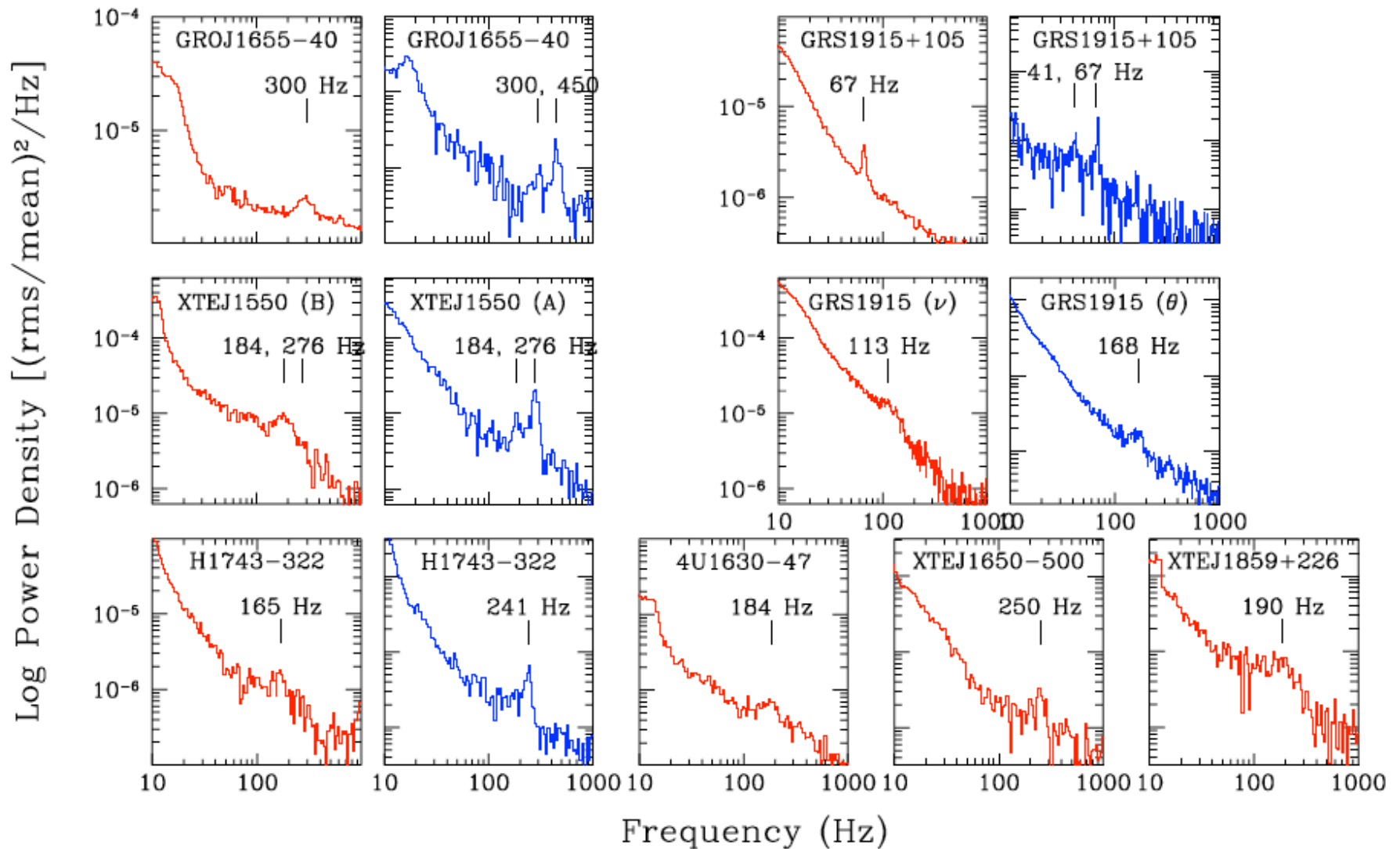


# **Global Oscillations of BH Accretion Flows and High-Frequency QPOs**

Dong Lai  
Cornell University

Prague Synergy 2013, “Relativistic Compact Objects”, 2013.11.26

# High-Frequency QPOs in BH X-Ray Binaries



Remillard & McClintock 2006; also Belloni et al. 2012

## Basic Facts about HFQPOs

- 40-450 Hz:  $\sim$  orbital frequency at  $r_{\text{isco}}$
- Frequency stable ( $<10\%$  change when  $\dot{M}$  doubles)
- Some systems:  $\sim 2:3$  ratio
- Only occur in “Transitional state” (=“very high state”) (Episodic jet)
- Weak QPOs:  $\sim 1\%$  flux variation (in hard X-rays),  $Q \sim 2-10$

Reviews: Remillard & McClintock 2006; Belloni, Sanna, Mendez 2012

Phenomenology will be improved by future missions (e.g. LOFT...)

# Theories/Models of HFQPOs

HFQPOs are related to (and thus probe) strong gravity...

**HOW ?**

**Many Ideas/Models of HFQPOs (?)**

# Ideas/Models of HFQPOs

- Orbiting blobs (hot spots) in disks (Stella et al '99; Schnittman & Bertschinger '04)

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## Weak nonlinear coupling between epicyclic modes in slender tori

J. Horák

$$\kappa_{ABC}^{(p)} = \frac{1}{2} \int_V p \left\{ (\gamma - 1)^2 \eta_A \eta_B \eta_C + 3(\gamma - 1) \eta_{(A} \eta_{BC)} + 2\eta_{(ABC)} \right\} dV, \quad (12)$$

$$\kappa_{ABC}^{(g)} = -\frac{1}{2} \int_V \rho \xi_A^i \xi_B^j \xi_C^k \nabla_i \nabla_j \nabla_k \Phi dV, \quad (13)$$

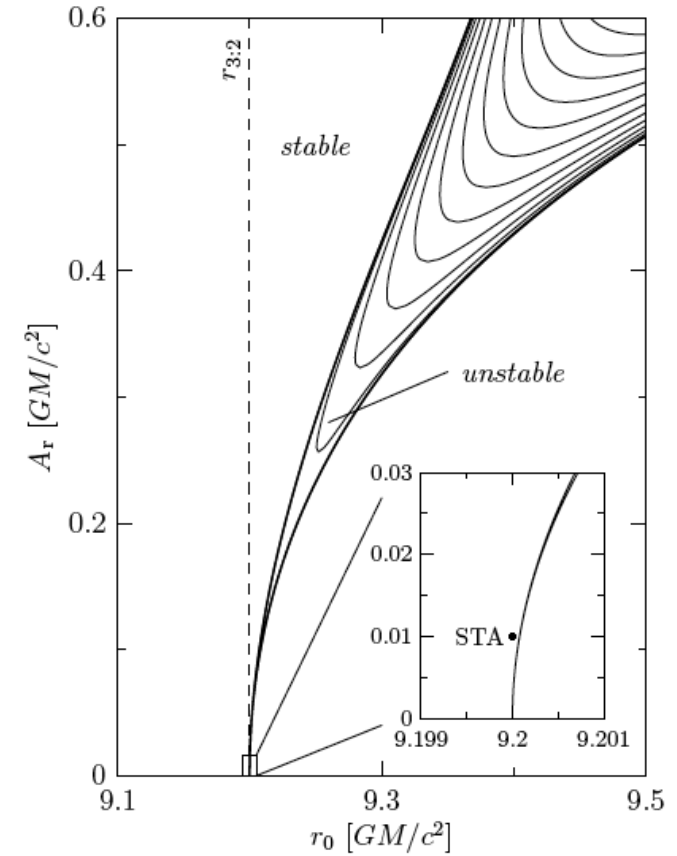
$$\kappa_{ABCD}^{(p)} = -\frac{1}{3!} \int_V \gamma p \left\{ (3 - 3\gamma + \gamma^2) \eta_A \eta_B \eta_C \eta_D + 8 \eta_{(A} \eta_{BCD)} + 6(\gamma - 2) \eta_{(A} \eta_{B} \eta_{CD)} + 3\eta_{(AB} \eta_{CD)} \right\} dV \quad (14)$$

