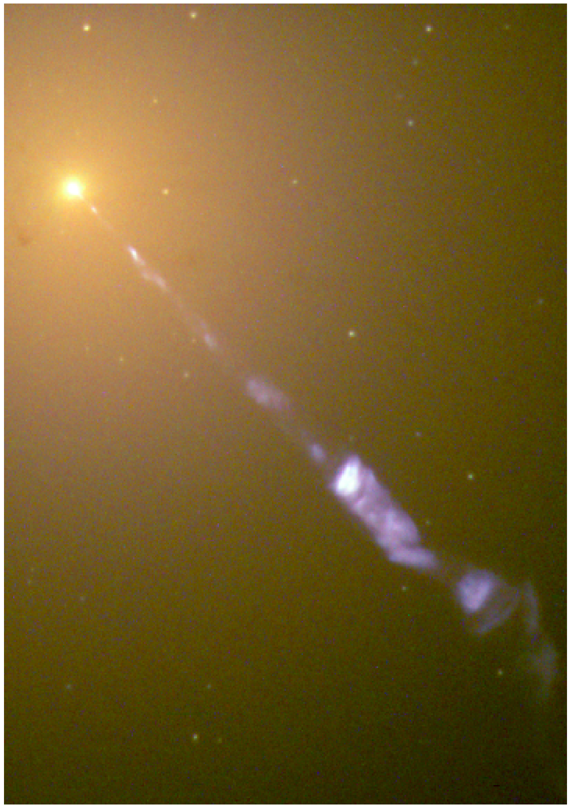
Probing Extreme Physics with Compact Objects: Black Holes



Dong Lai

Department of Astronomy
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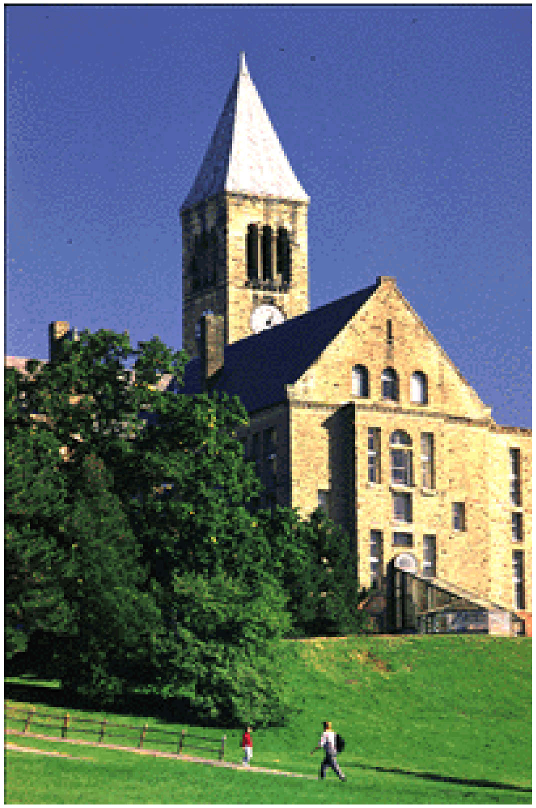
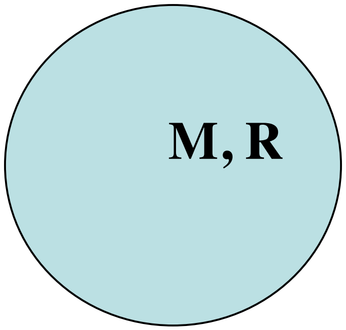


“Dark Star” Concept: John Michell (1783) Pierre Laplace (1795)



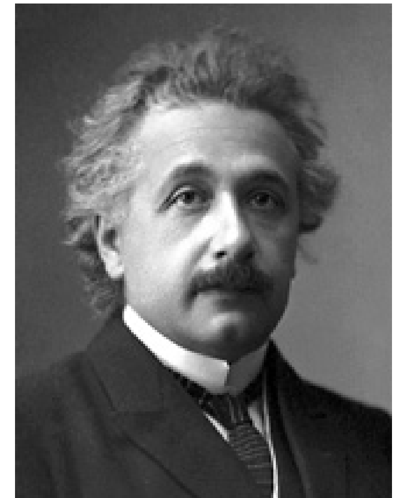
Escape velocity: $v_{\rm esc} = \left(\frac{2GM}{R}\right)^{1/2}$

$$v_{\rm esc} = c \quad \Longrightarrow$$

$$R = \frac{2GM}{c^2} = 3\ {\rm km}\left(\frac{M}{1\ M_\odot}\right)$$

- **Although “correct” answer, derivation/interpretation wrong**

"Black Hole" Concept:



- **Einstein (1915): General Relativity**

  Gravity is not a force, but rather it manifests as curvature of spacetime caused by matter and energy

  Einstein field equation: $G_{\mu\nu} = 8\pi T_{\mu\nu}$

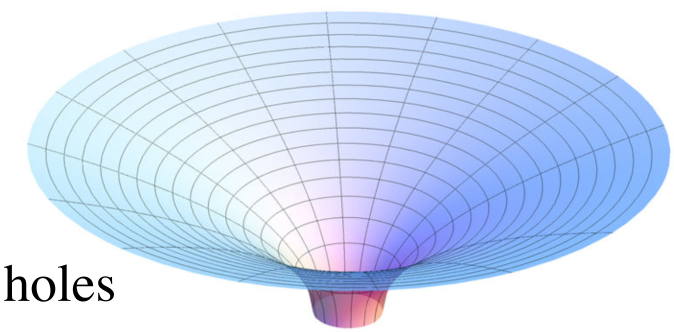
- **Karl Schwarzschild (1916):**

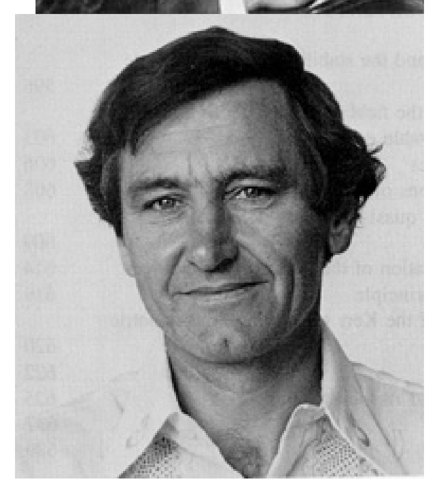
  The first exact solution to Einstein field equation

$$ds^2 = \left(1 - \frac{R_S}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{R_S}{r}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

The horizon radius (Schwarzschild radius):

$$R_S = \frac{2GM}{c^2}$$





- **Roy Kerr (1963):**

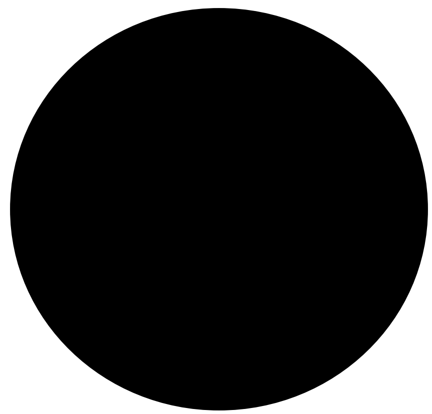
  Solution for spinning black holes

Astrophysical Black Holes: Kerr Metric

Two parameters: M, a=J/M

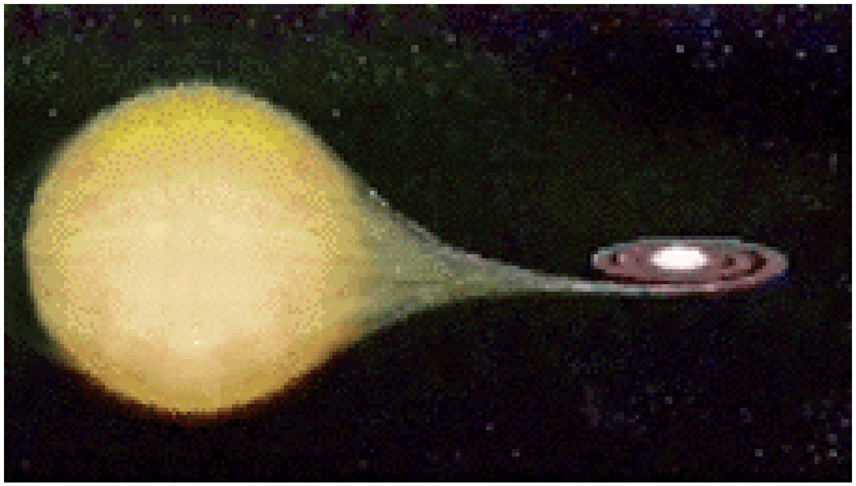
$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4aMr\sin^2\theta}{\rho^2} dt d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right) \sin^2\theta d\phi^2$$

Isolated Black Holes are “boring” (unobservable)

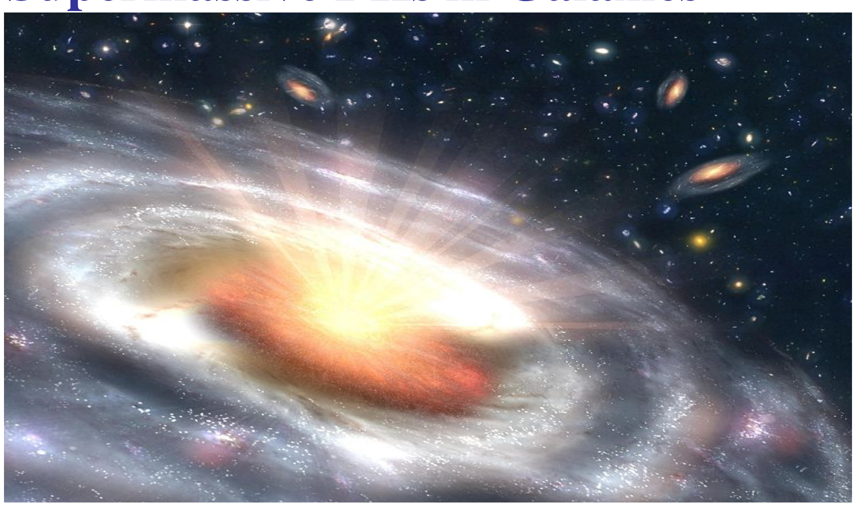


Accreting Black Holes

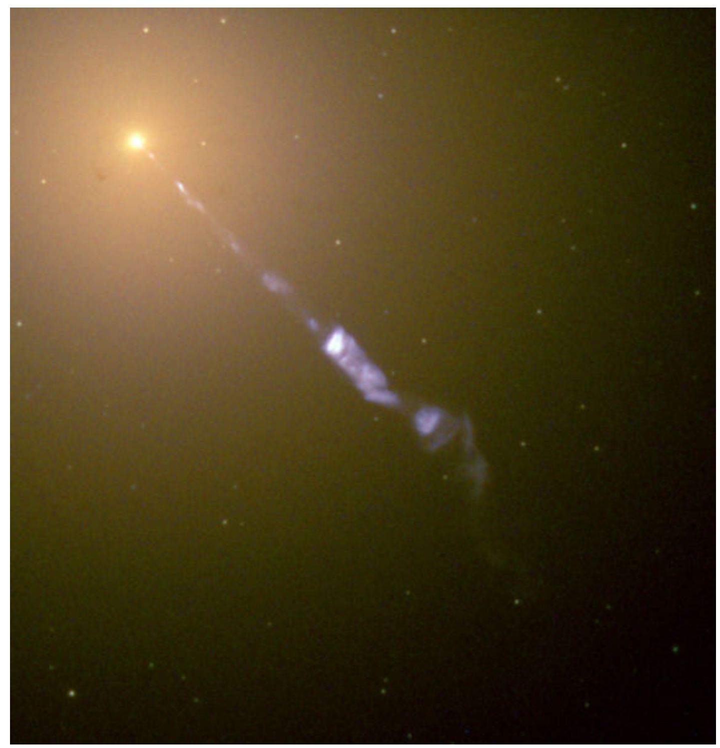
Stellar-mass BHs in binaries



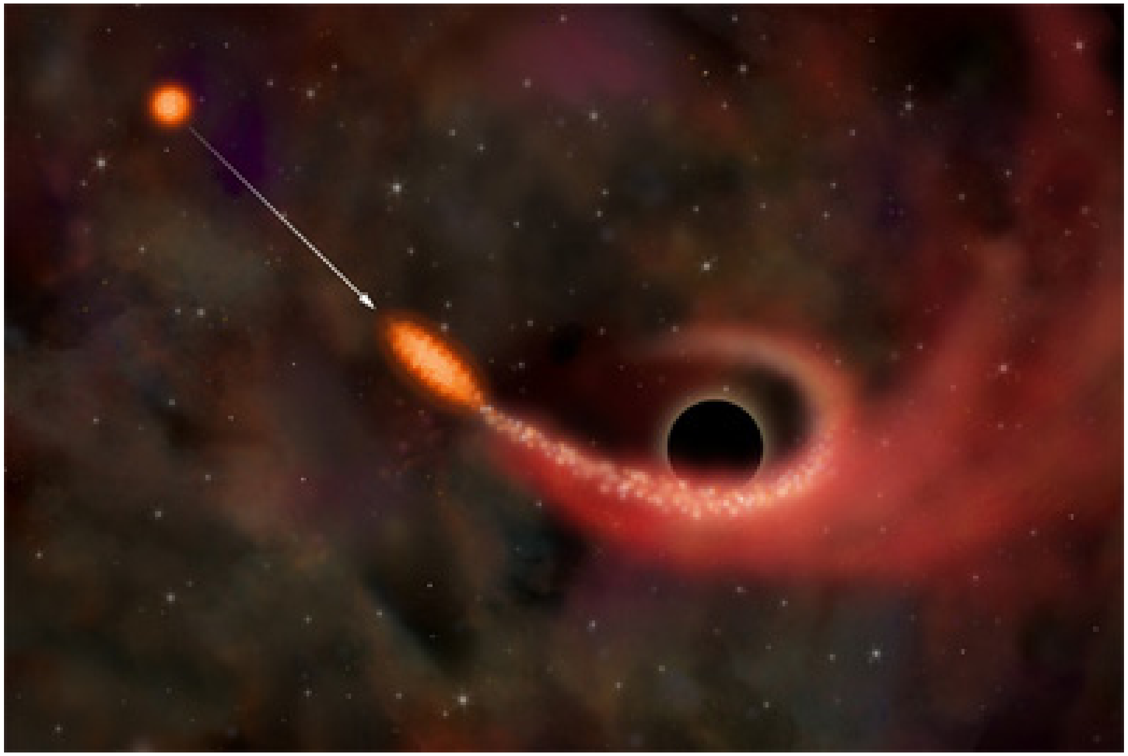
Supermassive BHs in Galaxies



Accreting gas has angular momentum
==> **Accretion disk**
==> Radiation from disk (x-rays)
Outflows (jets)



Tidal Disruption of Stars by BH

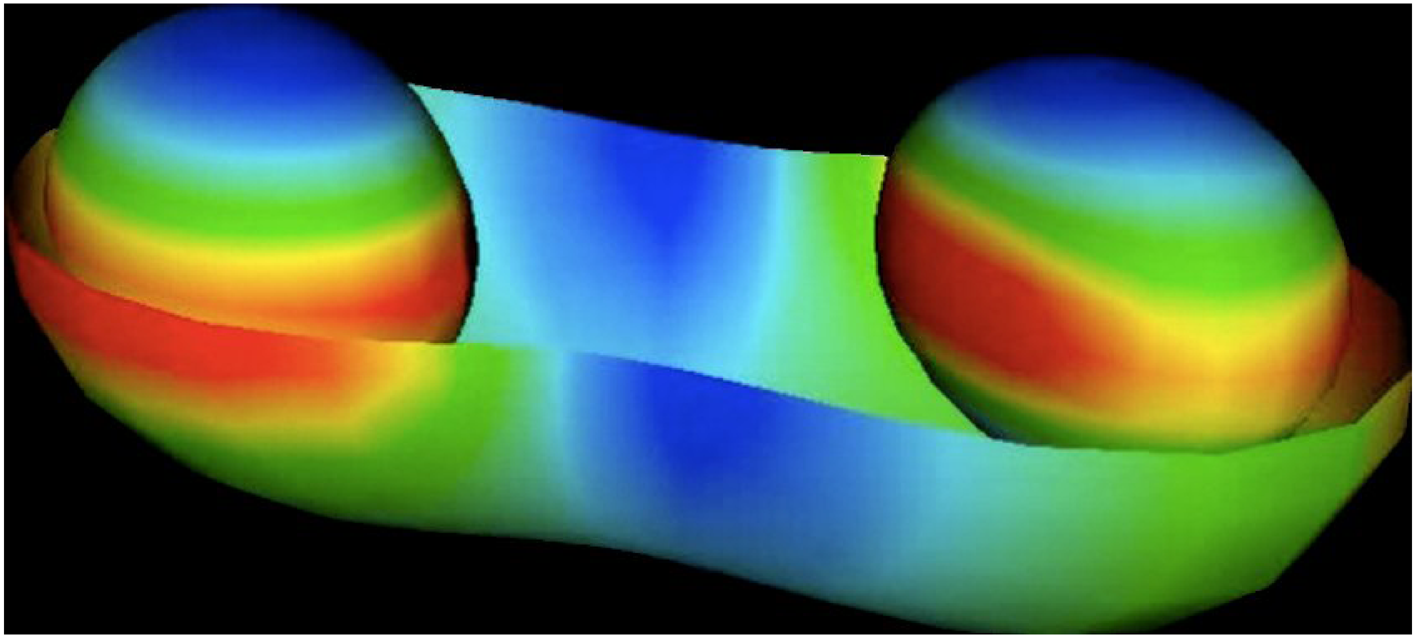


==> Electromagnetic Flares

BH - Neutron Star Binary Merger

==> (short) GRBs **?**

Binary BH Merger



==> Gravitational waves

Black Holes in Astrophysics

Stellar-mass BHs in X-ray Binaries
Supermassive BHs in active galaxies

Intermediate-mass BHs (ULXs) ?

