Probing Extreme Physics with Compact Objects: Black Holes

Dong Lai

Department of Astronomy
Cornell University
“Dark Star” Concept:  John Michell (1783)  Pierre Laplace (1795)

- Although “correct” answer, derivation/interpretation wrong

Escape velocity:  \[ v_{\text{esc}} = \left( \frac{2GM}{R} \right)^{1/2} \]

\[ v_{\text{esc}} = c \]

\[ R = \frac{2GM}{c^2} = 3 \text{ km} \left( \frac{M}{1\ M_\odot} \right) \]
“Black Hole” Concept:

• **Einstein (1915): General Relativity**
Gravity is not a force, but rather it manifests as curvature of spacetime caused by matter and energy

\[
G_{\mu\nu} = 8\pi T_{\mu\nu}
\]

• **Karl Schwarzschild (1916):**
The first exact solution to Einstein field equation

\[
ds^2 = \left(1 - \frac{R_S}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{R_S}{r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2)
\]

**The horizon radius (Schwarzschild radius):**

\[
R_S = \frac{2GM}{c^2}
\]

• **Roy Kerr (1963):**
Solution for spinning black holes
Astrophysical Black Holes: Kerr Metric

Two parameters: \( M, \quad a=J/M \)

\[
ds^2 = - \left( 1 - \frac{2Mr}{\rho^2} \right) dt^2 - \frac{4aMr \sin^2 \theta}{\rho^2} dt d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left( r^2 + a^2 + \frac{2Mr a^2 \sin^2 \theta}{\rho^2} \right) \sin^2 \theta d\phi^2
\]
Isolated Black Holes are “boring”
(unobservable)
Accreting Black Holes

Stellar-mass BHs in binaries

Accreting gas has angular momentum

$\Rightarrow$ Accretion disk

$\Rightarrow$ Radiation from disk (x-rays)

Outflows (jets)

Supermassive BHs in Galaxies
Tidal Disruption of Stars by BH

==> Electromagnetic Flares

BH - Neutron Star Binary Merger

==> (short) GRBs?
Binary BH Merger

==> Gravitational waves
Black Holes in Astrophysics

Stellar-mass BHs in X-ray Binaries
Supermassive BHs in active galaxies
  Intermediate-mass BHs (ULXs) ?

Tidal disruption of stars

BH/NS or BH/BH mergers
Black Hole Power in Astrophysics

(1) Accretion Power

\[ L_{\text{acc}} = \epsilon \dot{M} c^2 \]

\( \epsilon = \) Binding energy (per unit mass) at ISCO
- 5.7% for \( a=0 \)
- 42% for \( a=M \)

Inner-most stable circular orbit
- \( r_{\text{ISCO}} = 6M \) for \( a=0 \)
- \( =M \) for \( a=M \)

Note:
- The above applies to thin ("cold") disks (radiative efficient disks);
- "Radiative Inefficient Disks" (e.g. ADAF), efficiency is smaller…
Black Hole Power in Astrophysics

(2) Spin Power

Extracting spin energy from BH (Penrose Process)

BH area theorem -->

\[ M_{\text{irr}} = \left( \frac{A}{16\pi} \right)^{1/2} = \frac{M}{\sqrt{2}} \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right)^{1/2} \]

Maximum efficiency of energy extraction = \[ 1 - \frac{M_{\text{irr}}}{M} \]

= 29.3% for a=M

How to do it?
Interaction of BH with magnetized plasma (a la pulsar)…
Blandford-Znajek (1977)
Blandford-Znajek Process
Interaction of BH with magnetized plasma (a la pulsar)...