$$\kappa_{ABCD}^{(g)} = -\frac{1}{3!} \int_V \rho \xi_A^i \xi_B^j \xi_C^k \xi_D^l \nabla_i \nabla_j \nabla_k \nabla_l \Phi dV \quad (15)$$

and

$$\kappa_{ABCDE}^{(p)} = \frac{1}{4!} \int_V \gamma p \left\{ (1 + 6\gamma - 4\gamma^2 + 3\gamma^3) \eta_A \eta_B \eta_C \eta_D \eta_E + 10\gamma(\gamma - 3) \eta_{(A} \eta_{B} \eta_{C} \eta_{DE)} + 20\gamma \eta_{(A} \eta_{B} \eta_{CDE)} + 15(\gamma - 1) \eta_{(A} \eta_{BC} \eta_{DE)} + 20\eta_{(AB} \eta_{CDE)} \right\} dV, \quad (16)$$

$$\kappa_{ABCDE}^{(g)} = -\frac{1}{4!} \int_V \rho \xi_A^i \xi_B^j \xi_C^k \xi_D^l \xi_E^m \nabla_i \nabla_j \nabla_k \nabla_l \nabla_m \Phi dV, \quad (17)$$



➔ Resonance is very weak



# Ideas/Models of HFQPOs

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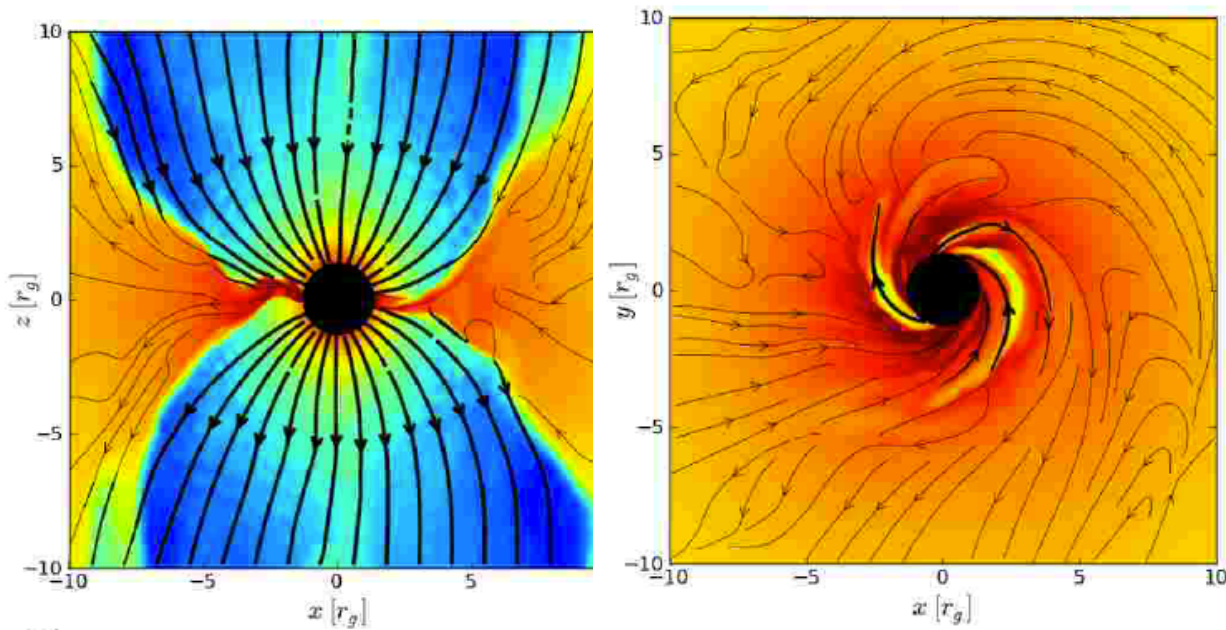
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- Disk/Magnetosphere Boundary Layer Oscillations (Li & Narayan '04; Tsang & DL '09; Fu & DL'12; see simulations by McKinney et al 2012)

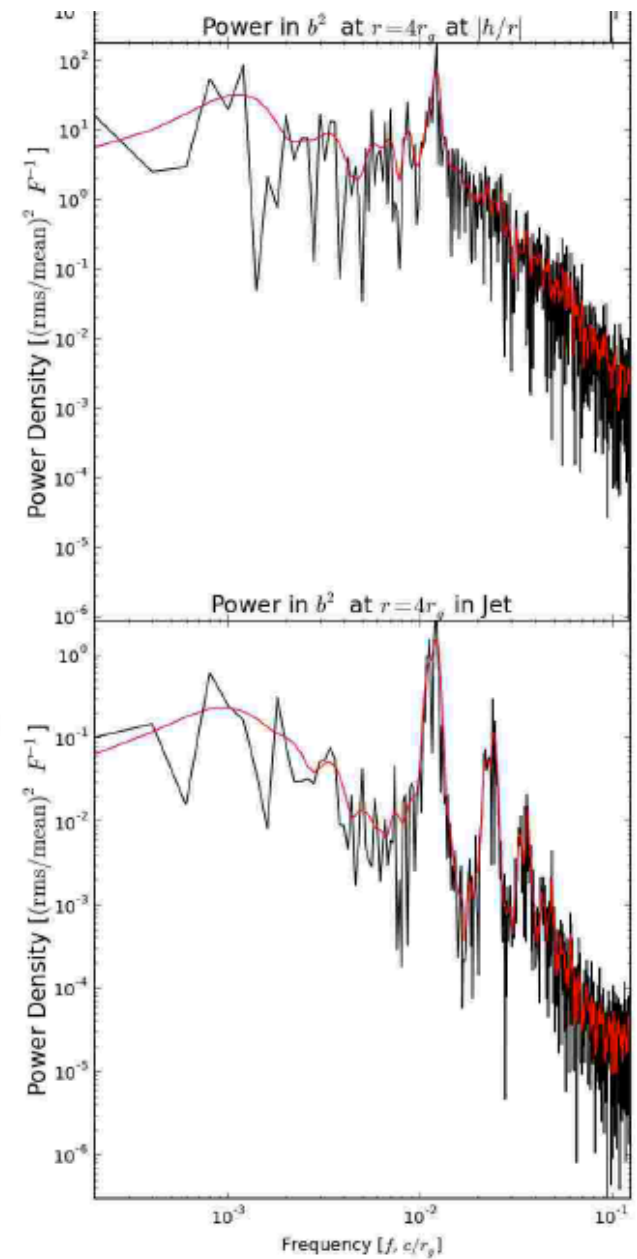
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- Oscillation modes in relativistic disks (Kato, Wagoner etc, Varniere, Tagger)

# GRMHD Simulations



Henisey et al. 2009; O'Neill et al. 2011;  
Dolence et al. 2012; [McKinney et al. 2012](#);  
Shcherbakov & McKinney 2013



# **Future theoretical progress on QPOs**

Requires

- Full GRMHD simulations
- Semi-analytic works (+Controlled studies)

# Ideas/Models of HFQPOs

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## Key issue: Excitation of the modes??