Tidal disruption of stars

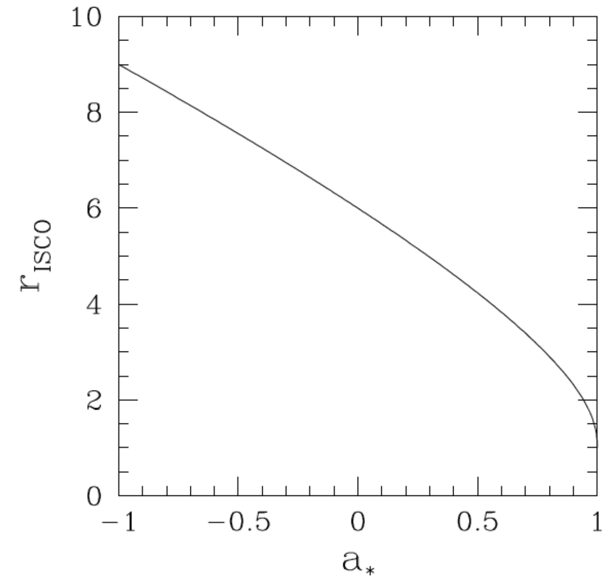
BH/NS or BH/BH mergers

Black Hole Power in Astrophysics

(1) Accretion Power

$$L_{\rm acc} = \epsilon \dot{M} c^2$$

ϵ = Binding energy (per unit mass) at ISCO
= 5.7% for a=0
42% for a=M



Inner-most stable circular orbit

r_{ISCO}=6M for a=0
=M for a=M

Note:
--The above applies to thin ("cold") disks (radiative efficient disks);
--"Radiative Inefficient Disks" (e.g. ADAF), efficiency is smaller…

Black Hole Power in Astrophysics

(2) Spin Power

Extracting spin energy from BH (Penrose Process)

BH area theorem --> $$M_{\rm irr} = \left(\frac{A}{16\pi}\right)^{1/2} = \frac{M}{\sqrt{2}}\left(1+\sqrt{1-\frac{a^2}{M^2}}\right)^{1/2}$$

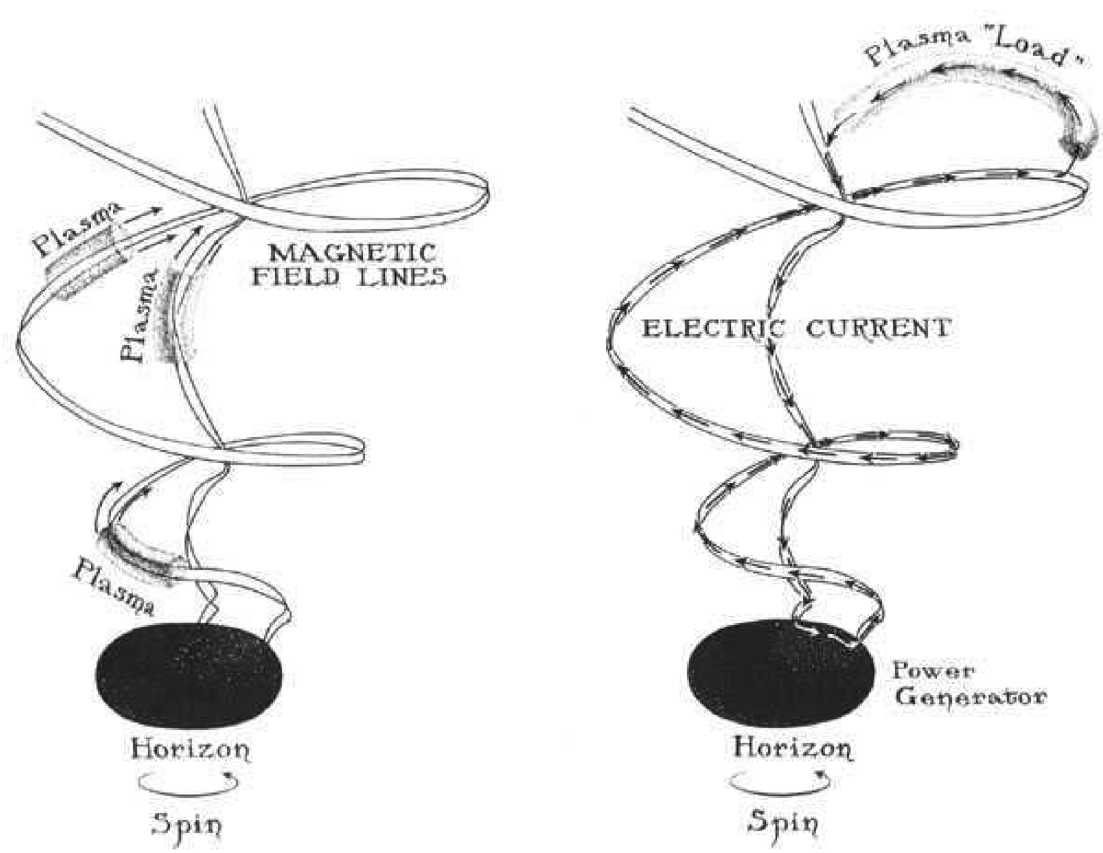
Maximum efficiency of energy extraction = $1 - M_{\rm irr}/M$

= 29.3% for a=M

How to do it?
Interaction of BH with magnetized plasma (a la pulsar)…
Blandford-Znajek (1977)

Blandford-Znajek Process

Interaction of BH with magnetized plasma (a la pulsar)…



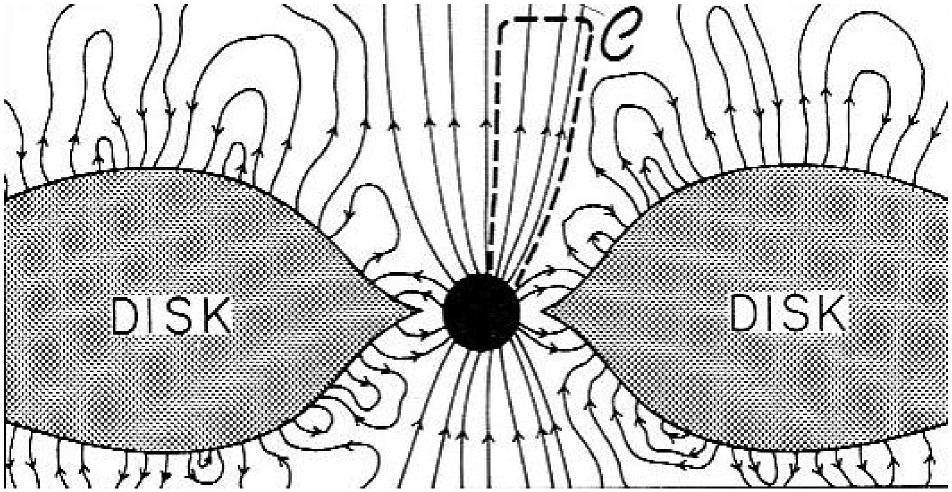
Kip Thorne

Black Hole Power in Astrophysics

(1) Accretion Power

(2) Spin Power

Relative importance ??



Highlight #1: Mass and Spin of BHs

Mass of BHs in X-ray binaries

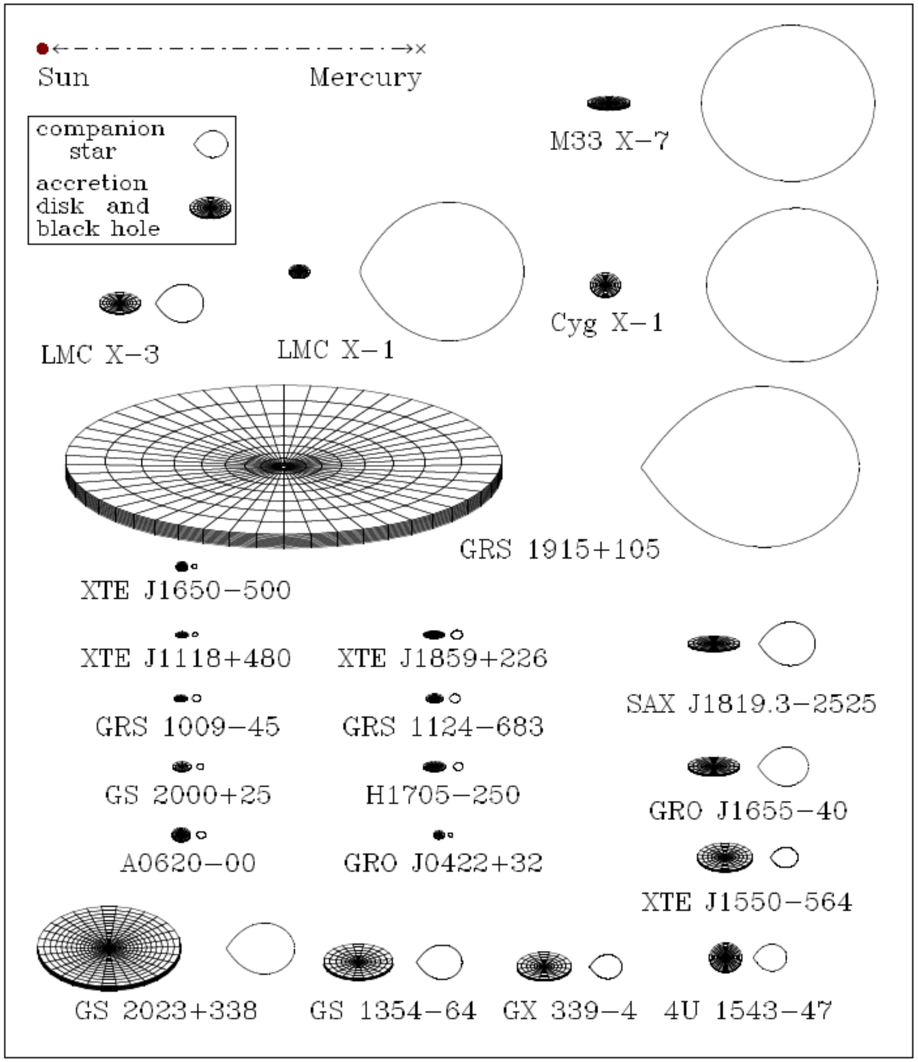
Well measured for 23 systems

(McClintock et al. 2011)

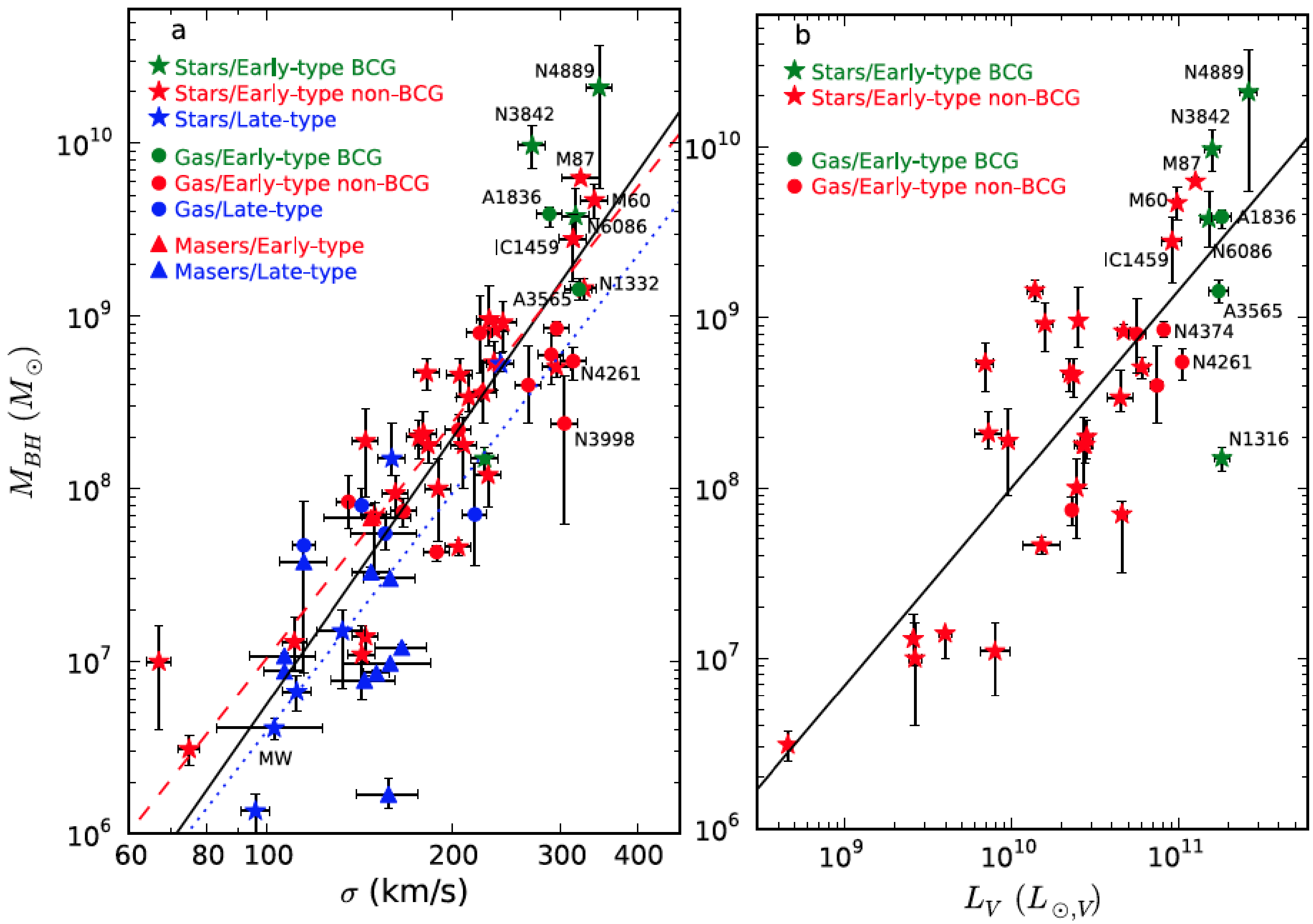
Note: BH mass gap: 2-5 M_{sun}

--> implication for supernova ?

(O'Connor & Ott 11)



Mass of Supermassive BHs



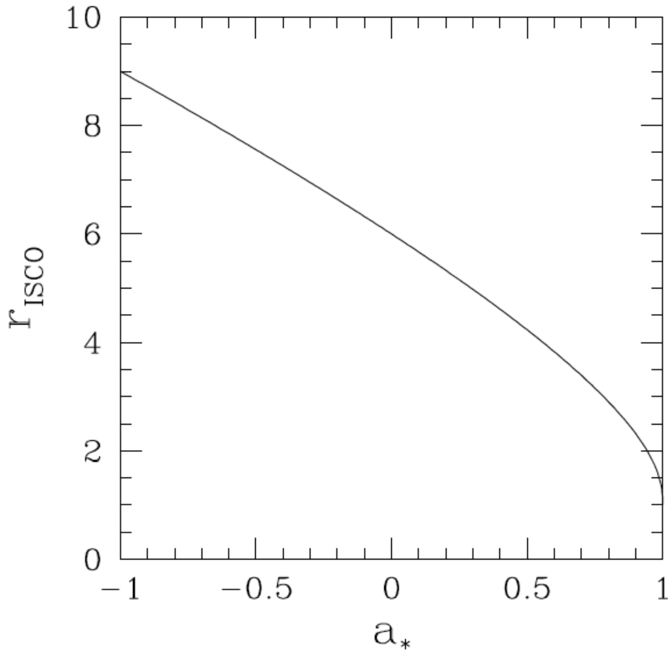
McConnell et al. 2011

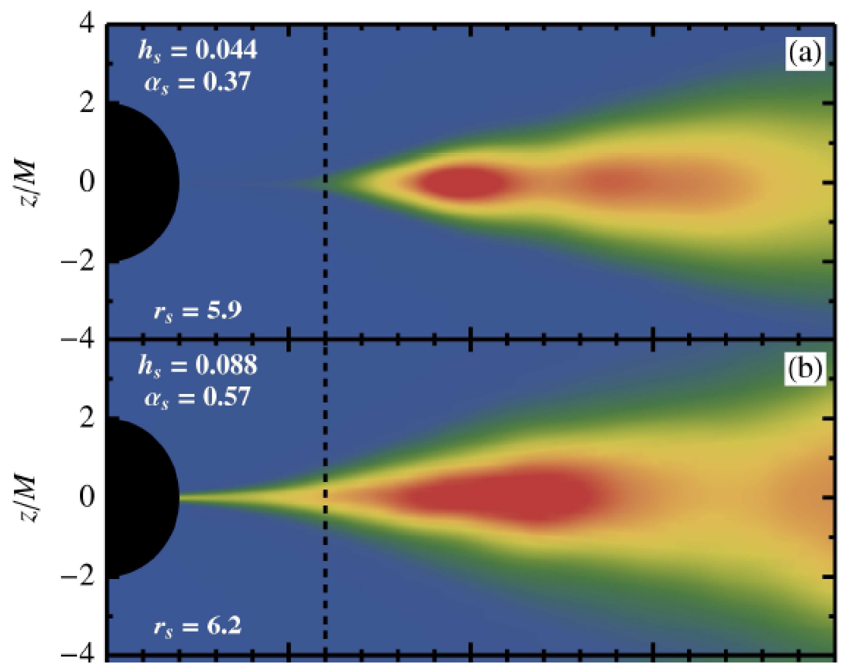
NGC 3842: 9.7x10^{10}M$_{sun}$
NGC 4889: ~10^{10}M$_{sun}$

