
\Omega_{L_{\text{out}}S_3} = \frac{GS_3 (4 + 3m_{12}/m_3)}{2c^2 a_{\text{out}}^3 (1 - e_{\text{out}}^2)^{3/2}}.
\]
2. de-Sitter-like Precession of $L_{\text{in}}$ around $L_{\text{out}}$

--- modifies the eccentricity growth indirectly

\[
\frac{dL_{\text{in}}}{dt} \bigg|_{L_{\text{in}}L_{\text{out}}} = \Omega_{L_{\text{in}}L_{\text{out}}} \hat{L}_{\text{out}} \times L_{\text{in}}
\]

\[
\Omega_{L_{\text{in}}L_{\text{out}}} = \Omega^{(N)}_{L_{\text{in}}L_{\text{out}}} + \Omega^{(GR)}_{L_{\text{in}}L_{\text{out}}}
\]

\[
\Omega^{(N)}_{L_{\text{in}}L_{\text{out}}} = -\frac{3}{4} \Omega_{L_{\text{K}}}(\hat{L}_{\text{out}} \cdot \hat{L}_{\text{in}}) \quad \text{(for $e_{\text{in}} = 0$)}.
\]

\[
\Omega^{(GR)}_{L_{\text{in}}L_{\text{out}}} = \frac{3}{2} \frac{G(m_3 + \mu_{\text{out}}/3)n_{\text{out}}}{c^2 a_{\text{out}}(1 - e_{\text{out}}^2)}
\]

Cross terms in PN (Will 2014,18)
Inclination Resonance

\[ \gamma \equiv \frac{\Omega_{\text{LinLout}}}{\Omega_{\text{LoutS}_3}} = \frac{\Omega_{\text{LinLout}}^{(N)}}{\Omega_{\text{LoutS}_3}} + \frac{\Omega_{\text{LinLout}}^{(GR)}}{\Omega_{\text{LoutS}_3}} \]
Summary
Formation Channels of Merging BH Binaries

**Standard isolated binary evolution channel:**
uncertain physics (common envelope…)

- circular mergers ($e_m=0$)
  - aligned spin-orbit angle

**Dynamical formation channels:**
“clean” physics, but “environmental” uncertainties

1. **Dense star clusters**
   - mostly circular mergers ?
     - expect random spin-orbit misalignments ?

2. **Tertiary-induced mergers**
   Perturbations from outer companion $\rightarrow$ Lidov-Kozai
   Spin-orbit coupling (de Sitter precession) important..

- 10% mergers have residual $e>0.1$ when entering LIGO band
  - Preference of $90^0$ spin-orbit misalignment, especially for circular mergers

Rates?  All potentially compatible…
LISA useful for probing dynamical formation