-- m=0 inertial modes excited by global disk deformation (e.g. warps)

(Kato '08; Ferreira & Ogilvie '08; Henissey et al.10; Kato 2012, 2013)

-- **Our effort:** Mode growth due to corotation resonance, nonlinear effect, magnetic field, turbulence, etc.

(DL & Tsang '09; Tsang & DL '08,'09a,b; Fu & DL '09,'11,12; Horak & DL'13; Miranda et al.'13; Yu & DL '13)

# Global Oscillations & Their Excitations in BH Accretion Disks

with [David Tsang](#) (Cornell→Caltech/McGill)  
[Wen Fu](#) (Cornell→Rice)  
[Jiri Horak](#) (Prague)  
[Cong Yu](#) (YNAO/CAS)  
[Ryan Miranda](#) (Cornell)

## Main points of our works:

### P-modes (“inertial-acoustic modes”, “spiral density modes”)

- Trapped (partially) in the innermost region of disk
- Frequencies can be calculated: robust, agree largely with observations
- Can grow due to corotation resonance (“corotational instability”)  
GR plays an important role
- Large-scale B field may enhance the mode growth  
(=> “transitional state”, connection with jets)
- Turbulent viscosity may reduce or enhance mode growth  
Also excite quasi-normal modes of disk

**References:** DL & Tsang 09; Tsang & DL 08, 09; Fu & DL 09,11,12  
Horak & DL 13; Yu & DL (in prep); Miranda, Horak & DL (in prep)



## Waves in 2D disks (Spiral density waves):

$$\delta v, \delta \Sigma \propto \exp(im\varphi - i\omega t)$$

Can propagate only in the region:

$$r < r_{\text{ILR}} \quad \text{or} \quad r > r_{\text{OLR}}$$

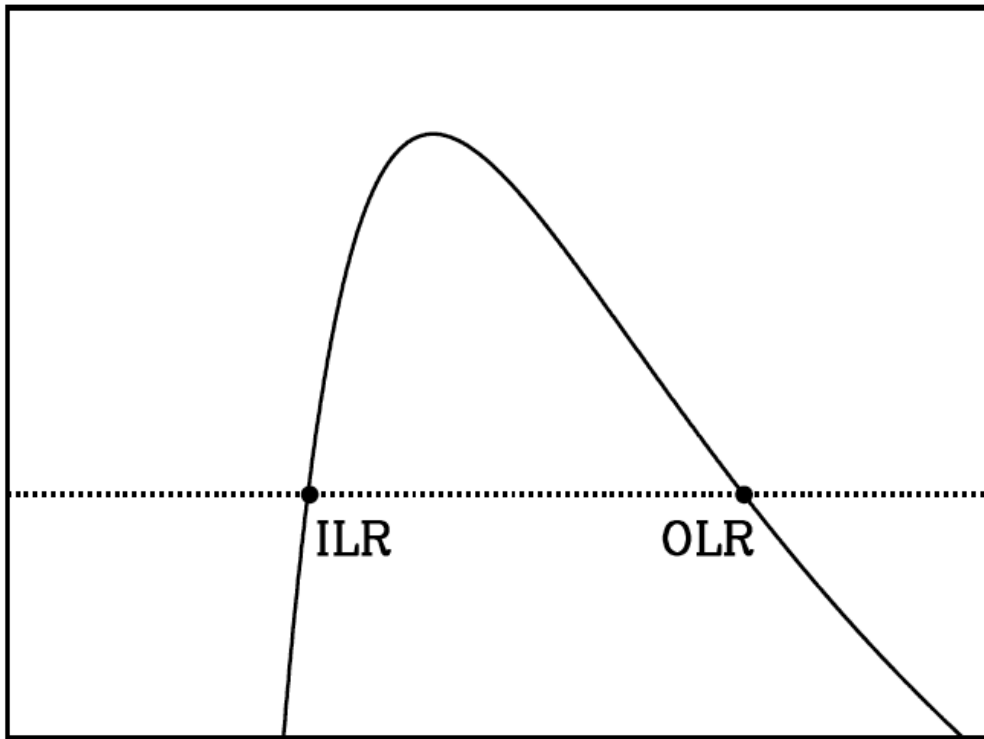
Lindblad Resonances:  $\omega - m\Omega(r) = \pm\kappa(r)$

where  $\Omega(r)$  = disk rotation rate

$\kappa(r)$  = radial epicyclic frequency

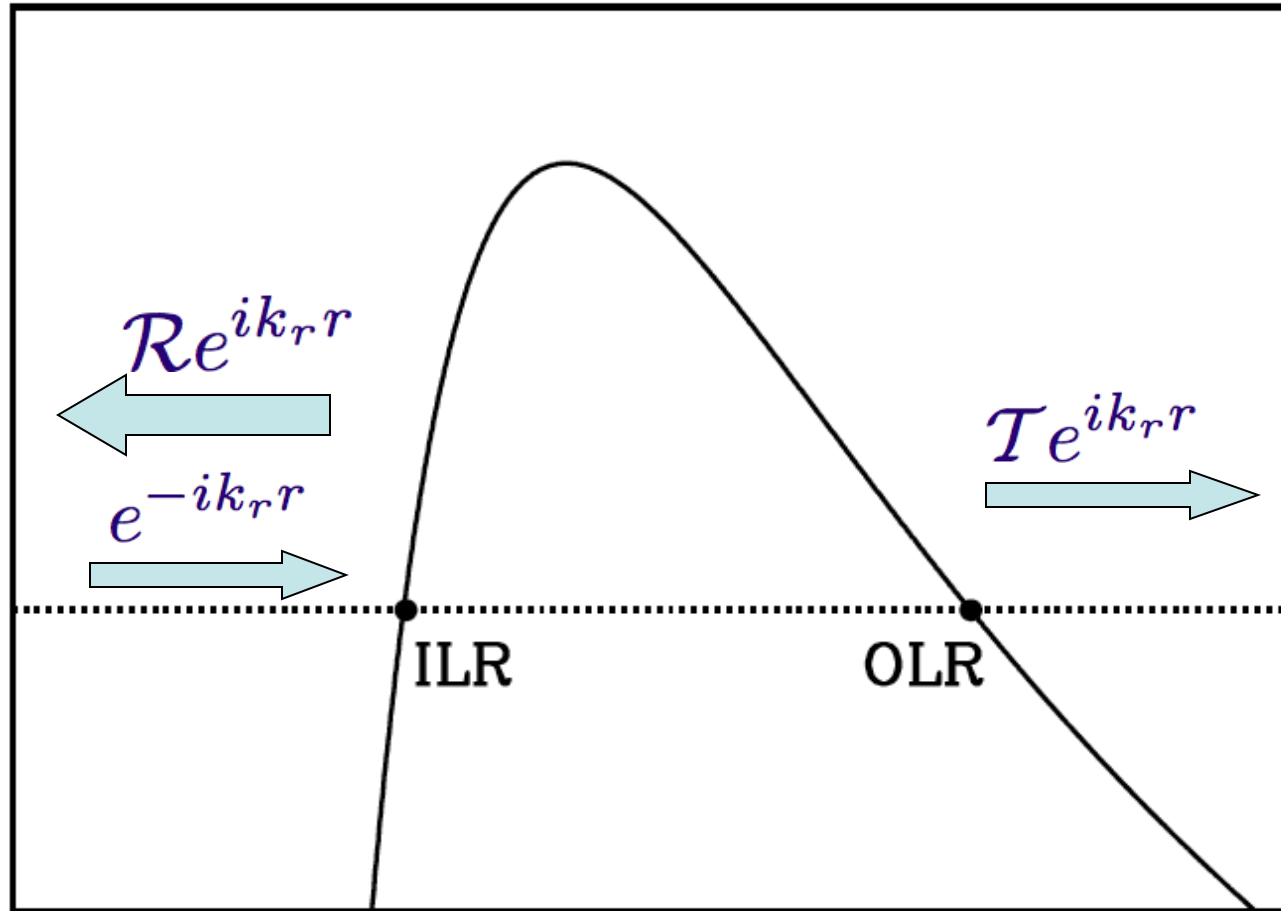
$$\kappa^2 = \frac{2\Omega}{r} \frac{d}{dr}(r^2\Omega)$$