Spin of BHs in X-ray Binaries

- **Method 1: Continuum Fitting**
  Measure the temperature of the inner disk in thermal state (thin disk)
  ==> Radius of the inner edge of the disk
  ==> BH spin

Key assumption:
disk inner edge at ISCO
(no radiation inside ISCO)





Penna et al. 2011

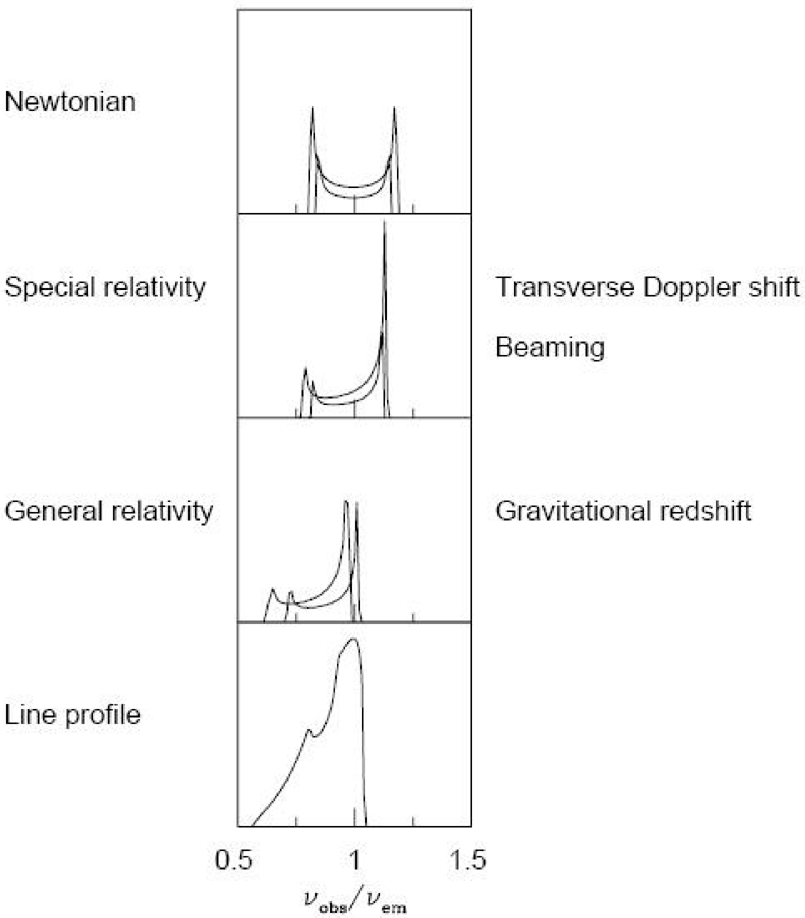
BH spin measurement using continuum fitting method

	Source	Spin a_*
1	GRS 1915+105	> 0.98
2	LMC X–1	$0.92^{+0.05}_{-0.07}$
4	M33 X–7	0.84 ± 0.05
3	4U 1543–47	0.80 ± 0.05
5	GRO J1655–40	0.70 ± 0.05
6	XTE J1550–564	$0.34^{+0.20}_{-0.28}$
7	LMC X–3	< 0.3[b]
8	A0620–00	0.12 ± 0.18

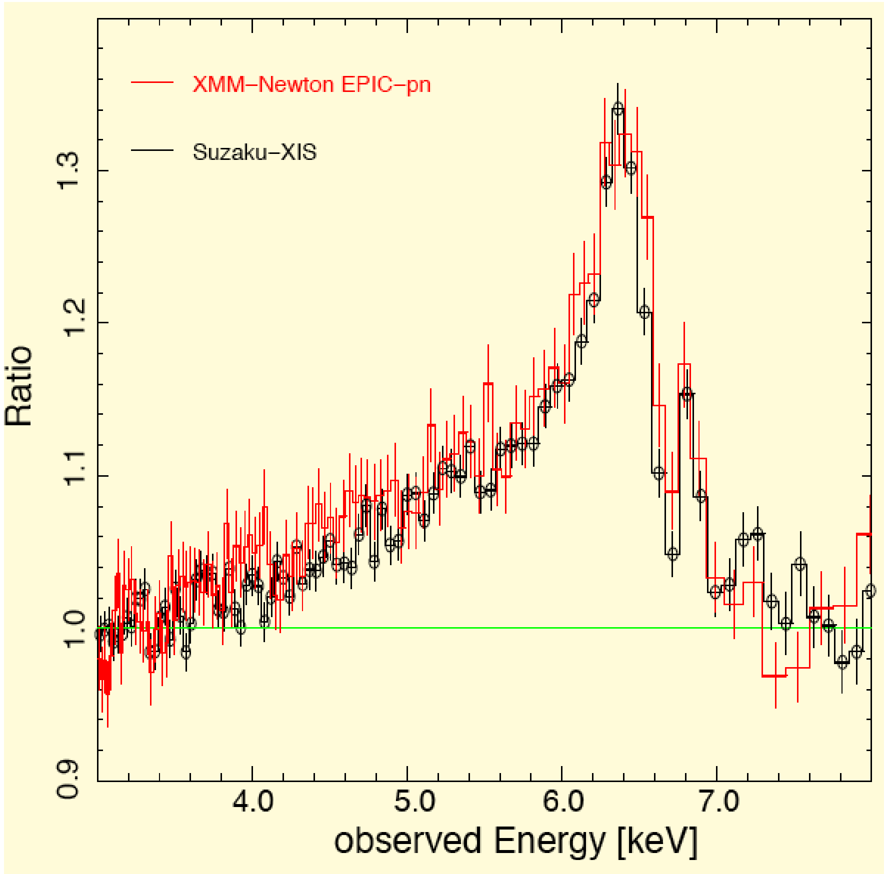
McClintock et al 2011

Spin of BHs

- **Method 2: Broad Fe K line shape**



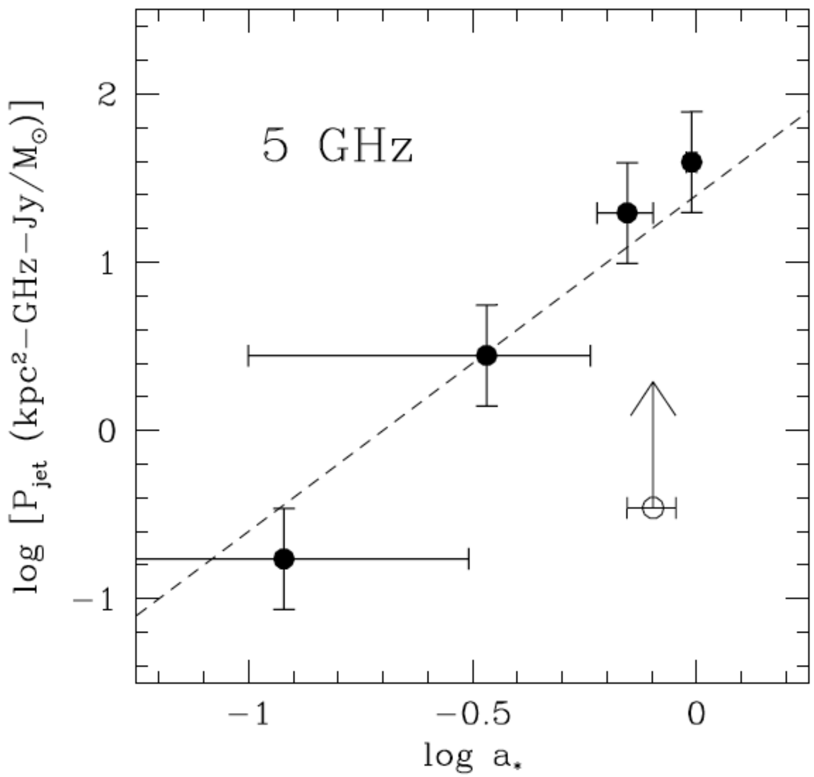
Fabian et al 2000



Miniutti et al 2007

MGC-6-30-15: a>0.989 (?)

Evidence for jet powered by BH Spin ?

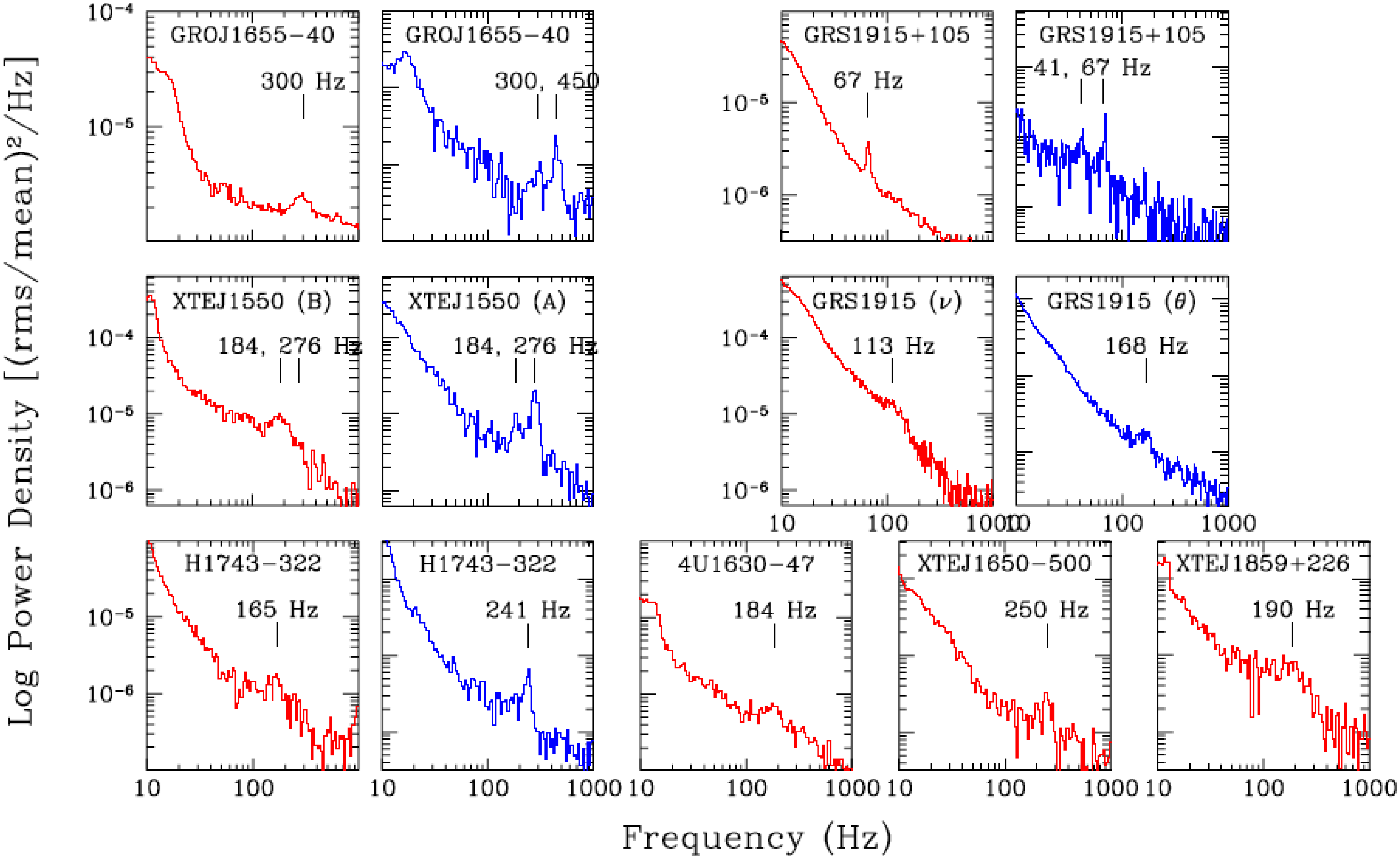


Narayan & McClintock ‘11

BH Binary	a_*	M $(M_\odot)$	D (kpc)	i (deg)	$(S_\nu)_{\max,5\mathrm{GHz}}$ (Jy)
A0620–00	0.12 ± 0.19	6.61 ± 0.25	1.06 ± 0.12	51.0 ± 0.9	0.203
XTE J1550–564	0.34 ± 0.24	9.10 ± 0.61	4.38 ± 0.50	74.7 ± 3.8	0.265
GRO J1655–40	0.7 ± 0.1	6.30 ± 0.27	3.2 ± 0.5	70.2 ± 1.9	2.42
GRS 1915+105	0.975 ± 0.025	14.0 ± 4.4	11.0 ± 1.0	66.0 ± 2.0	0.912
4U 1543–47	0.8 ± 0.1	9.4 ± 1.0	7.5 ± 1.0	20.7 ± 1.5	$> 1.16 \times 10^{-2}$

Highlight #2:
Rapid Variabilities of Accreting BHs and Dynamics of Inner Disks

High-Frequency QPOs in BH X-Ray Binaries

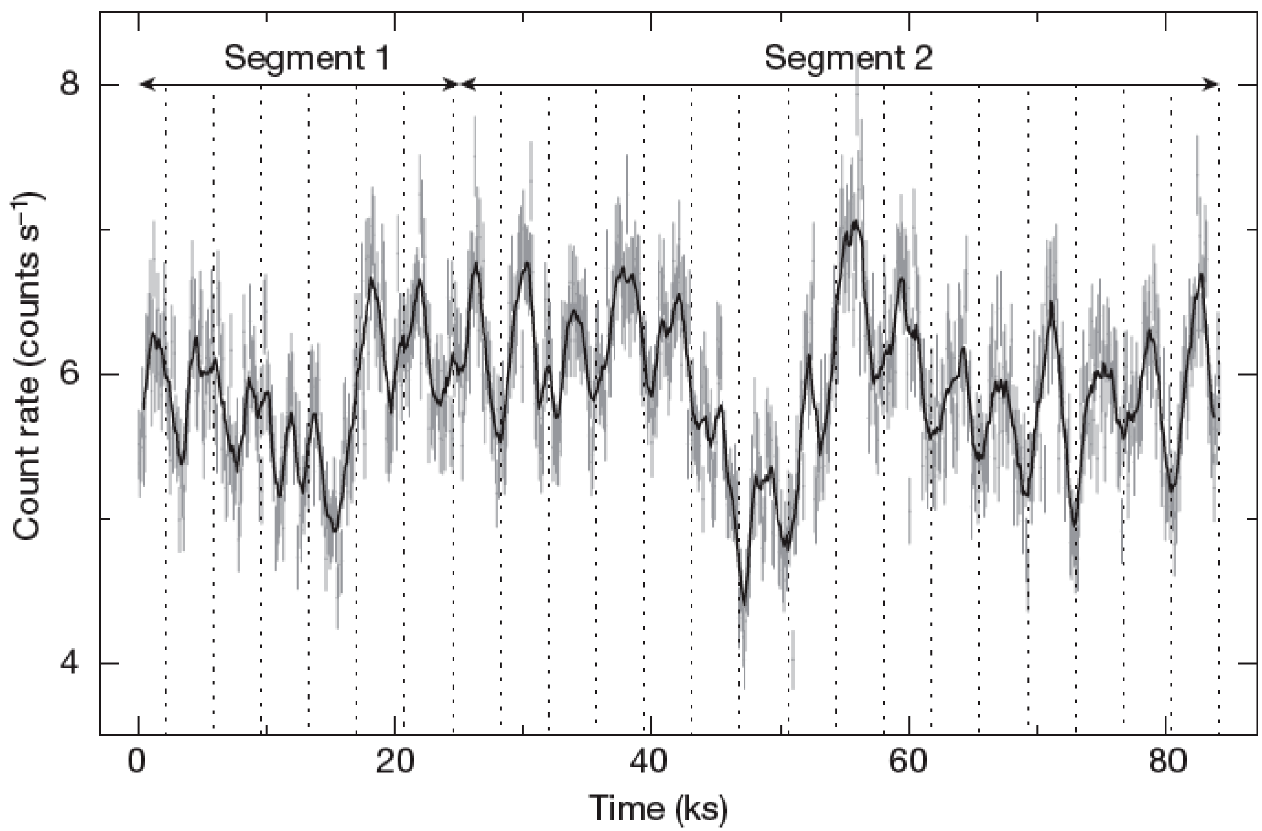


Remillard & McClintock 2006

Basic Facts about HFQPOs

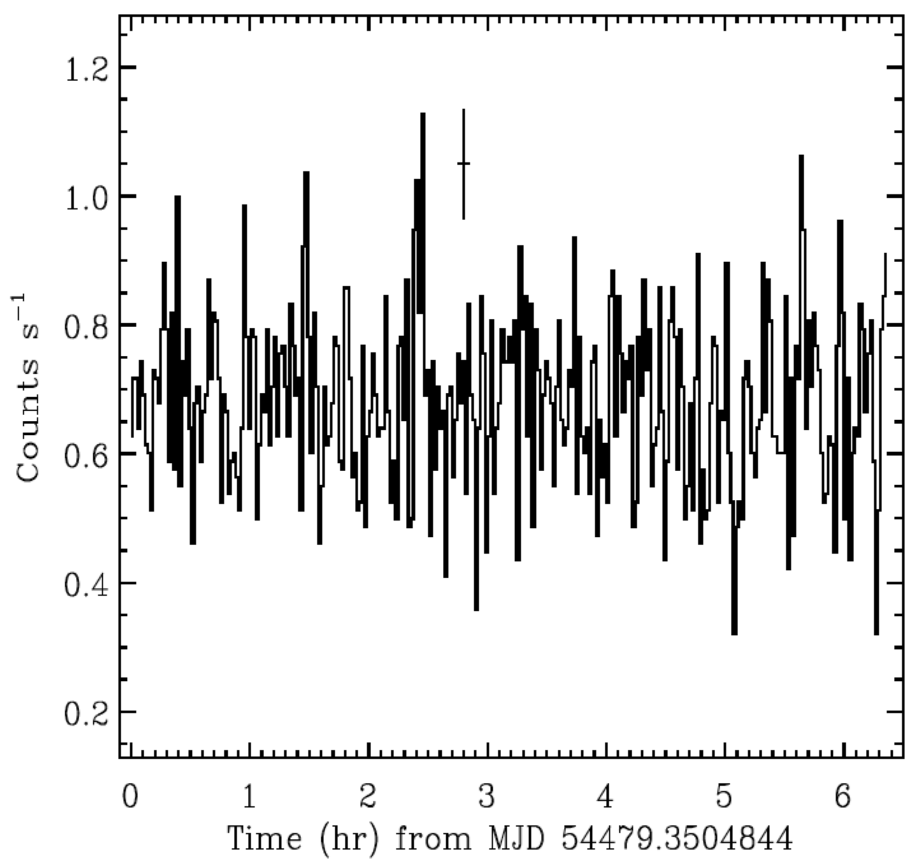
- 40-450 Hz: ~ orbital frequency at r_{isco}
- Frequency stable (<10% change when Mdot doubles)
- Some systems: ~2:3 ratio
- Weak QPOs: ~1% flux variation (in hard X-rays), Q~2-10
- Only occur in “Transitional state” (Episodic jet)