Kip Thorne
Black Hole Power in Astrophysics

(1) Accretion Power

(2) Spin Power

Relative importance ??
Highlight #1: Mass and Spin of BHs
Mass of BHs in X-ray binaries

Well measured for 23 systems
(McClintock et al. 2011)

Note: BH mass gap: 2-5 $M_{\text{sun}}$
--> implication for supernova?  
(O’Connor & Ott 11)
Mass of Supermassive BHs

NGC 3842: $9.7 \times 10^{10} M_{\odot}$  
NGC 4889: $\sim 10^{10} M_{\odot}$  

McConnell et al. 2011
Spin of BHs in X-ray Binaries

• **Method 1: Continuum Fitting**
  Measure the temperature of the inner disk in thermal state (thin disk)
  \[\Rightarrow\] Radius of the inner edge of the disk
  \[\Rightarrow\] BH spin

**Key assumption:**
  disk inner edge at ISCO
  (no radiation inside ISCO)

Penna et al. 2011
BH spin measurement using continuum fitting method

<table>
<thead>
<tr>
<th>Source</th>
<th>Spin $a_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GRS 1915+105</td>
<td>&gt; 0.98</td>
</tr>
<tr>
<td>2 LMC X–1</td>
<td>0.92$^{+0.05}_{-0.07}$</td>
</tr>
<tr>
<td>4 M33 X–7</td>
<td>0.84 ± 0.05</td>
</tr>
<tr>
<td>3 4U 1543–47</td>
<td>0.80 ± 0.05</td>
</tr>
<tr>
<td>5 GRO J1655–40</td>
<td>0.70 ± 0.05</td>
</tr>
<tr>
<td>6 XTE J1550–564</td>
<td>0.34$^{+0.20}_{-0.28}$</td>
</tr>
<tr>
<td>7 LMC X–3</td>
<td>&lt; 0.3$^b$</td>
</tr>
<tr>
<td>8 A0620–00</td>
<td>0.12 ± 0.18</td>
</tr>
</tbody>
</table>

McCintock et al 2011
Spin of BHs

• Method 2: Broad Fe K line shape

Miniutti et al 2007
MGC-6-30-15: $a > 0.989$ (?)
Evidence for jet powered by BH Spin?

Narayan & McClintock ‘11

<table>
<thead>
<tr>
<th>BH Binary</th>
<th>$a_*$</th>
<th>$M$ ($M_\odot$)</th>
<th>$D$ (kpc)</th>
<th>$i$ (deg)</th>
<th>$(S_\nu)_{\text{max},5\text{GHz}}$ (Jy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0620-00</td>
<td>$0.12 \pm 0.19$</td>
<td>$6.61 \pm 0.25$</td>
<td>$1.06 \pm 0.12$</td>
<td>$51.0 \pm 0.9$</td>
<td>$0.203$</td>
</tr>
<tr>
<td>XTE J1550–564</td>
<td>$0.34 \pm 0.24$</td>
<td>$9.10 \pm 0.61$</td>
<td>$4.38 \pm 0.50$</td>
<td>$74.7 \pm 3.8$</td>
<td>$0.265$</td>
</tr>
<tr>
<td>GRO J1655–40</td>
<td>$0.7 \pm 0.1$</td>
<td>$6.30 \pm 0.27$</td>
<td>$3.2 \pm 0.5$</td>
<td>$70.2 \pm 1.9$</td>
<td>$2.42$</td>
</tr>
<tr>
<td>GRS 1915+105</td>
<td>$0.975 \pm 0.025$</td>
<td>$14.0 \pm 4.4$</td>
<td>$11.0 \pm 1.0$</td>
<td>$66.0 \pm 2.0$</td>
<td>$0.912$</td>
</tr>
<tr>
<td>4U 1543–47</td>
<td>$0.8 \pm 0.1$</td>
<td>$9.4 \pm 1.0$</td>
<td>$7.5 \pm 1.0$</td>
<td>$20.7 \pm 1.5$</td>
<td>$&gt;1.16 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Highlight #2: Rapid Variabilities of Accreting BHs and Dynamics of Inner Disks
High-Frequency QPOs in BH X-Ray Binaries

Remillard & McClintock 2006
Basic Facts about HFQPOs

- 40-450 Hz: ~ orbital frequency at $r_{isco}$
- Frequency stable (<10% change when Mdot doubles)
- Some systems: ~2:3 ratio
- Weak QPOs: ~1% flux variation (in hard X-rays), Q~2-10
- Only occur in “Transitional state” (Episodic jet)
X-ray QPO (P ~ 1 hr) from active galaxy RE J1034+396

Gierlinski et al 2008, Nature
QPOs from Ultra-Luminous X-ray Source NGC 5408 X-1 (an Intermediate-mass BH?)