## Wave propagation diagram (effective potential)



wave at  $r > r_{\text{OLR}}$  :  $\omega/m > \Omega \Rightarrow$  positive energy

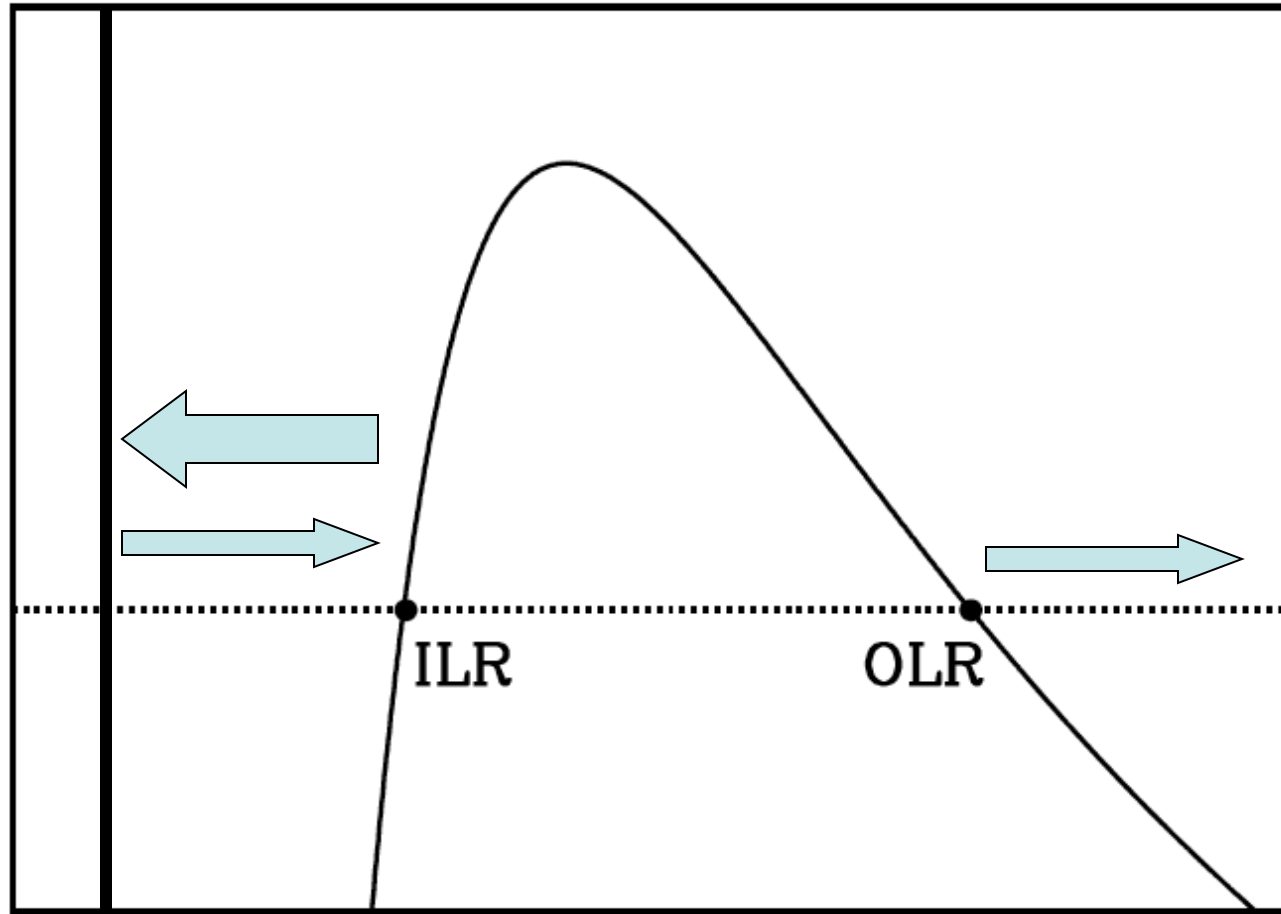
wave at  $r < r_{\text{ILR}}$  :  $\omega/m < \Omega \Rightarrow$  negative energy