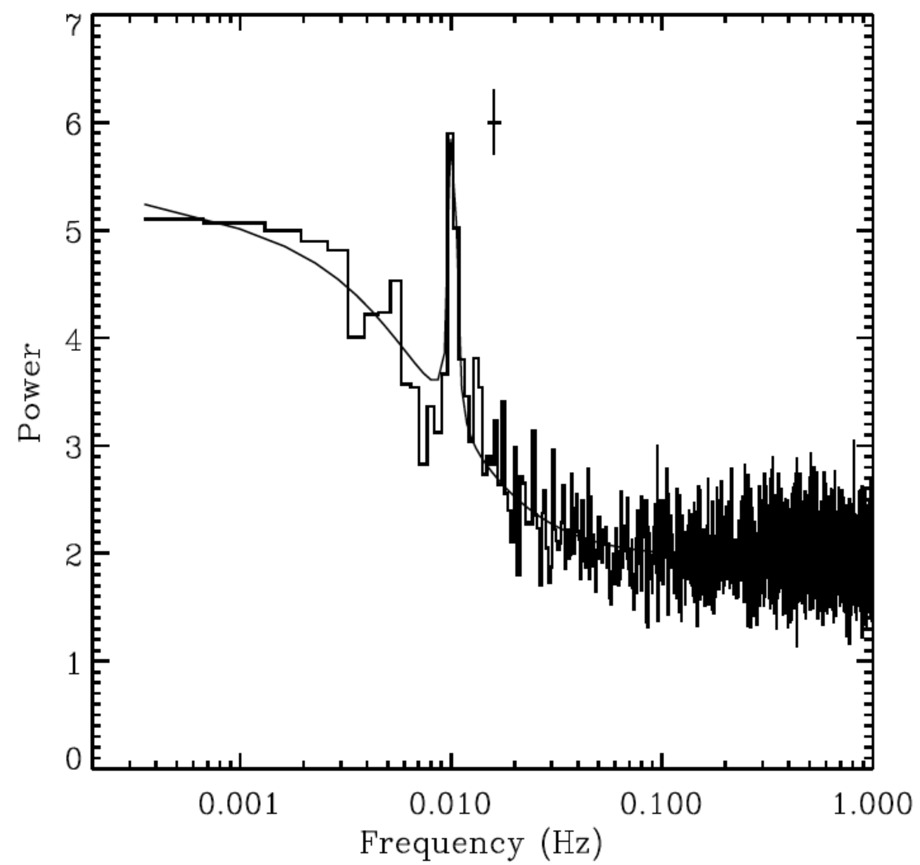
X-ray QPO (P ~ 1 hr) from active galaxy RE J1034+396



Gierlinski et al 2008, Nature

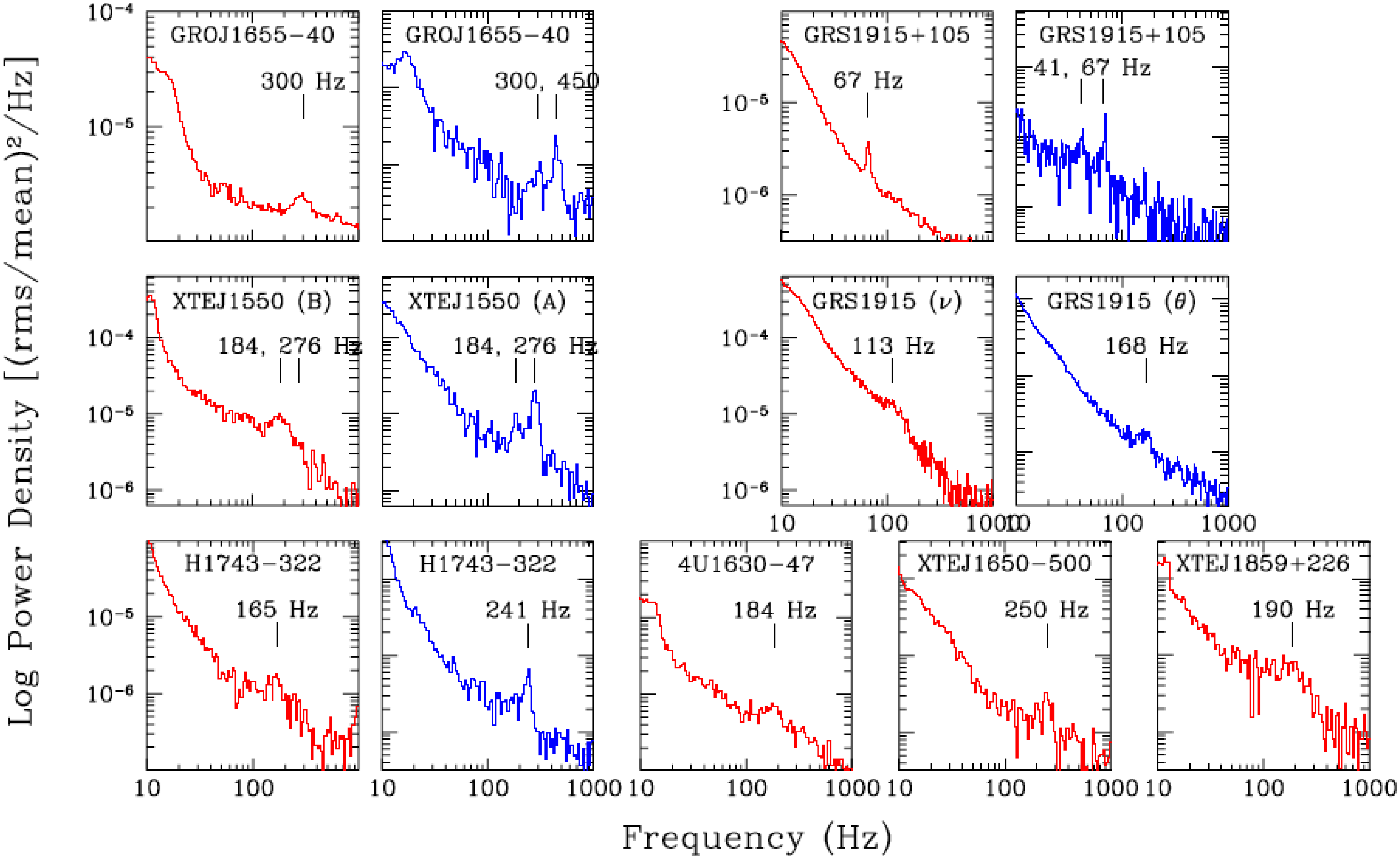
QPOs from Ultra-Luminous X-ray Source NGC 5408 X-1
(an Intermediate-mass BH?)





Strohmayer & Mushotzky 2009

High-Frequency QPOs in BH X-Ray Binaries



Remillard & McClintock 2006

Ideas/Models of HFQPOs

• Orbiting blobs (hot spots) in disks (Stella et al '99; Schnittman & Bertschinger '04)

• Nonlinear resonances of some kind (Abramowicz, Kluzniak, Horak, Rebusco)

• Acoustic modes in torus (Rezzolla el al '03; Lee, Abramowicz & Kluzniak '04; Blaes et al. '07; Sramkova et al '07; Horak'08)

• Disk/Magnetosphere Boundary Layer Oscillations (Li & Narayan '04; Tsang & DL '09)

• Oscillation modes in relativistic disks (Kato; Wagoner & collaborators)

-- m=0 inertial modes excited by global disk deformation (e.g. warps) (Kato '03,'08; Ferreira & Ogilvie '08; Henisey et al.10)

-- Rossby modes trapped in special region of a magnetic disk (Tagger & Varniere '06; see also Tagger & Pallet '99; Varniere & Tagger'02)

-- Cornell effort: Mode growth due to corotational resonance, magnetic fields (DL & Tsang '09; Tsang & DL '08,'09a,b; Fu & DL '09,'11a,b)

P-modes of BH Accretion Disks
"inertial-acoustic modes", "spiral density modes"

-- Trapped (partially) in the innermost region of disk

-- Frequencies can be calculated: robust, agree with observations

-- Can grow due to corotation resonance ("corotational instability")

GR plays an important role

-- B field effect

with David Tsang (Cornell Ph.D. 09 --> Caltech)
Wen Fu (Cornell Ph.D. student)

References: DL & Tsang 2009; Tsang & DL 2008, 2009; Fu & DL 2009,2011
Fu & DL 2012

Waves in 2D disks (Spiral density waves):

$$\delta v, \delta\Sigma \propto \exp(im\varphi - i\omega t)$$

Can propagate only in the region:

$$r < r_{\rm ILR} \qquad \text{or} \qquad r > r_{\rm OLR}$$

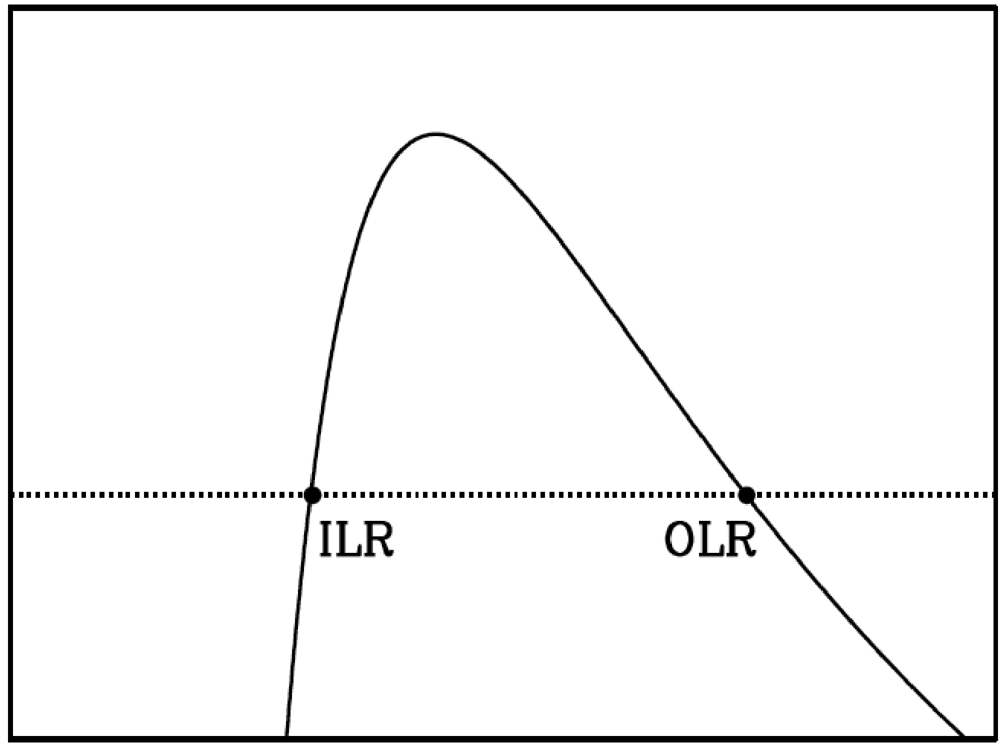
Lindblad Resonances: $\omega - m\Omega(r) = \pm\kappa(r)$

where $\Omega(r) = \text{disk rotation rate}$

$\kappa(r) = \text{radial epicyclic frequency}$

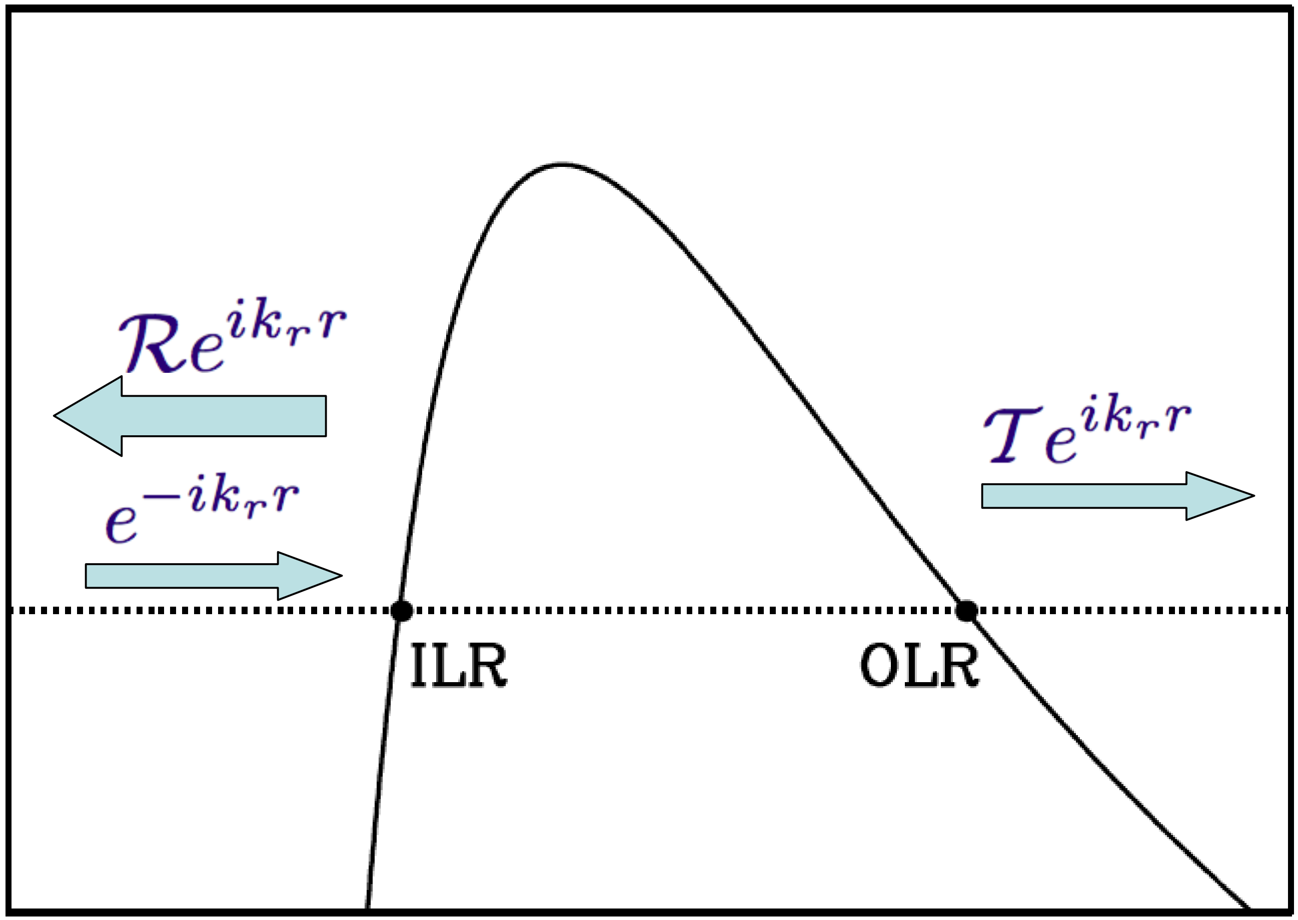
$$\kappa^2 = \frac{2\Omega}{r}\frac{d}{dr}(r^2\Omega)$$

Wave propagation diagram (effective potential)



$$\text{wave at } r > r_{\rm OLR}: \quad \omega/m > \Omega \;\Rightarrow\; \text{positive energy}$$