Strohmayer & Mushotzky 2009
High-Frequency QPOs in BH X-Ray Binaries

Remillard & McClintock 2006
Ideas/Models of HFQPOs

• Orbiting blobs (hot spots) in disks (Stella et al ‘99; Schnittman & Bertschinger ‘04)

• Nonlinear resonances of some kind (Abramowicz, Kluzniak, Horak, Rebusco)

• Acoustic modes in torus (Rezzolla el al ‘03; Lee, Abramowicz & Kluzniak ‘04; Blaes et al. ‘07; Sramkova et al ‘07; Horak’08)

• Disk/Magnetosphere Boundary Layer Oscilations
  (Li & Narayan ‘04; Tsang & DL ‘09)

• Oscillation modes in relativistic disks (Kato; Wagoner & collaborators)
  -- m=0 inertial modes excited by global disk deformation (e.g. warps)
    (Kato ‘03,’08; Ferreira & Ogilvie ‘08; Henisey et al.10)
  -- Rossby modes trapped in special region of a magnetic disk
    (Tagger & Varniere ‘06; see also Tagger & Pallet ‘99; Varniere & Tagger’02)
  -- Cornell effort: Mode growth due to corotational resonance, magnetic fields
    (DL & Tsang ‘09; Tsang & DL ‘08,’09a,b; Fu & DL ‘09,’11a,b)
P-modes of BH Accretion Disks
“inertial-acoustic modes”, “spiral density modes”

-- Trapped (partially) in the innermost region of disk
-- Frequencies can be calculated: robust, agree with observations
-- Can grow due to corotation resonance (“corotational instability”)

GR plays an important role
-- B field effect

with David Tsang (Cornell Ph.D. 09 --> Caltech)
Wen Fu (Cornell Ph.D. student)

Fu & DL 2012
Waves in 2D disks (Spiral density waves):

\[ \delta v, \delta \Sigma \propto \exp(i m \varphi - i \omega t) \]

Can propagate only in the region:

\[ r < r_{\text{ILR}} \quad \text{or} \quad r > r_{\text{OLR}} \]

Lindblad Resonances:

\[ \omega - m \Omega(r) = \pm \kappa(r) \]

where \( \Omega(r) = \) disk rotation rate

\( \kappa(r) = \) radial epicyclic frequency

\[ \kappa^2 = \frac{2 \Omega}{r} \frac{d}{dr} (r^2 \Omega) \]
Wave propagation diagram (effective potential)

Wave at $r > r_{OLR}$: $\omega/m > \Omega \Rightarrow$ positive energy
Wave at $r < r_{ILR}$: $\omega/m < \Omega \Rightarrow$ negative energy
\[ (-1) = (-1)|R|^2 + |T|^2 \] 
\[ \Rightarrow |R|^2 = 1 + |T|^2 > 1 \] 

Super-reflection
Trapped mode between $r_{\text{in}}$ and $r_{\text{ILR}}$: overstable
Even more interesting…

**Corotation resonance**, where

\[
\frac{\omega}{m} = \Omega
\]
\[ (-1) = (-1)|R|^2 + |T|^2 + D_c \]
\[ \Rightarrow |R|^2 = 1 + |T|^2 + D_c \]