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2$$

$$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 > 1$$

**Super-reflection**

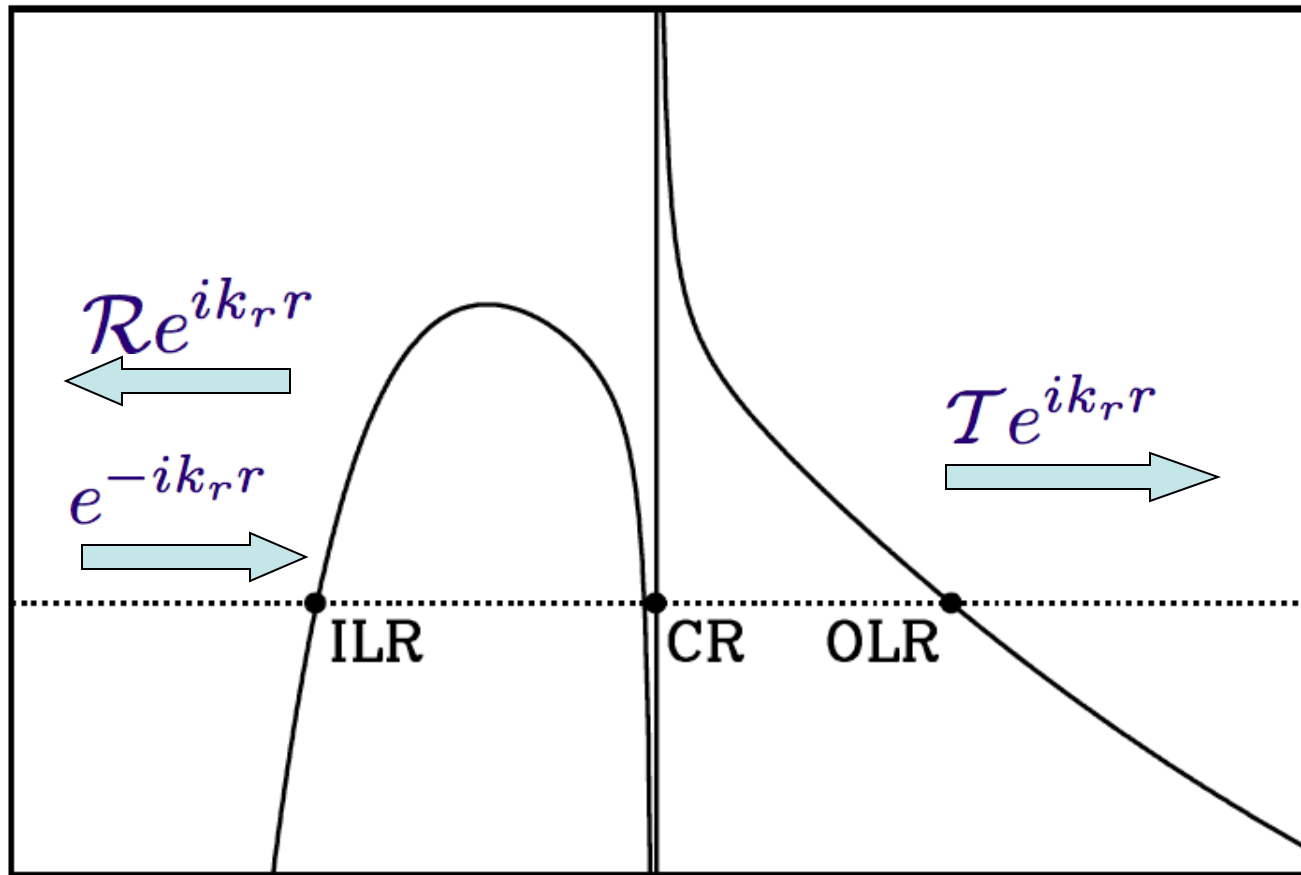


Trapped mode between  $r_{\text{in}}$  and  $r_{\text{ILR}}$  : overstable

Even more important/interesting...

**Corotation resonance**, where

$$\omega/m = \Omega$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2 + \mathcal{D}_c$$

$$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c$$

Wave absorption at corotation

Can have both signs !

**Reflectivity at ILR:**  $|\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c$

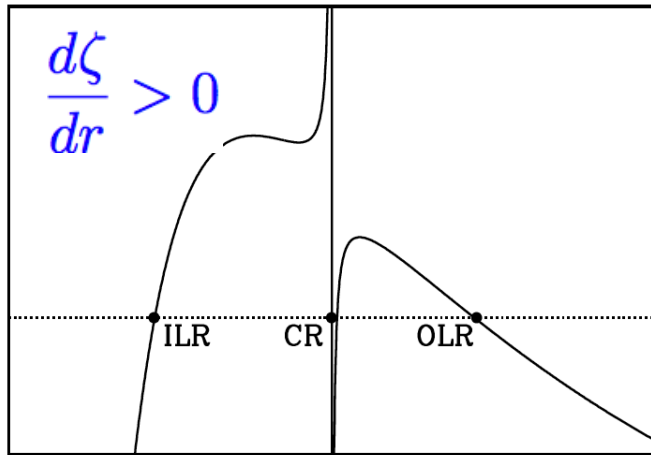
Sign depends on sign of  $d\zeta/dr$

$$\zeta = \frac{\kappa^2}{2\Omega\Sigma} \quad (\text{vortensity})$$

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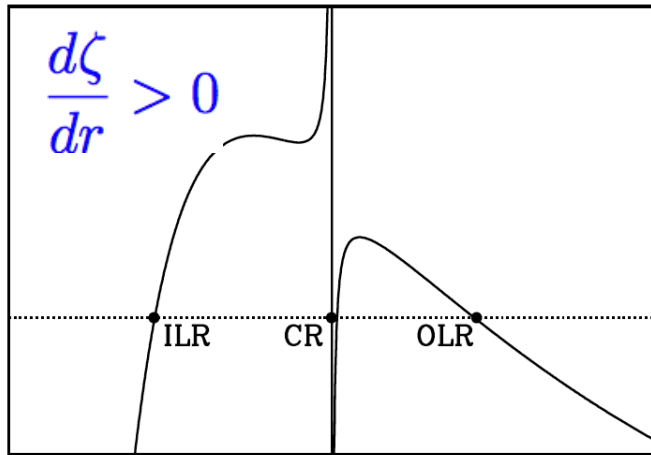
$$\Rightarrow \mathcal{D}_c > 0$$



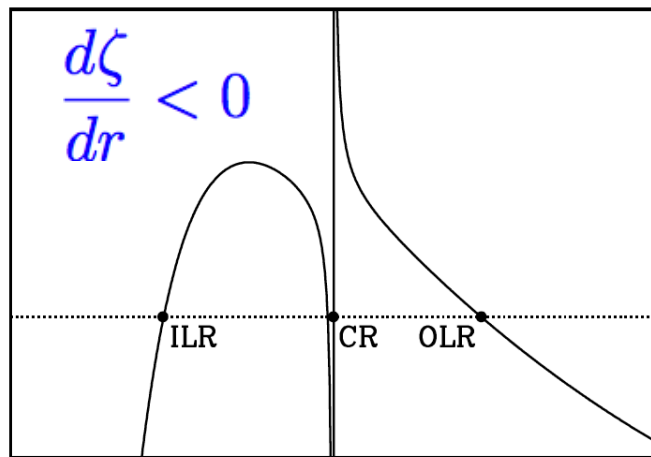
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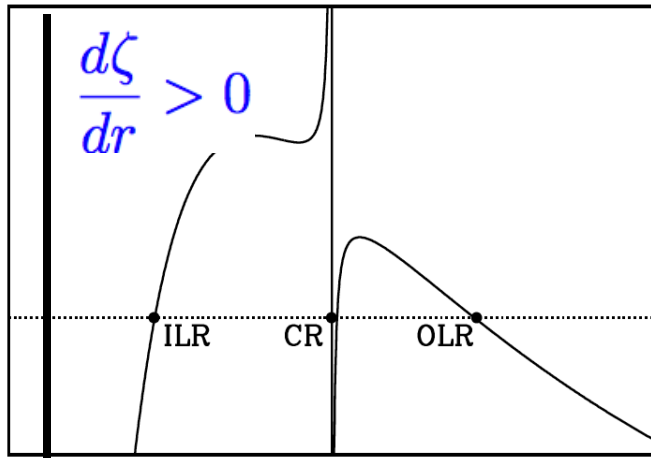


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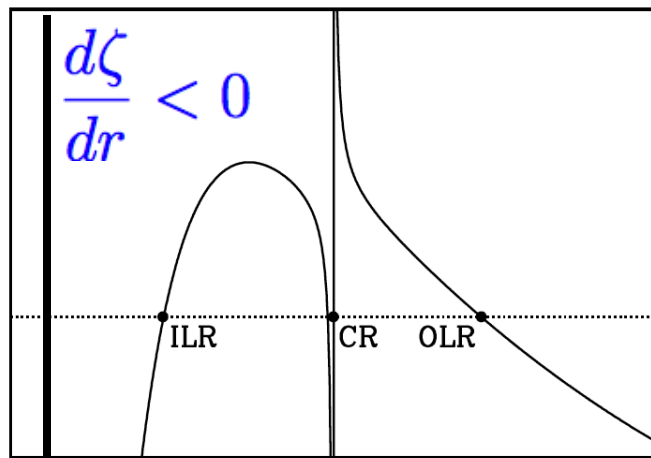
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$$\Rightarrow \mathcal{D}_c > 0$$