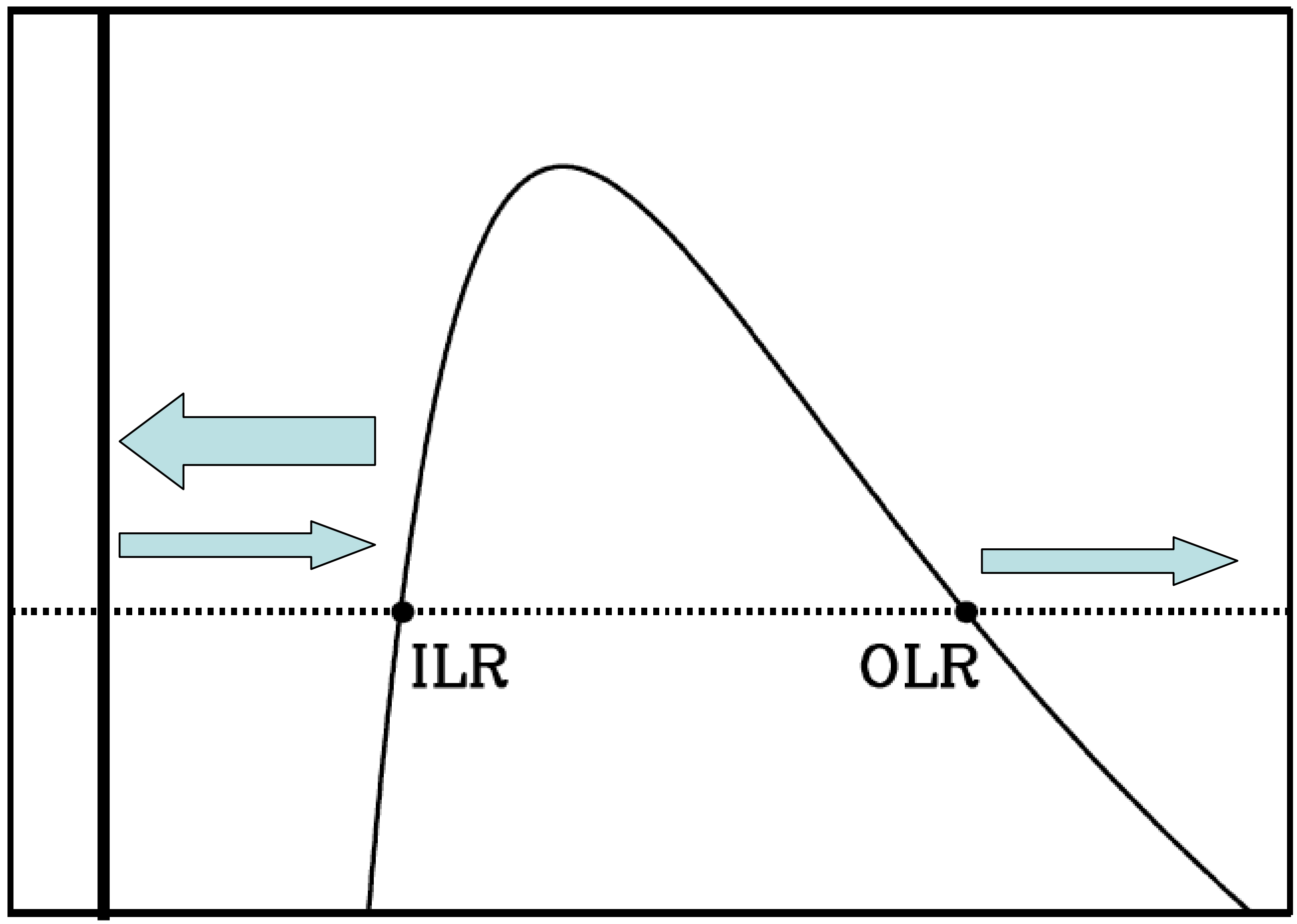
$$\text{wave at } r < r_{\rm ILR}: \quad \omega/m < \Omega \;\Rightarrow\; \text{negative energy}$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2$$

$$\Rightarrow \ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 > 1$$

Super-reflection

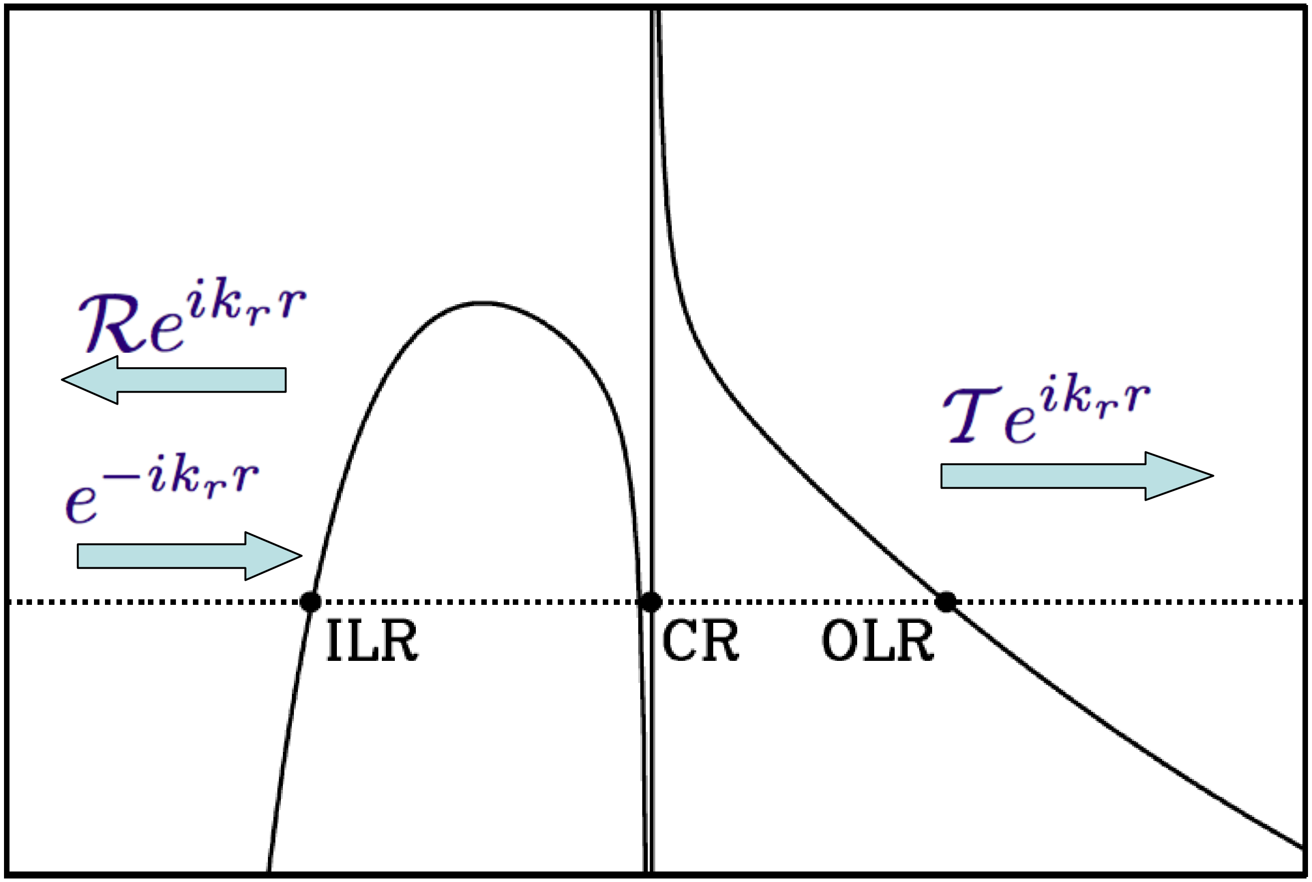


Trapped mode between $r_{\rm in}$ and $r_{\rm ILR}$: overstable

Even more interesting…

Corotation resonance, where

$$\omega/m = \Omega$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2 + \mathcal{D}_c$$

$$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c$$

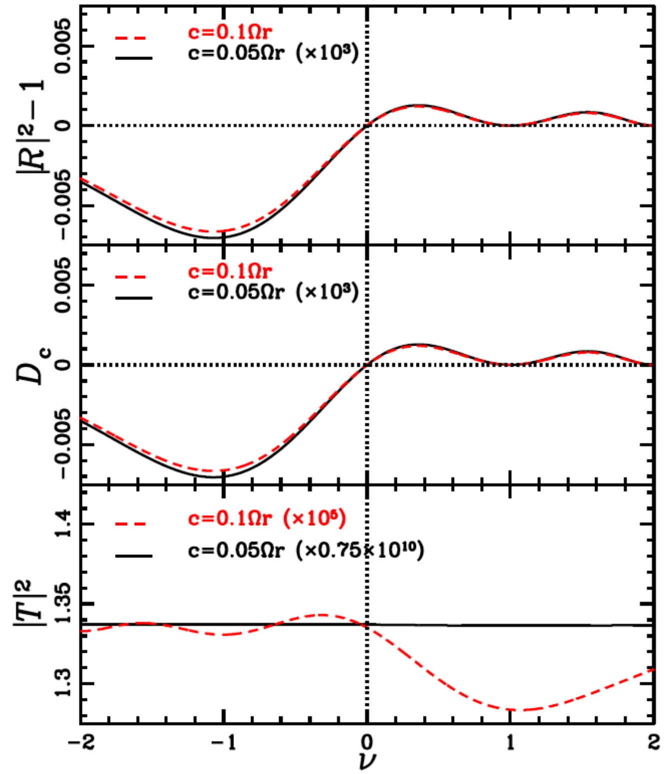
Wave absorption at corotation

Can have both signs !

Calculations of reflectivity/transmission:

$$\delta h = \sqrt{S/k}\left[\exp\left(-i\int_{r_{\rm IL}}^{r} kdr + \frac{\pi}{4}\right) + \mathcal{R}\exp\left(i\int_{r_{\rm IL}}^{r} kdr - \frac{\pi}{4}\right)\right]$$

$$\delta h = \sqrt{S/k}\,\mathcal{T}\exp\left(i\int_{r_{\rm OL}}^{r} kdr + \frac{\pi}{4}\right)$$



- Solve wave equation in different regions
- Match the solutions using asymptotic expansions
- Around corotation: Whittaker function; Stokes phenomenon

Tsang & DL

$$\mathcal{R} = \frac{1 + \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} + \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}{1 - \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

$$\mathcal{T} = \frac{ie^{-2\Theta_{\rm II}}e^{i\pi\nu}}{1 - \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

$$\Theta_{\rm IIa} = \int_{r_{\rm IL}}^{r_c}|k|\,dr \qquad \Theta_{\rm IIb} = \int_{r_c}^{r_{\rm OL}}|k|\,dr$$

Reflectivity at ILR: $|\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c$

Sign depends on sign of $d\zeta/dr$

$$\zeta = \frac{\kappa^2}{2\Omega\Sigma}$$ (vortensity)

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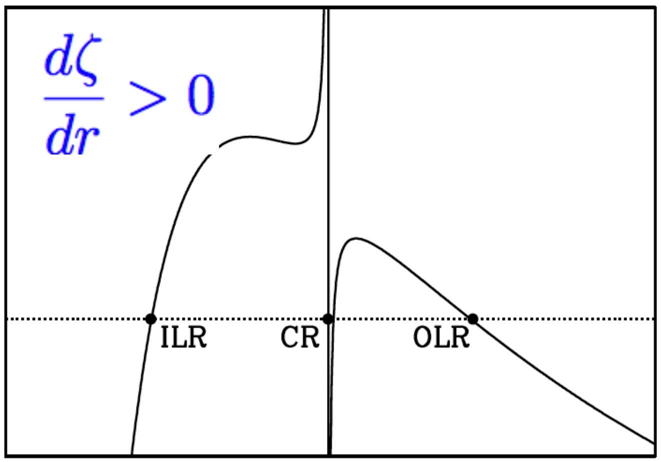

$$\Rightarrow \mathcal{D}_c > 0$$

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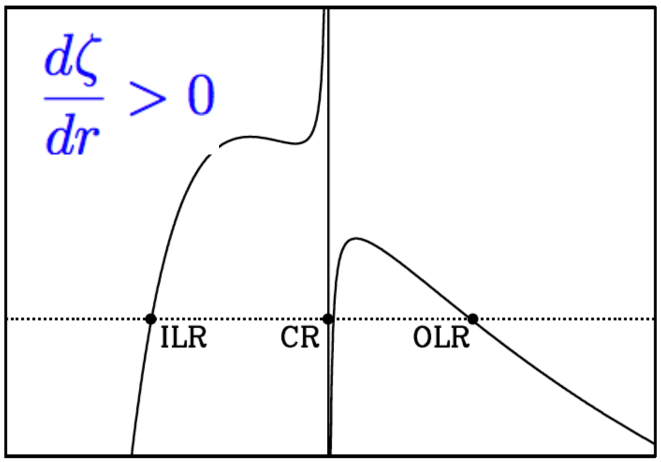
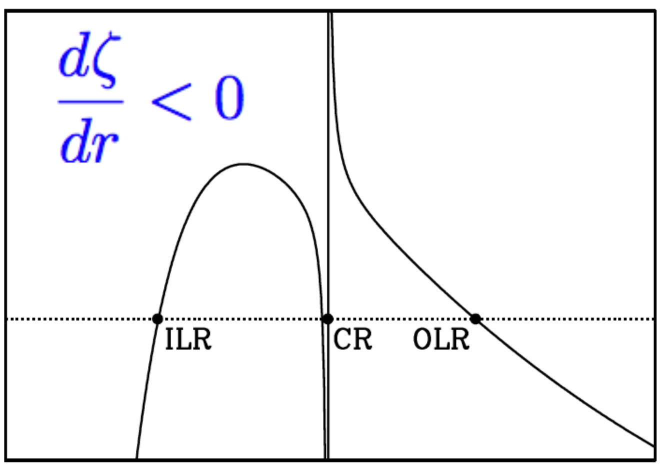


$$\Rightarrow \mathcal{D}_c > 0$$



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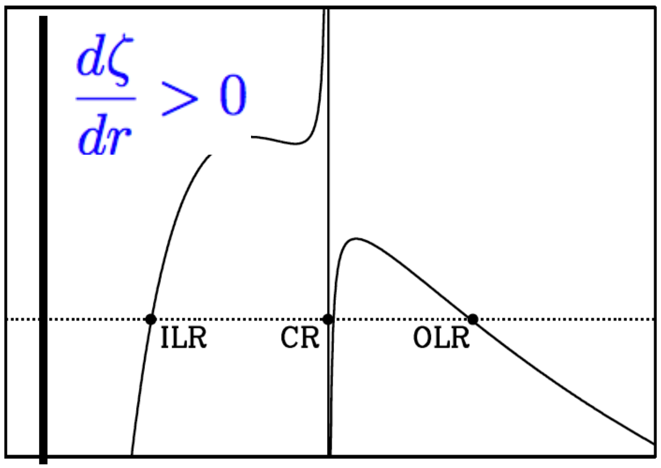
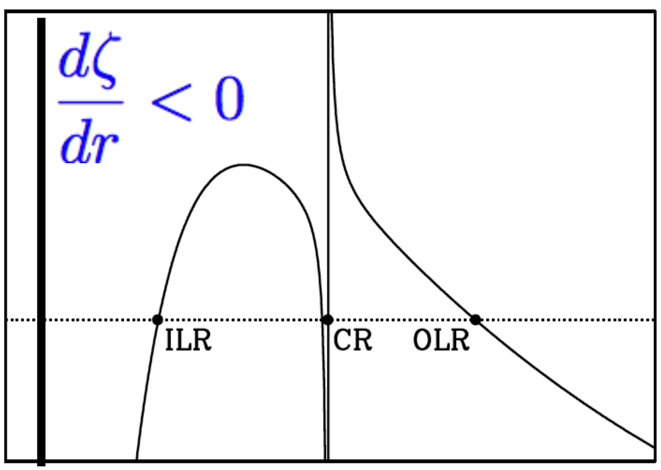