Wave absorption at corotation

Can have both signs!
Calculations of reflectivity/transmission:

\[ \delta h = \sqrt{S/k} \left[ \exp \left( -i \int_{r_{IL}}^{r} k \, dr + \frac{\pi}{4} \right) + R \exp \left( i \int_{r_{IL}}^{r} k \, dr - \frac{\pi}{4} \right) \right] \]

\[ \delta h = \sqrt{S/k} \, T \exp \left( i \int_{r_{OL}}^{r} k \, dr + \frac{\pi}{4} \right) \]

- Solve wave equation in different regions
- Match the solutions using asymptotic expansions
- Around corotation: Whittaker function; Stokes phenomenon

\[ R = \frac{1 + \frac{1}{4} \left( e^{-i2\pi\nu} + \sin^2 \pi\nu \right) e^{-2\Theta_{II}} + \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIb}}}{(\Gamma(1+\nu))^2}}{1 - \frac{1}{4} \left( e^{-i2\pi\nu} + \sin^2 \pi\nu \right) e^{-2\Theta_{II}} - \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIb}}}{(\Gamma(1+\nu))^2}} \]

\[ T = \frac{ie^{-2\Theta_{II}}e^{i\pi\nu}}{1 - \frac{1}{4} \left( e^{-i2\pi\nu} + \sin^2 \pi\nu \right) e^{-2\Theta_{II}} - \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2} \frac{e^{-2\Theta_{IIb}}}{(\Gamma(1+\nu))^2}} \]

\[ \Theta_{IIa} = \int_{r_{IL}}^{r_c} |k| \, dr \quad \Theta_{IIb} = \int_{r_c}^{r_{OL}} |k| \, dr \]
Reflectivity at ILR: \[ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + D_c \simeq 1 + D_c \]

Sign depends on sign of \( d\zeta/dr \)

\[ \zeta = \frac{\kappa^2}{2\Omega \Sigma} \] (vortensity)
Reflectivity at ILR: \[ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + D_c \simeq 1 + D_c \]

Sign depends on sign of \( \frac{d\zeta}{dr} \)

\[ \zeta = \frac{\kappa^2}{2\Omega \Sigma} \] (vortensity)

\[ \Rightarrow D_c > 0 \]
Reflectivity at ILR: \[ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c \]

Sign depends on sign of \( \frac{d\zeta}{dr} \)

\[ \zeta = \frac{\kappa^2}{2\Omega\Sigma} \] (vortensity)

\[ \Rightarrow \mathcal{D}_c > 0 \]

\[ \Rightarrow \mathcal{D}_c < 0 \]
Reflectivity at ILR: \(|\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c\)

Sign depends on sign of \(\frac{d\zeta}{dr}\)

\[\zeta = \frac{\kappa^2}{2\Omega\Sigma}\]

(vortensity)

\[\Rightarrow \mathcal{D}_c > 0\]

Overstable mode

\[\Rightarrow \mathcal{D}_c < 0\]

Damped mode
General Relativity Effect

GR makes \( d\zeta/dr > 0 \) in the Inner-most disk region

\( \implies \text{makes the mode grow!} \)

Vortensity \( \zeta = \frac{\kappa^2}{2\Omega \Sigma} \)

ISCO
Linear Mode Calculation (Mode freq. and growth rate)

\[ \Sigma \propto r^{-1}, \quad c_s = 0.1r \Omega, \quad m = 2 \]
\[ \omega_r = 0.93 \Omega_{\text{ISCO}}, \quad \omega_i/\omega_r = 0.0029 \]

 DL & Tsang 2009,2010
Nonlinear Simulation (2D) of Growing Modes

Wen Fu & DL 2012, in prep
Properties of Overstable Disk P-Modes:

Low-order p-modes trapped between inner disk edge and ILR

\[ \omega \simeq \beta m \Omega(r_{\text{in}}) \]

\[ \beta = 0.55 - 0.75 \text{ depending on disk models and inner BC} \]

- Mode frequencies robust, consistent with known BH mass (and spin)
- Frequency ratio approximately: 1:2:3:4… (not exactly)

Grow due to corotation resonance (GR plays important role)