Overstable mode

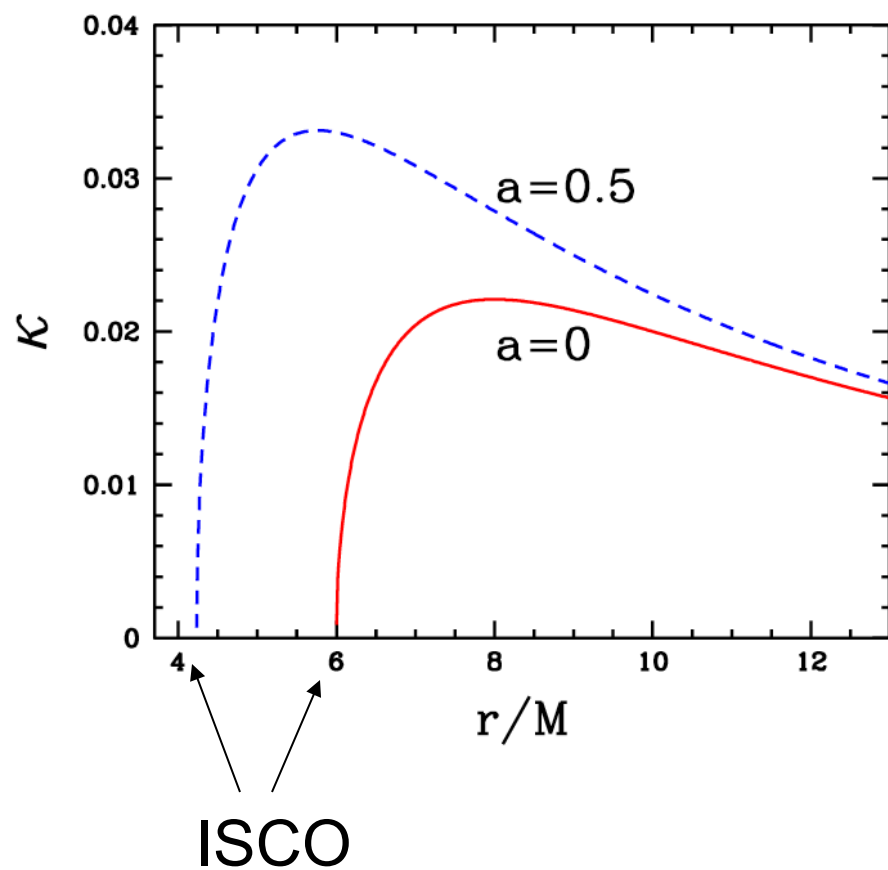


$$\Rightarrow \mathcal{D}_c < 0$$

Damped mode

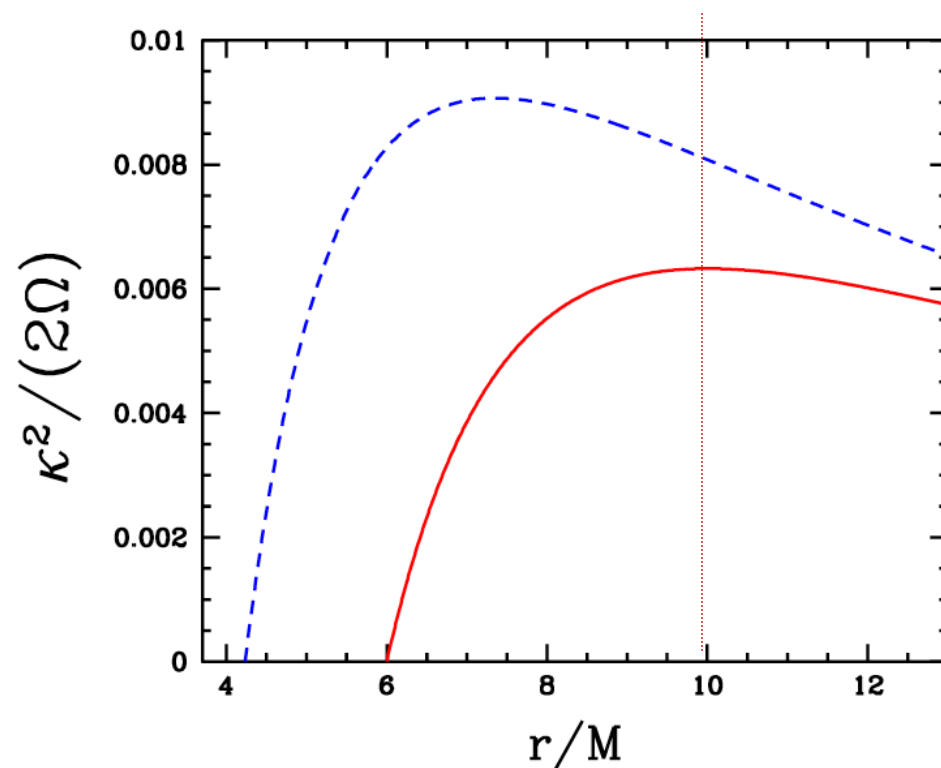
# General Relativity Effect

See Horak's talk

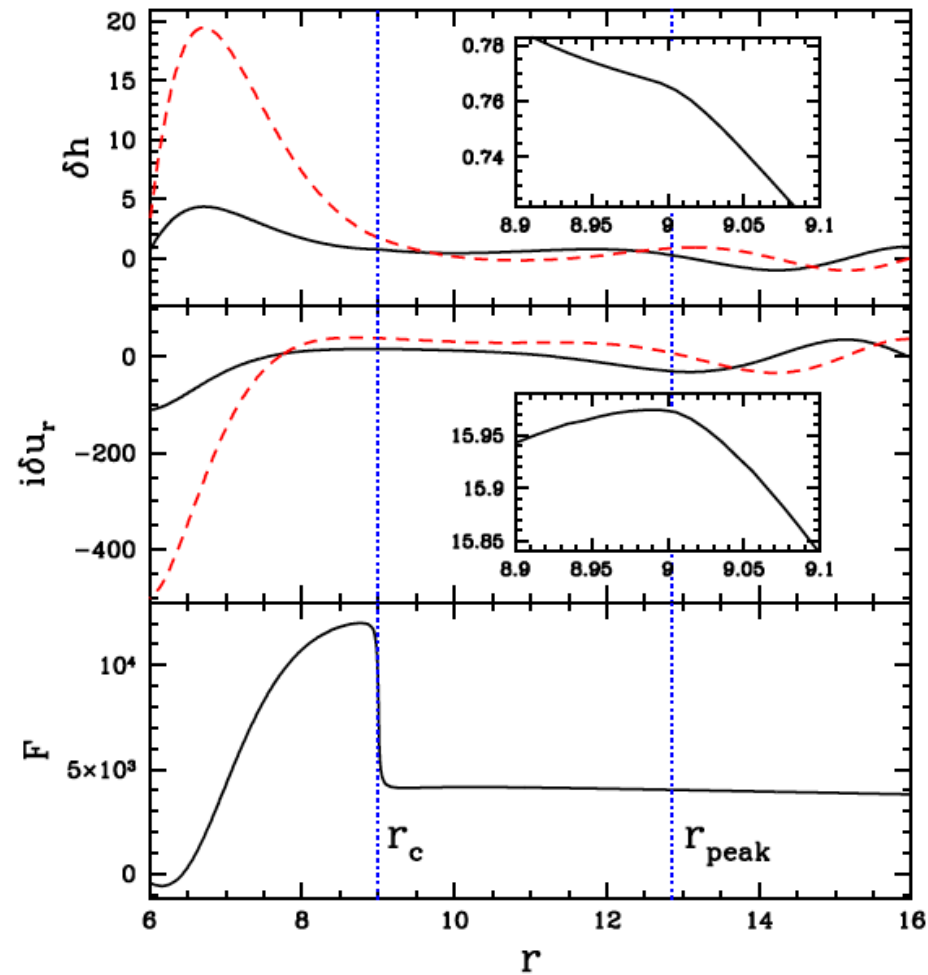


Vortensity  $\zeta = \frac{\kappa^2}{2\Omega\Sigma}$

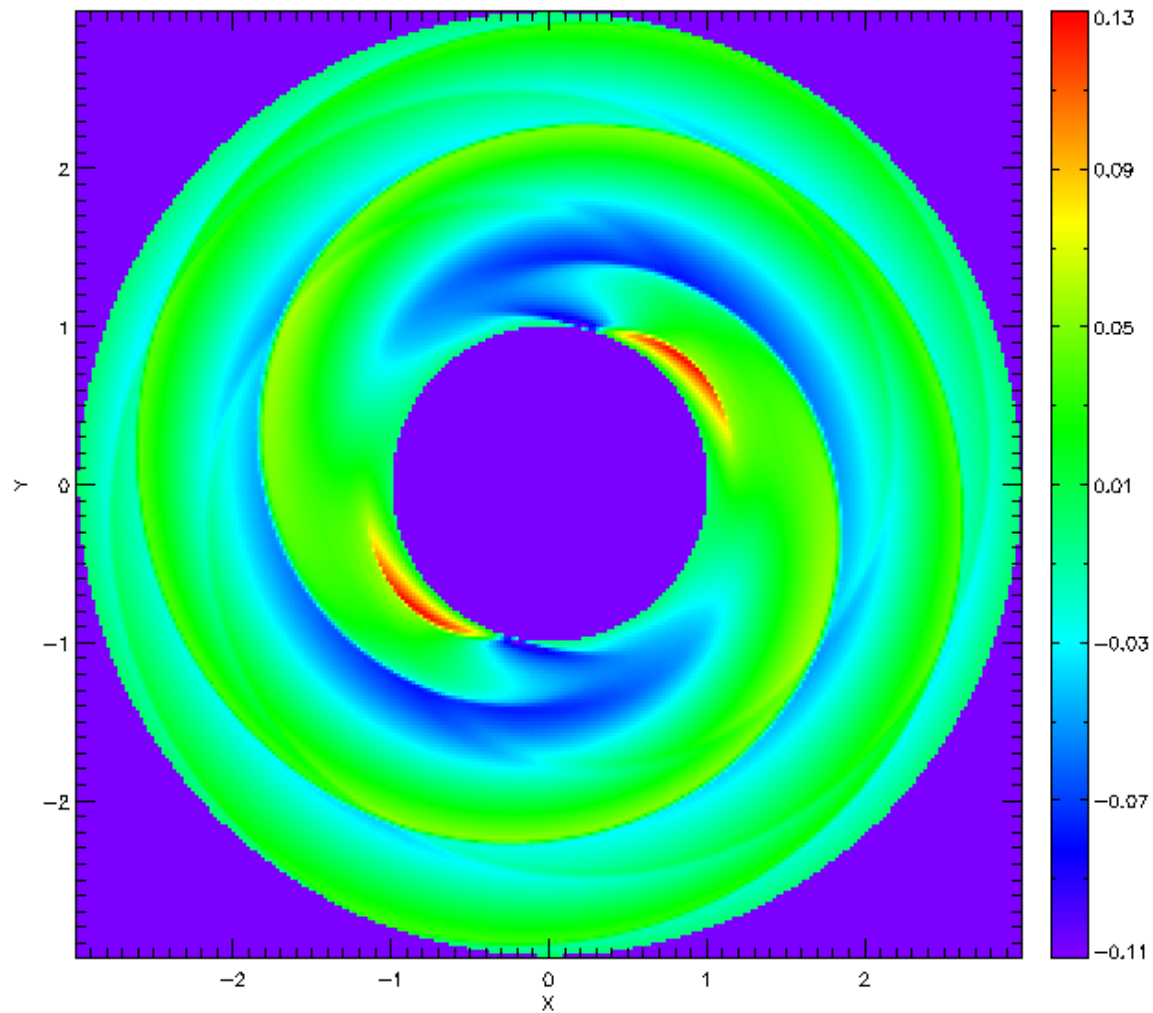
GR makes  $d\zeta/dr > 0$   
in the Inner-most disk region  
==> makes the mode grow !



## Linear Mode Calculation (Mode freq. and growth rate)



## Nonlinear Simulation (2D) of Growing Modes



Wen Fu & DL, 2012

# Results: Overstable Disk P-Modes

## Mode Frequencies:

- (Largely) consistent with known BH mass (and spin)
- varies ( $\sim 10\%$ ) as  $\dot{M}$  changes ( $\sim 3$ )
- Frequency ratio approximately: 1:2:3:4... (not exactly)

Mode growth due to corotation resonance  
(GR plays an important role)

A possible candidate for HFQPOs

# **Complications:**

## Mode damping due to radial infall

For standard thin (SS) disks, radial velocity increases rapidly near  $r_{\text{ISCO}}$ .

==> Tends to damp the p-modes;  