$\Rightarrow \mathcal{D}_c > 0$

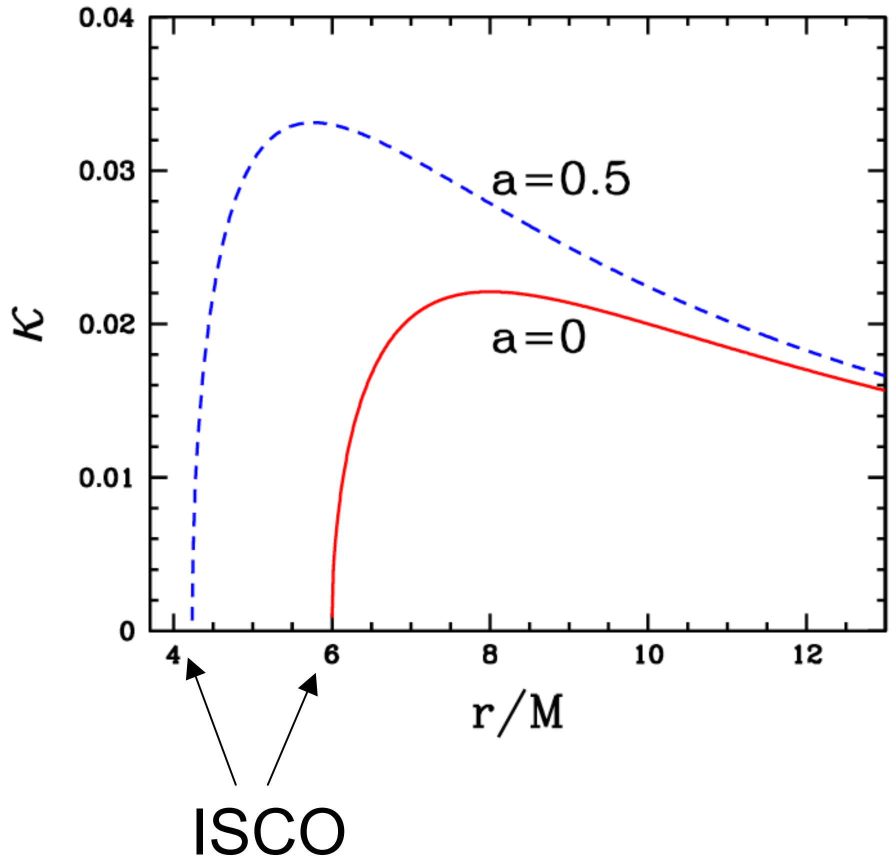
Overstable mode



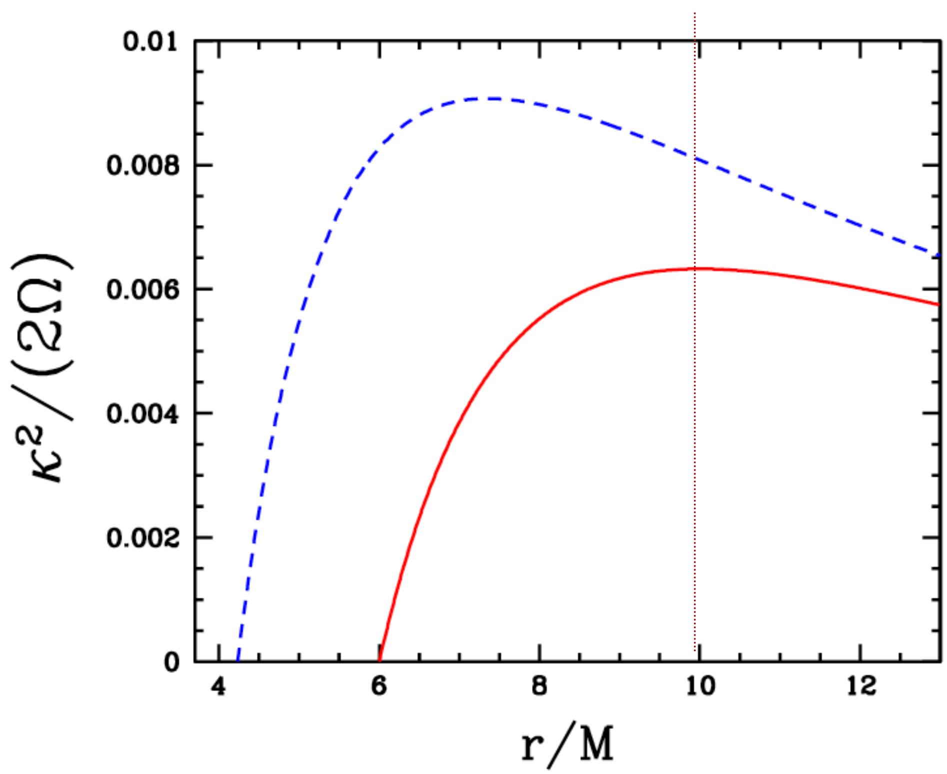
$\Rightarrow \mathcal{D}_c < 0$

Damped mode

General Relativity Effect

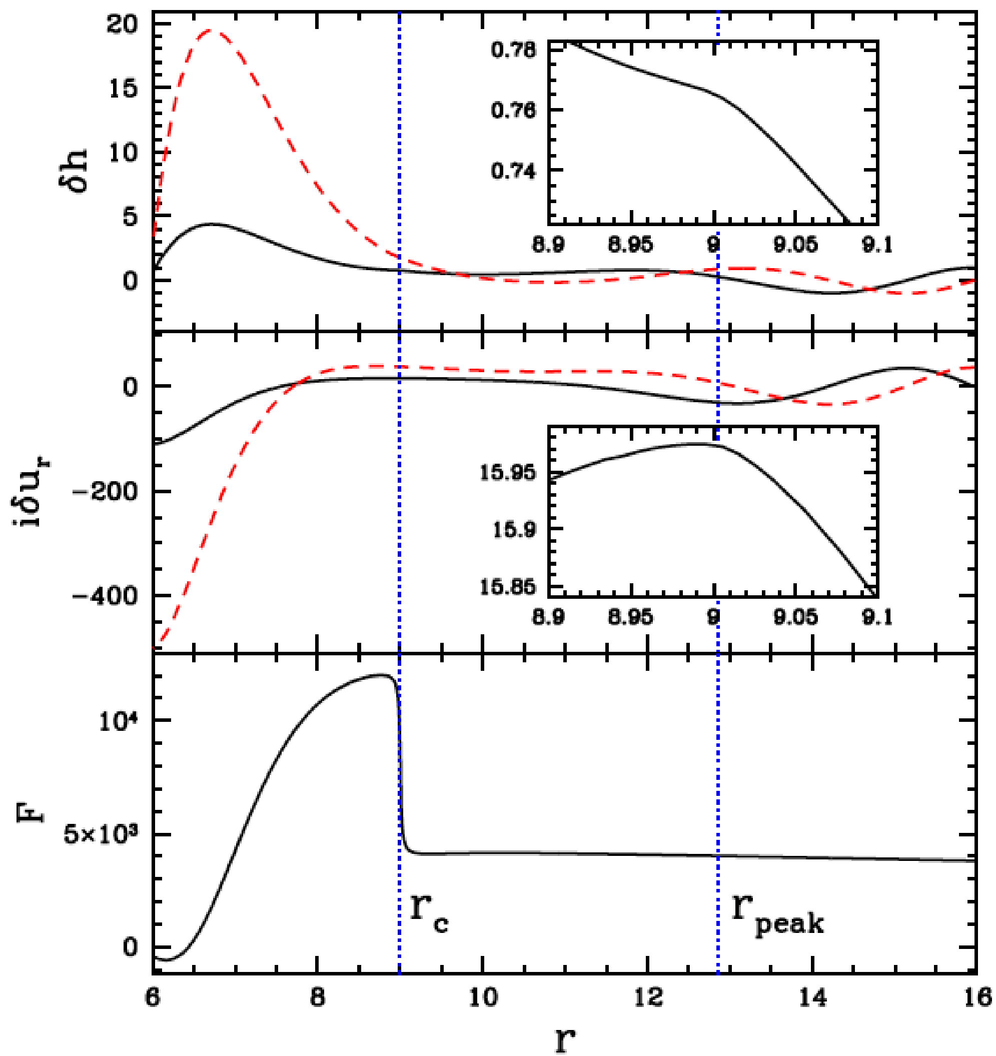


GR makes $d\zeta/dr > 0$
in the Inner-most disk region
==> makes the mode grow !



Vortensity $\zeta = \frac{\kappa^2}{2\Omega\Sigma}$

Linear Mode Calculation (Mode freq. and growth rate)

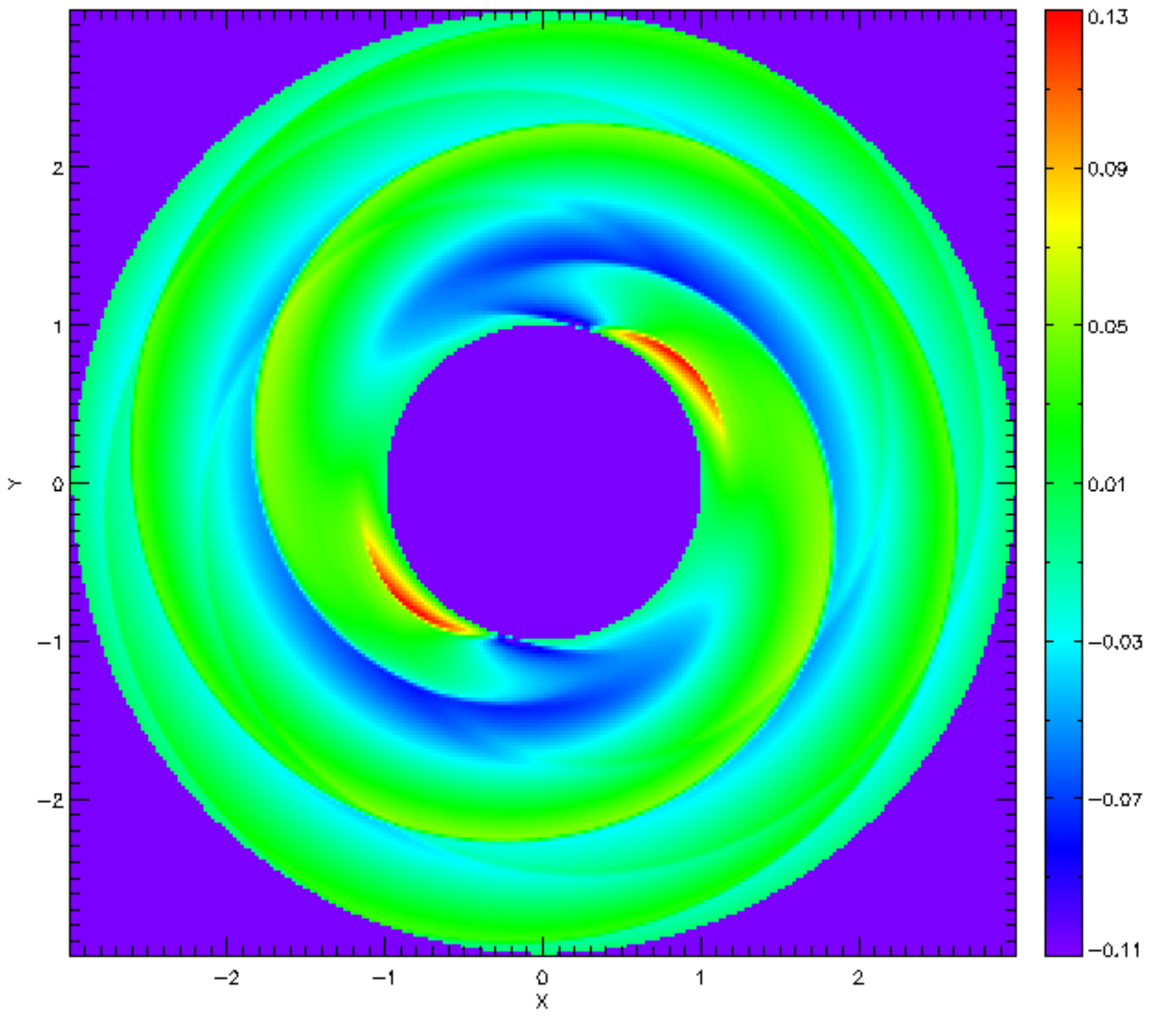


$$\Sigma \propto r^{-1}, \quad c_s = 0.1 r\Omega, \quad m = 2$$

$$\omega_r = 0.93\,\Omega_{\rm ISCO}, \quad \omega_i/\omega_r = 0.0029$$

DL & Tsang 2009,2010

Nonlinear Simulation (2D) of Growing Modes



Wen Fu & DL 2012, in prep

Properties of Overstable Disk P-Modes:

Low-order p-modes trapped between inner disk edge and ILR

$$\omega \simeq \beta m \Omega(r_{\rm in})$$

$\beta = 0.55$–0.75 depending on disk models and inner BC

- Mode frequencies robust,
  consistent with known BH mass (and spin)
- Frequency ratio approximately: 1:2:3:4… (not exactly)

Grow due to corotation resonance (GR plays important role)

A promising candidate for HFQPOs

Complications…

Complications…

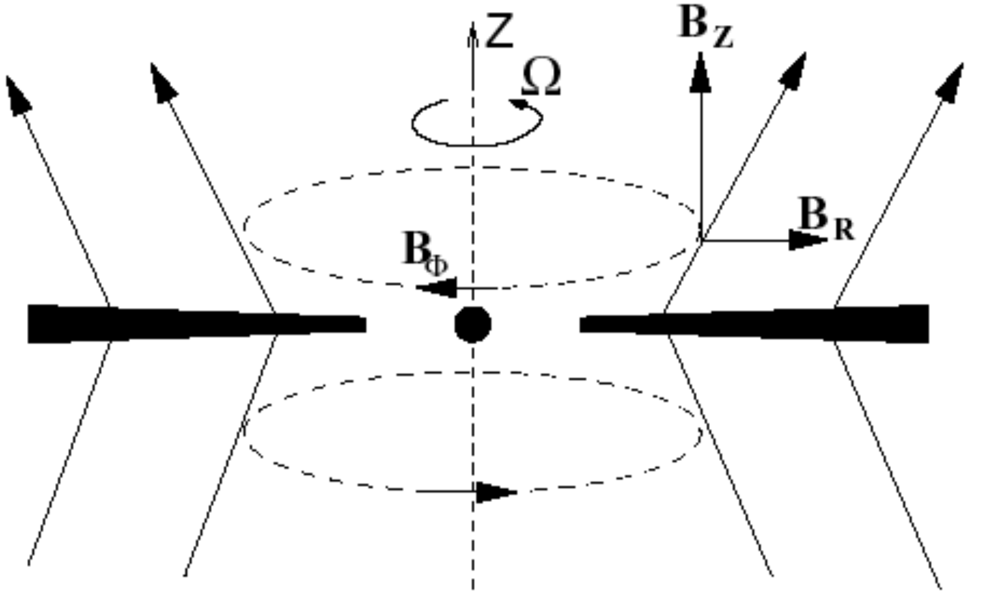
- **Mode damping due to radial infall**

  Competition: mode growth (due to corotation) and damping
  ==> HFQPOs do not always appear
  e.g. in thermal state (standard thin disk) no QPOs observed

- **Effects of magnetic fields**

  -- Mode frequencies are slightly affected
  -- Large-scale B field enhances the growth rate

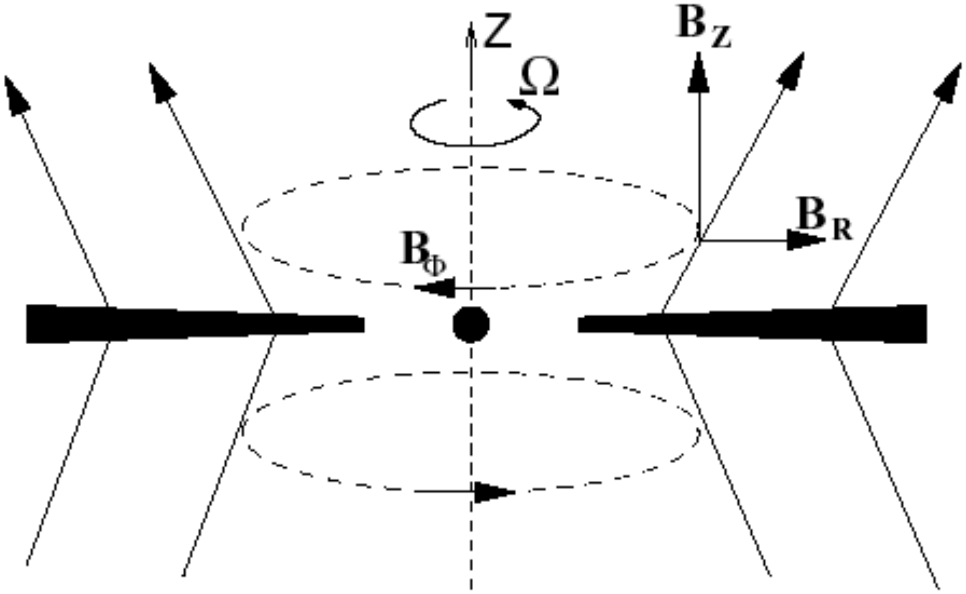
Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)



-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together,
the “clock” is mainly set by disk

Disks threaded by large-scale poloidal magnetic fields (embedded in a corona)



-- Increase the p-mode growth rate due to corotation resonance

Disk + Corona (coupled by B field) oscillate together,
the “clock” is mainly set by disk

-- Such large-scale field is ideal for producing jets/outflows

QPOs are observed at the same time as episodic jets

Recap of HFQPOs

- Intriguing puzzle --- Dynamics of inner-most region of BH accretion disks (No standard models yet)

- P-modes (spiral density modes) partially trapped in the inner-most region of disks is promising candidate:

  -- Frequencies can be calculated from first principle,

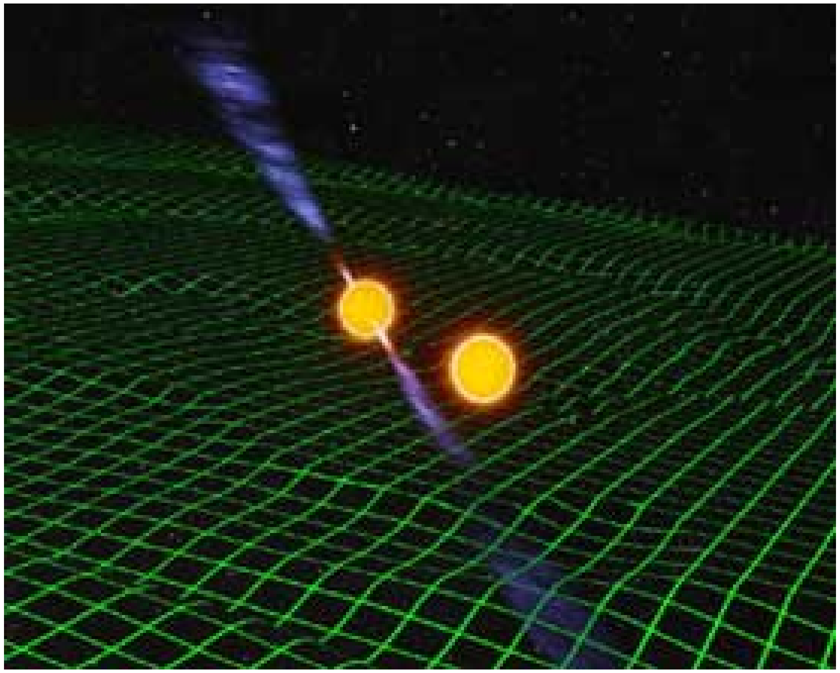
  robust, agree with observations (consistent mass, spin)

  -- Can grow naturally due to corotation resonance (GR important)

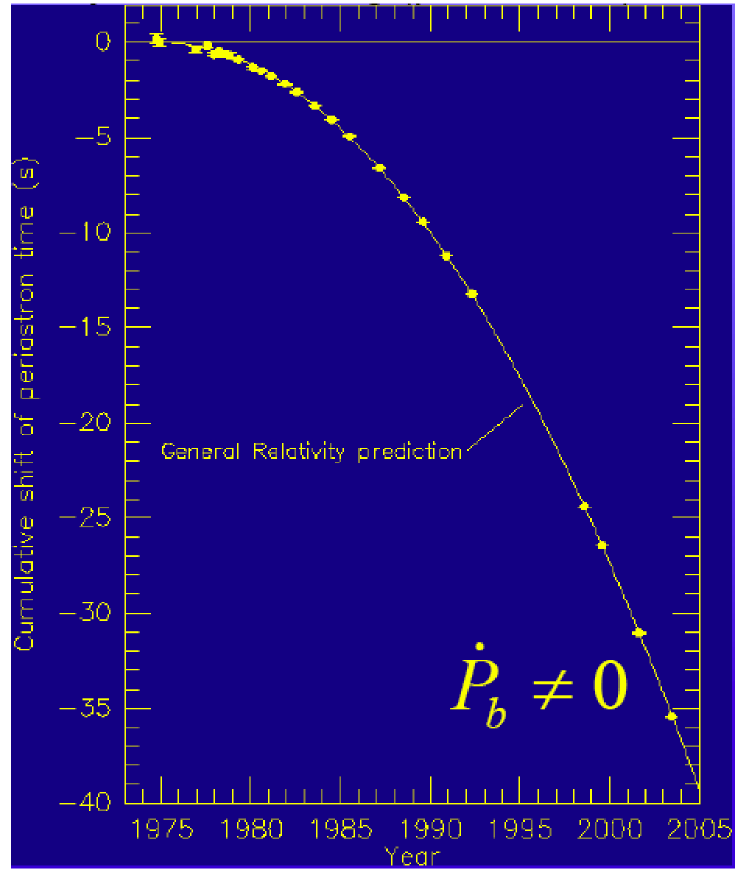
Incomplete: Complications, other issues (turbulence)…