A promising candidate for HFQPOs
Complications...
Complications…

• Mode damping due to radial infall
  
  Competition: mode growth (due to corotation) and damping
  
  ==> HFQPOs do not always appear
  
  e.g. in thermal state (standard thin disk) no QPOs observed

• Effects of magnetic fields
  
  -- Mode frequencies are slightly affected
  
  -- Large-scale B field enhances the growth rate
Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)

-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together, the “clock” is mainly set by disk
Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)

-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together, the “clock” is mainly set by disk

-- Such large-scale field is ideal for producing jets/outflows

QPOs are observed at the same time as episodic jets
Recap of HFQPOs

• Intriguing puzzle --- Dynamics of inner-most region of BH accretion disks (No standard models yet)

• P-modes (spiral density modes) partially trapped in the inner-most region of disks is promising candidate:
  -- Frequencies can be calculated from first principle, robust, agree with observations (consistent mass, spin)
  -- Can grow naturally due to corotation resonance (GR important)

Incomplete: Complications, other issues (turbulence)…
Highlight #3: Merging Binary Black Holes/Neutron Stars
Merging Neutron Stars:

Nobel Prize 1993

Taylor & Weisberg 2005

Nobel Prize 1993
NS-NS Merger

Shibata et al. 2006
BH-NS Merger

F. Foucart et al (Cornell) 2011
Merging NSs (NS/BH or NS/NS) as Central Engine of (short/hard) GRBs
The last few minutes: Gravitational Waveform
Gravitational Waves

- Warpage of Spacetime
- Generated by time-dependent quadrupoles
- Detector response to passage of GWs:
Gravitational Wave Interferometer

\[ \Delta L = h L \lesssim 4 \times 10^{-16} \text{ cm} \]

\[ \lesssim 10^{-21} \]

4 km

Kip Thorne
Final merger wave form probes NS EOS
Probe NS EOS using Inspiral Waveform

**Idea:**

- For point masses, the number of GW cycles is known exactly.

- Rosonant tidal excitations of NS oscillation modes during inspiral
  $\Rightarrow$ transfer orbital energy to NS
  $\Rightarrow$ **Missing GW cycles**
Resonant Excitations of NS Modes During Binary Inspiral

Non-rotating NS:
  G-mode (Reisenegger & Goldreich 1994; DL 1994)

Rotating NS:
  G-mode, F-mode, R-mode (Wynn Ho & DL 1999)
  Inertial modes (DL & Yanqin Wu 2006)
  R-mode (excited by gravitomagnetic force; Racine & Flanagan 2006)

Results:
• For R=10 km NS, the number of missing cycles < 0.1, unlikely measurable
  (unless NS is rapidly rotating)
• Number of missing cycles \( \Delta N \propto R^4 \) (g mode) or \( R^{3.5} \) (r mode)
  Important for larger NS
• Crustal modes: important? Could shatter crust, pre-cursor of short GRB
  (D. Tsang et al. 2011)
BH-BH Merger

Cornell-Caltech collaboration
Summary

• Compact Objects (White dwarfs, Neutron stars and Black Holes) have diverse observational manifestations
can be studied in many different ways: radio -- gamma rays, GWs

• They present a rich set of astrophysics/physics problems
  Ideal laboratory for probing physics under extreme conditions
Obrigado !!
Black Hole Power in Astrophysics

Accretion Power

\[ L_{\text{acc}} = \epsilon \dot{M} c^2 \]

\( \epsilon \) = Binding energy (per unit mass) at ISCO
= 5.7% for a=0
42% for a=M

“Spin” Power

Extracting spin energy from BH (Penrose)

BH area theorem -->

\[ M_{\text{irr}} = \left( \frac{A}{16\pi} \right)^{1/2} = \frac{M}{\sqrt{2}} \left( 1 + \sqrt{1 - \frac{a^2}{M^2}} \right)^{1/2} \]

Maximum efficiency of energy extraction = 1 − \( M_{\text{irr}}/M \)
= 29.3% for a=M