(but not completely, due to sharp density gradient around  $r_{\text{in}}$ )

**Competition:** mode growth (due to corotation) and damping (due to infall)

==> **Net result:**

In standard thin disk, p-modes are likely damped

==> No HFQPOs in thermal state



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==> No HFQPOs in thermal state

Real disks (in “intermediate state”) are more complicated...

**Magnetic fields...**

# Effects of Magnetic Fields on P-Modes

- Mode frequencies are slightly/modestly affected: Robust...
- Mode growth rates can be significantly affected:

## 1. Magnetic fields may accumulate inside $r_{\text{isco}}$

(e.g. Lovelace et al 2009; simulations by Hawley, Krolik etc)

==> The inner disk edge may be more reflective than standard SS disk (Tsang & DL 2009b; Fu & DL 2012)

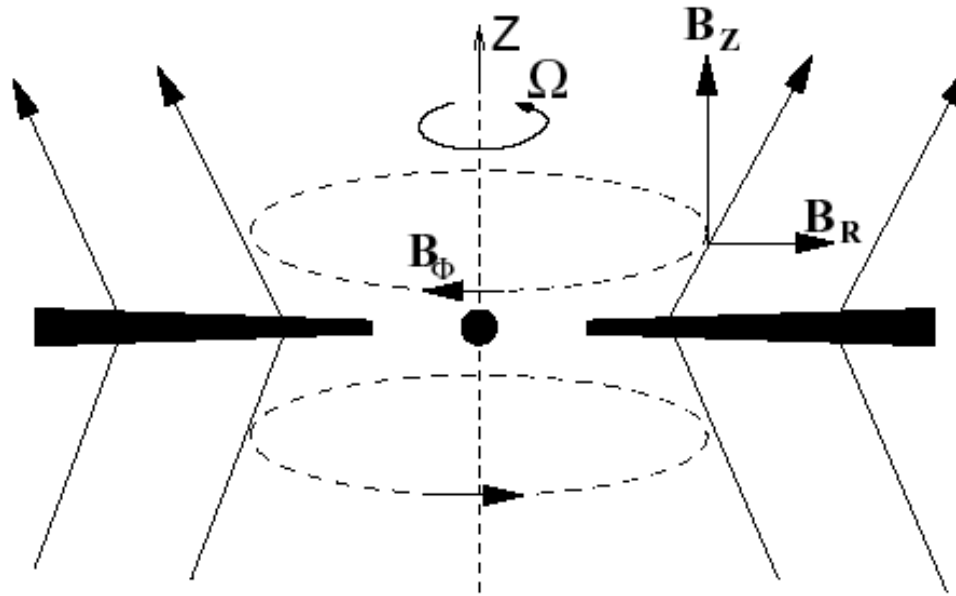
==> Enhance net growth of p-modes

## 2. Large-scale poloidal field can reduce mode frequency and enhance corotational instability

==> Enhance growth of p-modes

(Yu & DL, in prep)