Highlight #3:
Merging Binary Black Holes/Neutron Stars

Merging Neutron Stars:

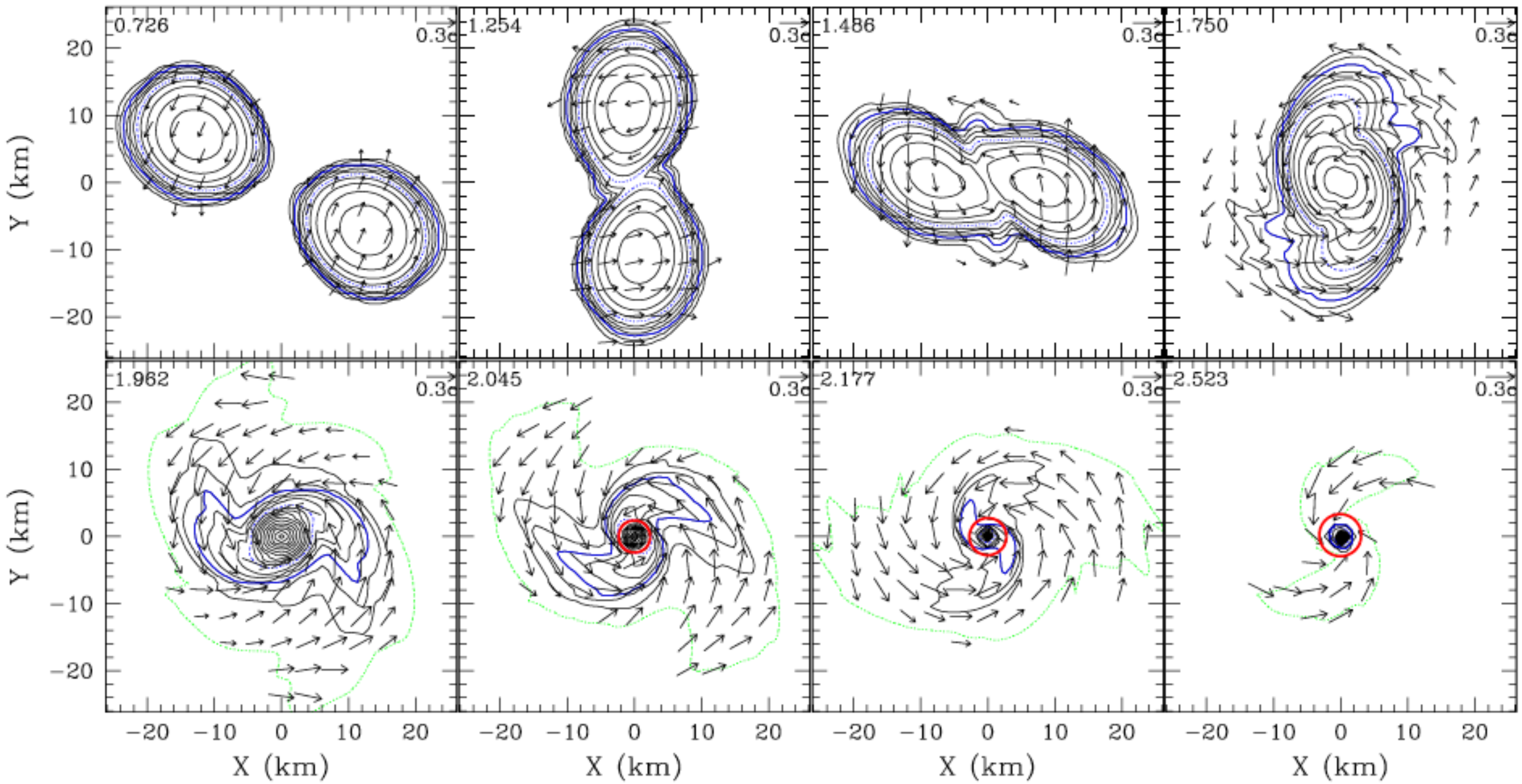


Nobel Prize 1993



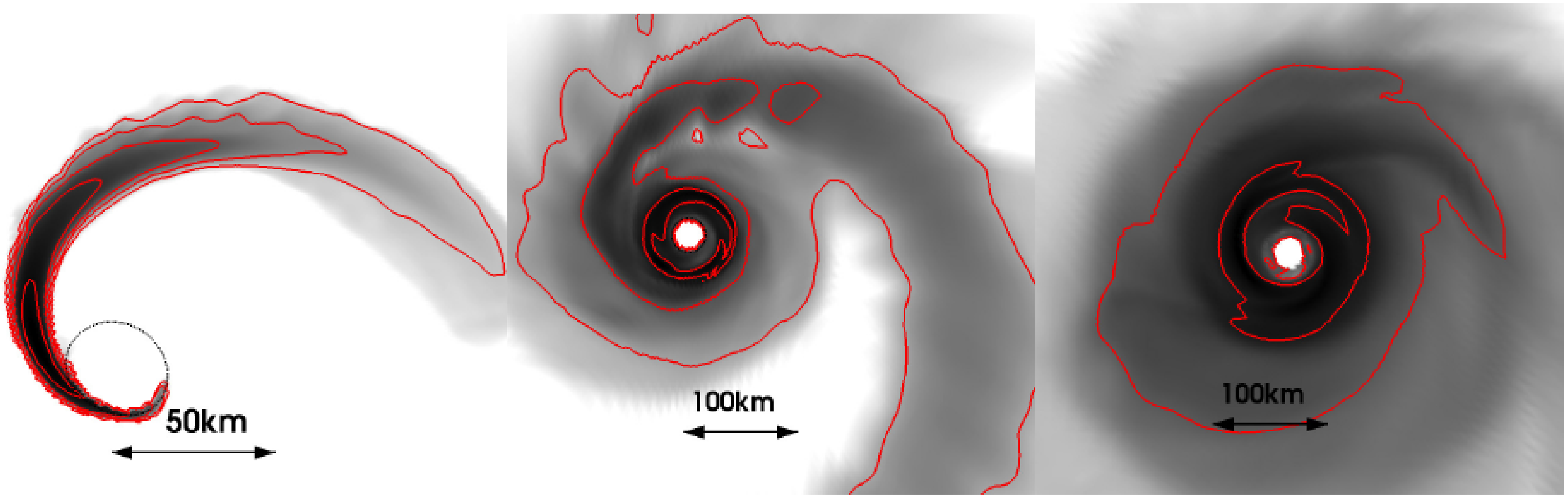
Taylor & Weisberg 2005

NS-NS Merger



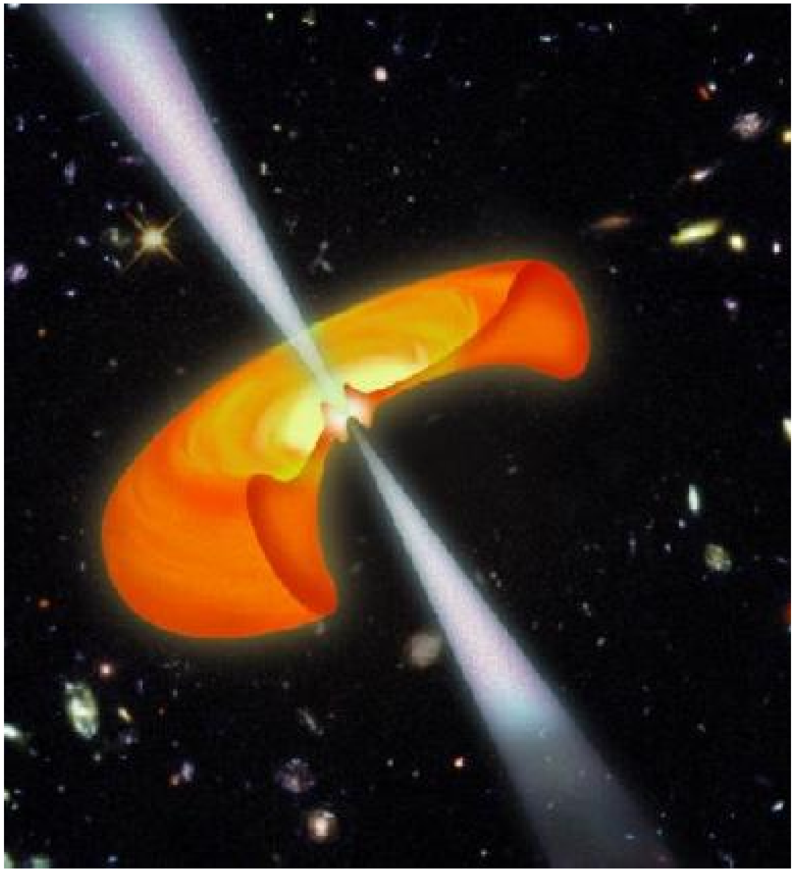
Shibata et al. 2006

BH-NS Merger

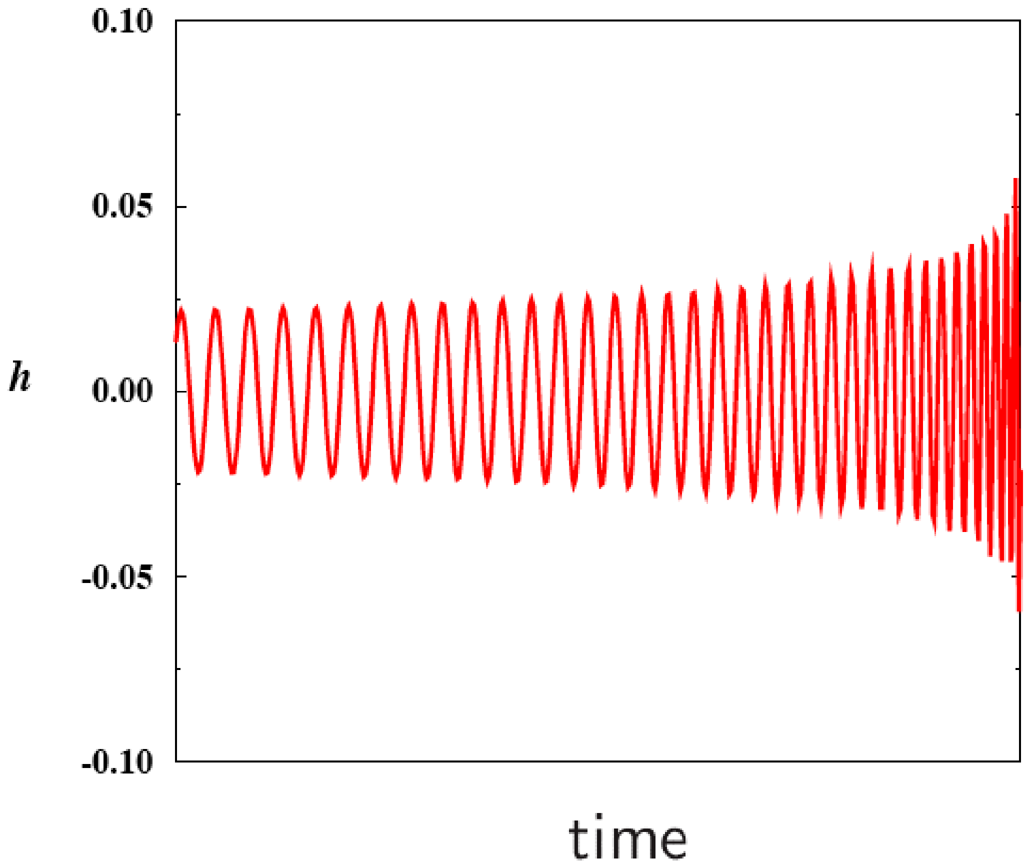


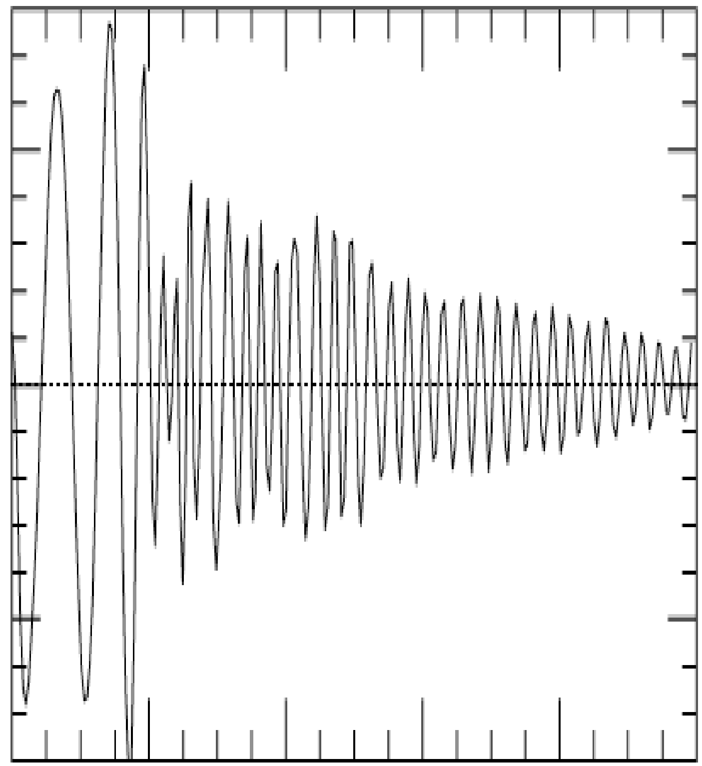
F. Foucart et al (Cornell) 2011

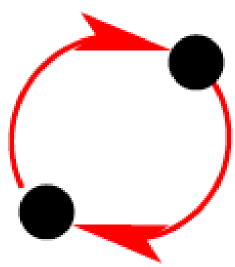
Merging NSs (NS/BH or NS/NS) as Central Engine of (short/hard) GRBs

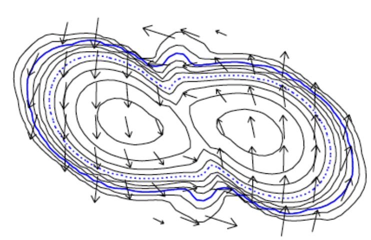


The last few minutes: Gravitational Waveform





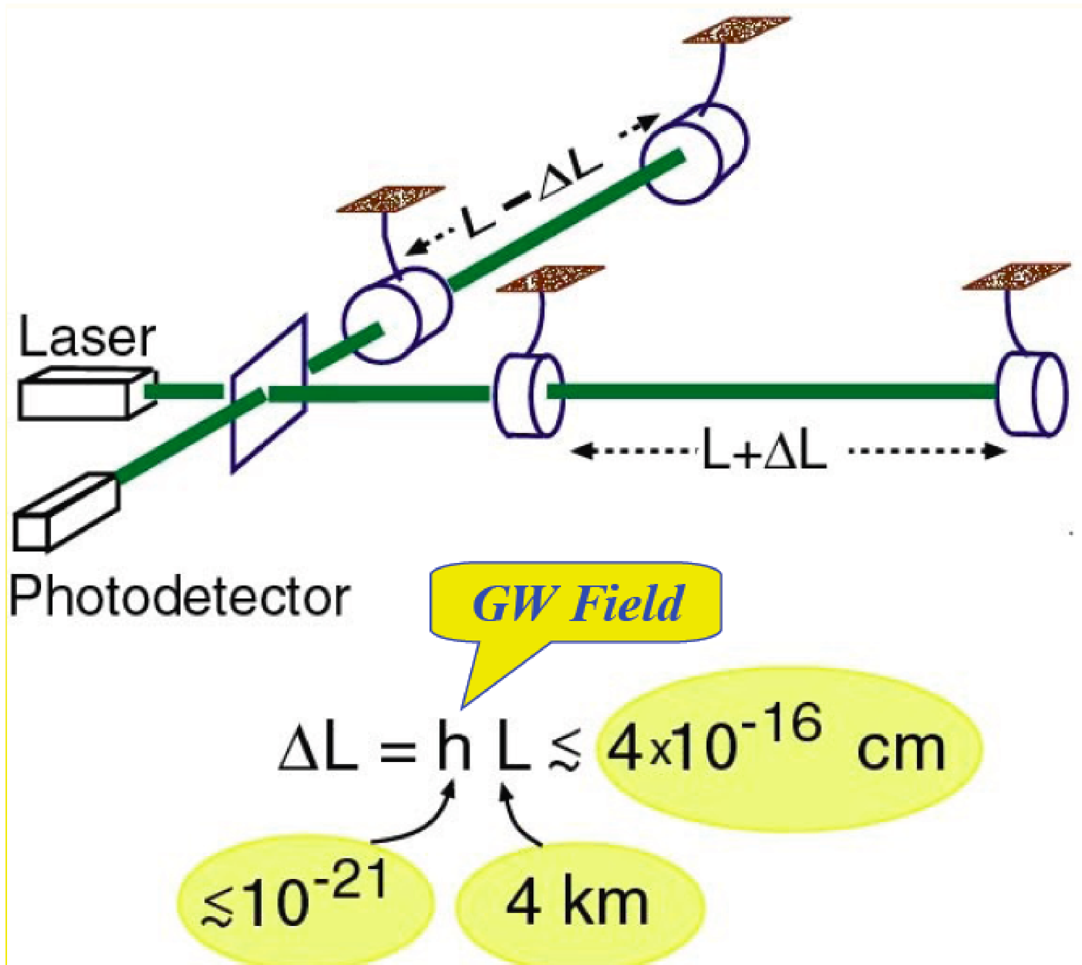




Gravitational Waves

- Warpage of Spacetime
- Generated by time-dependent quadrupoles
- Detector response to passage of GWs:

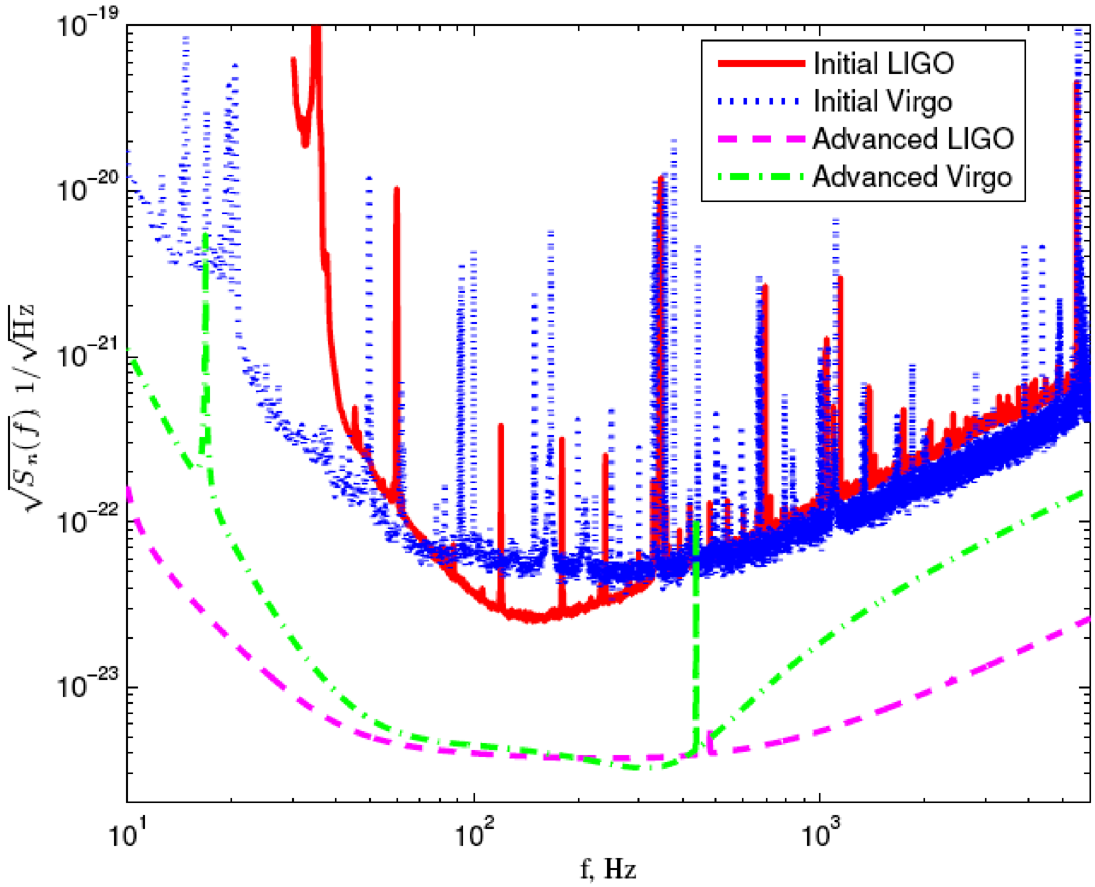
Gravitational Wave Interferometer



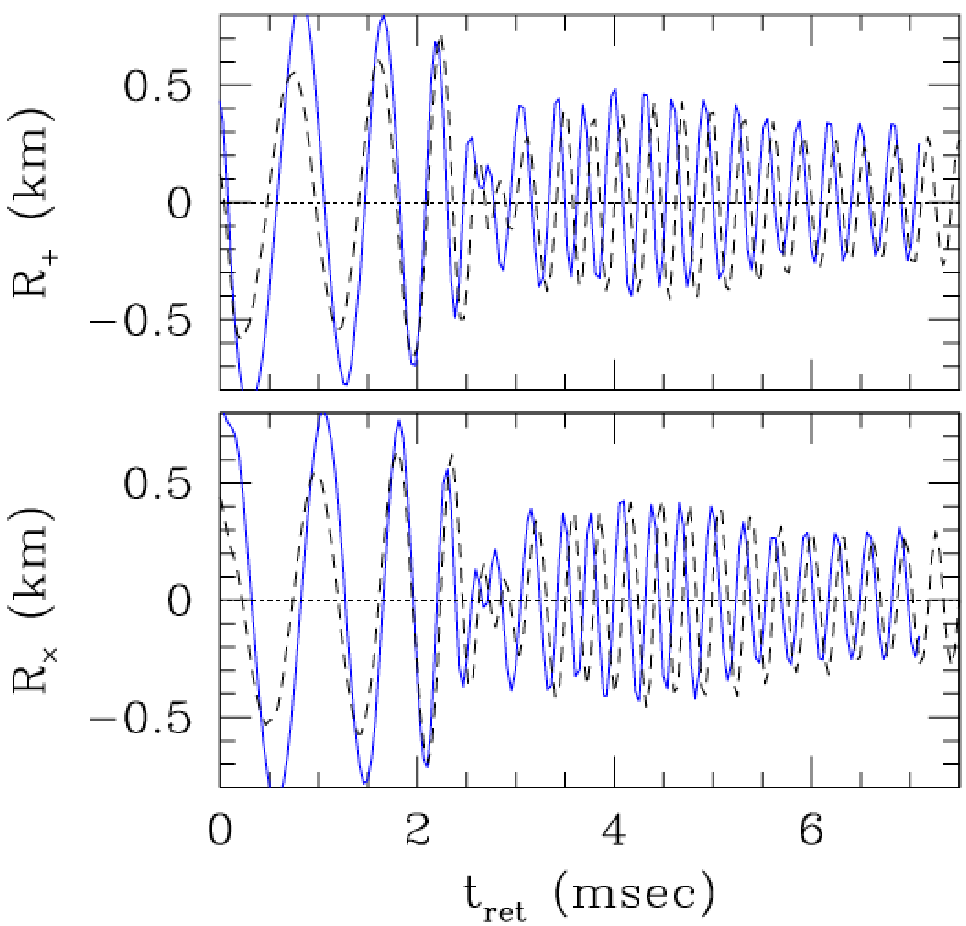
Kip Thorne

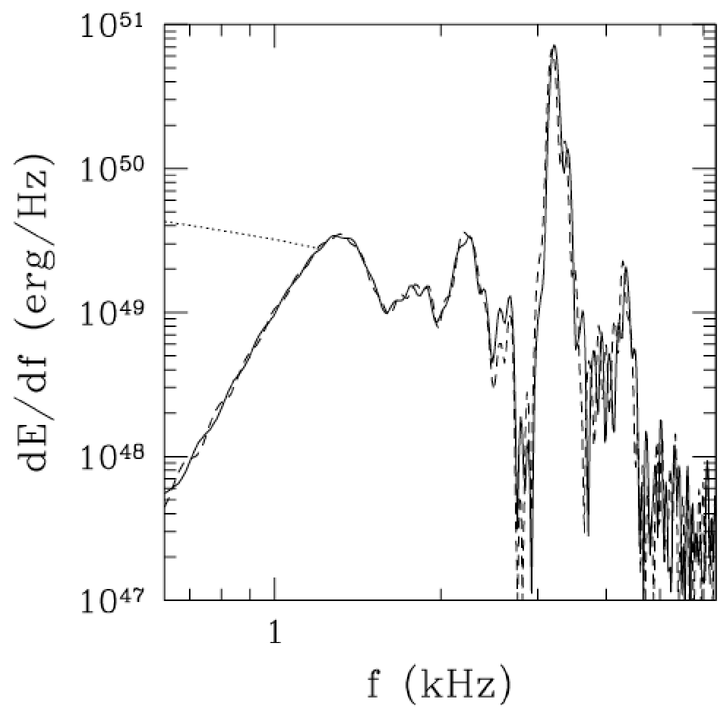

Hanford Washington


Livingston,
Louisiana

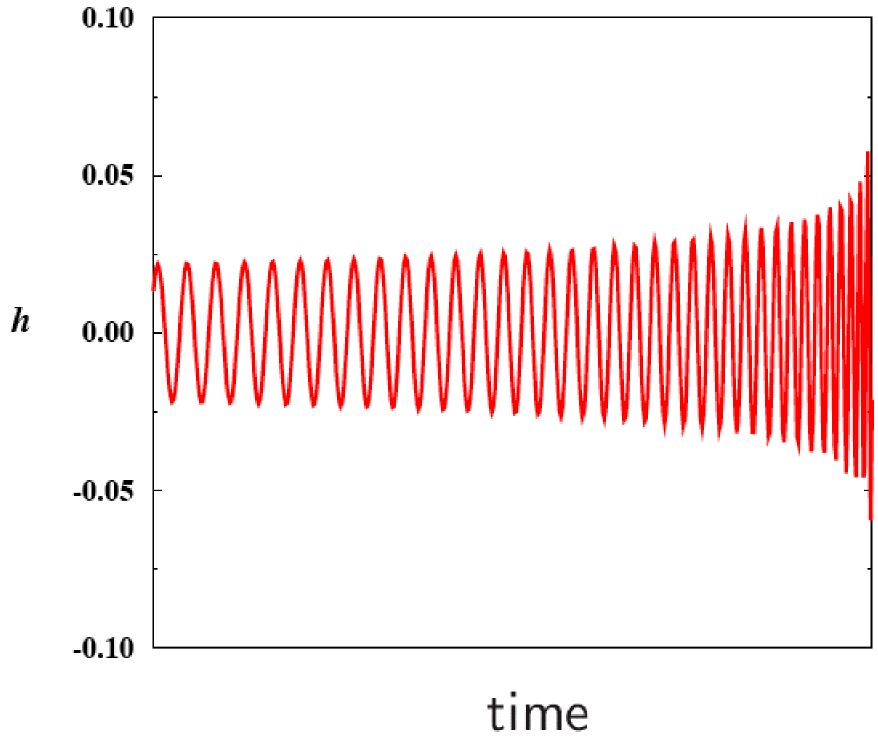

Initial LIGO
Initial Virgo
Advanced LIGO
Advanced Virgo
$\sqrt{S_n(f)}$ $1/\sqrt{\text{Hz}}$
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{1}
10^{2}
10^{3}
f, Hz

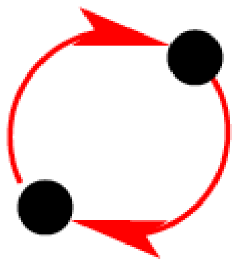
Final merger wave form probes NS EOS





Probe NS EOS using Inspiral Waveform





Idea:

- For point masses, the number of GW cycles is known exactly

- Rosonant tidal excitations of NS oscillation modes during inspiral
  ==> transfer orbital energy to NS
  ==> **Missing GW cycles**

Resonant Excitations of NS Modes During Binary Inspiral

Non-rotating NS:

G-mode (Reisenegger & Goldreich 1994; DL 1994)

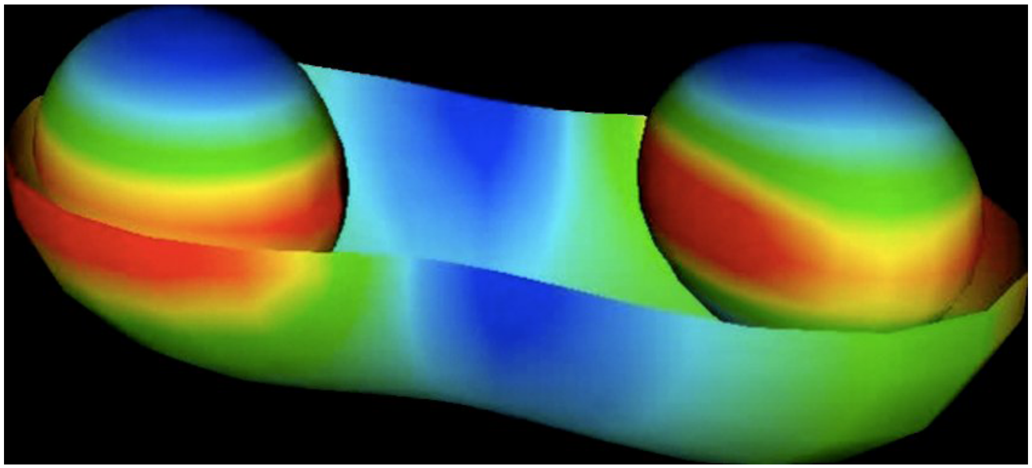
Rotating NS:

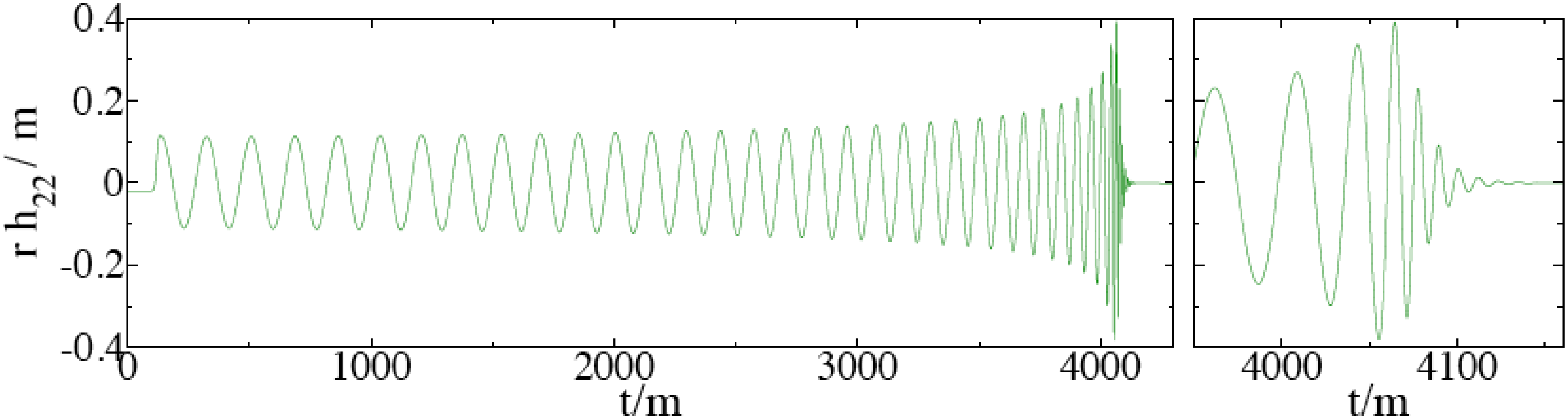
G-mode, F-mode, R-mode (Wynn Ho & DL 1999)
Inertial modes (DL & Yanqin Wu 2006)
R-mode (excited by gravitomagnetic force; Racine & Flanagan 2006)

Results:

- For R=10 km NS, the number of missing cycles < 0.1, unlikely measurable (unless NS is rapidly rotating)
- Number of missing cycles $\Delta N \propto R^4$ (g mode) or $R^{3.5}$ (r mode)
  Important for larger NS
- Crustal modes: important? Could shatter crust, pre-cursor of short GRB (D. Tsang et al. 2011)

BH-BH Merger





Cornell-Caltech collaboration

Summary

- **Compact Objects (White dwarfs, Neutron stars and Black Holes) have diverse observational manifestations**
  can be studied in many different ways: radio -- gamma rays, GWs

- **They present a rich set of astrophysics/physics problems**
  Ideal laboratory for probing physics under extreme conditions

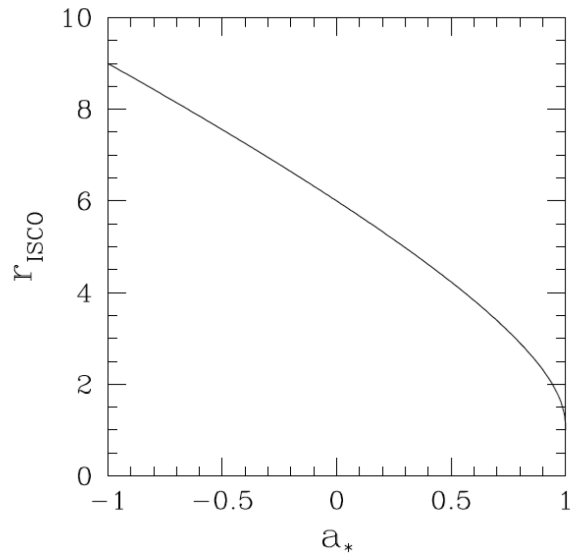
Obrigado !!

Black Hole Power in Astrophysics

Accretion Power

$$L_{\rm acc} = \epsilon \dot{M} c^2$$

ϵ = Binding energy (per unit mass) at ISCO
= 5.7% for a=0
42% for a=M



Inner-most stable circular orbit

r_{ISCO}=6M for a=0
=M for a=M

"Spin" Power

Extracting spin energy from BH (Penrose)

BH area theorem --> $$M_{\rm irr} = \left(\frac{A}{16\pi}\right)^{1/2} = \frac{M}{\sqrt{2}}\left(1+\sqrt{1-\frac{a^2}{M^2}}\right)^{1/2}$$

Maximum efficiency of energy extraction = $1 - M_{\rm irr}/M$
= 29.3% for a=M