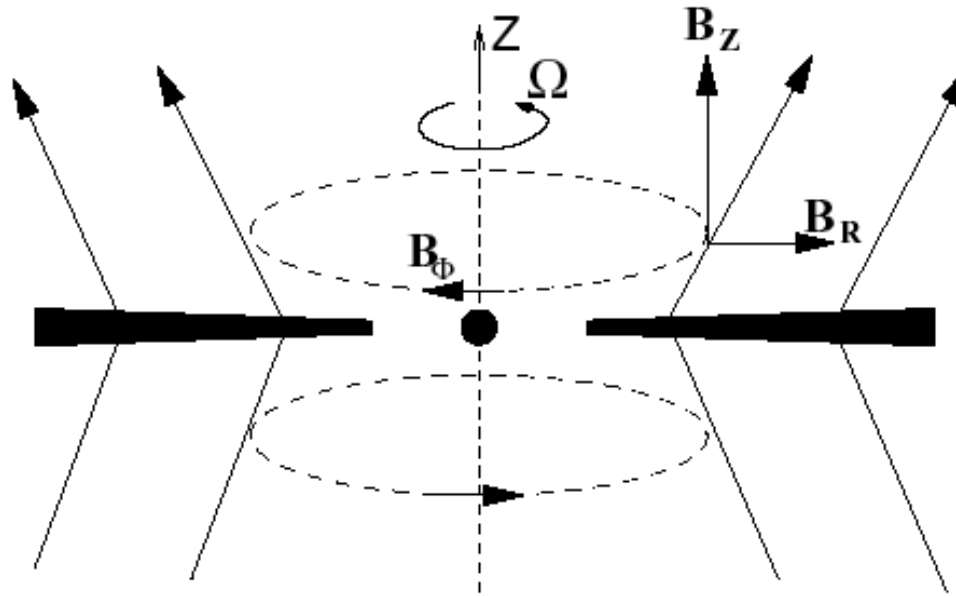
## Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)



-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together,  
the “clock” is mainly set by disk

## **Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)**

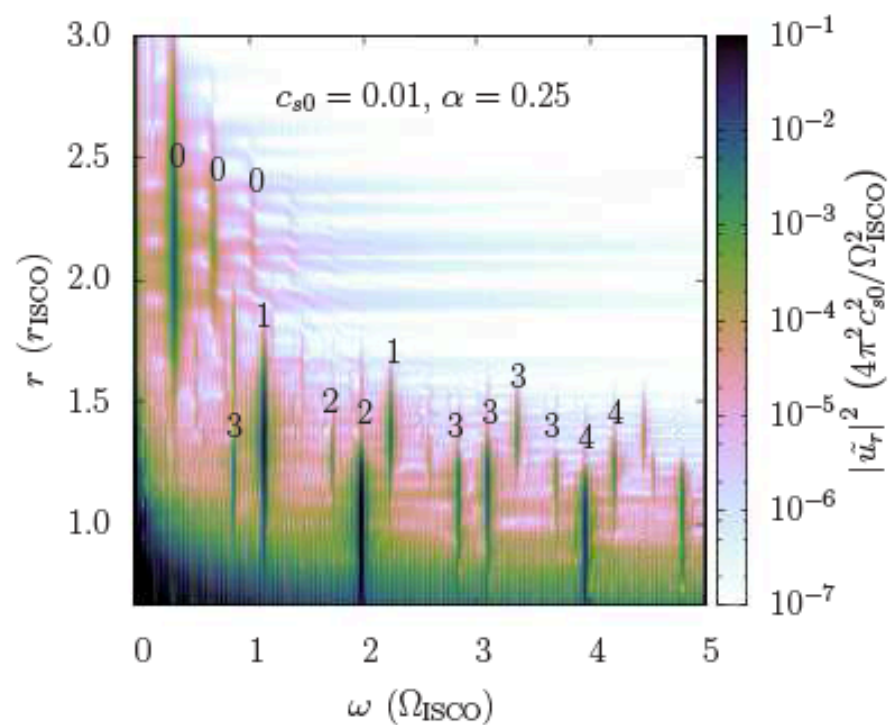
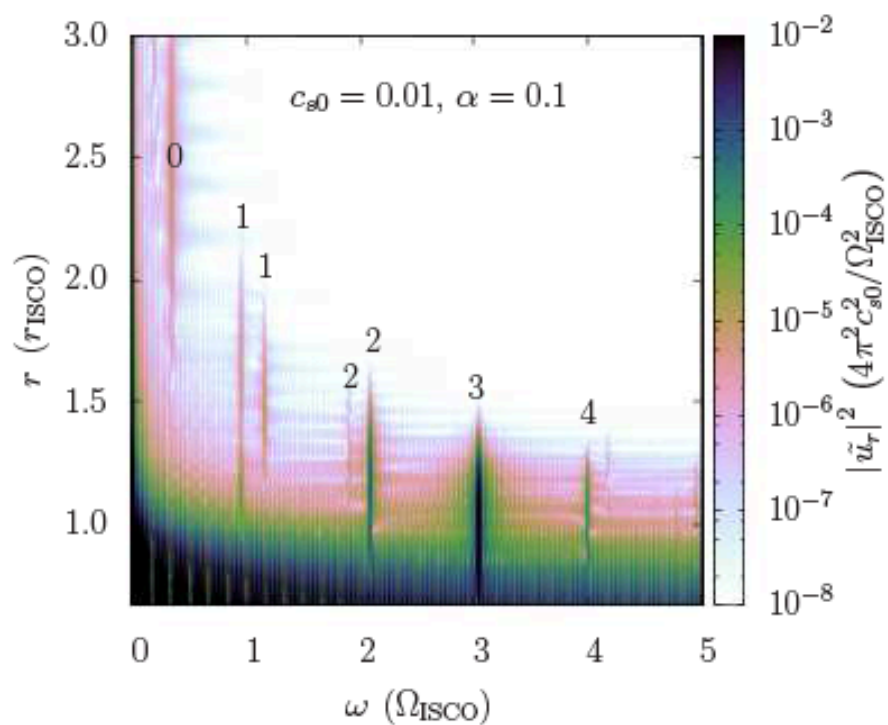


-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together,  
the “clock” is mainly set by disk

-- Such large-scale field is ideal for producing jets/outflows  
 (“Intermediate state”)

# Quasi-normal Disk Oscillation Modes excited by turbulent viscosity



Ryan Miranda, Horak & DL 2014

# Summary

- There are many concepts/ideas theoretical models for HFQPOs, but many have not been worked out... Theorists need to work hard
- P-modes (spiral density modes) partially trapped in the inner-most region of disks is possible candidate:
  - Frequencies can be calculated from first principle  
robust, largely agree with observations (consistent mass, spin)
  - Can grow naturally due to corotation resonance (GR important)
  - In this theory, there are good reasons to expect that
    - (i) They are not present in thermal state (mode damping due to infall)
    - (ii) They are present in intermediate state (effect of large-scale B field)
- \* Turbulent excitations of quasi-normal oscillation modes

**Thanks!**