Misaligned Planets/Stars, Disks and Rings

Dong Lai

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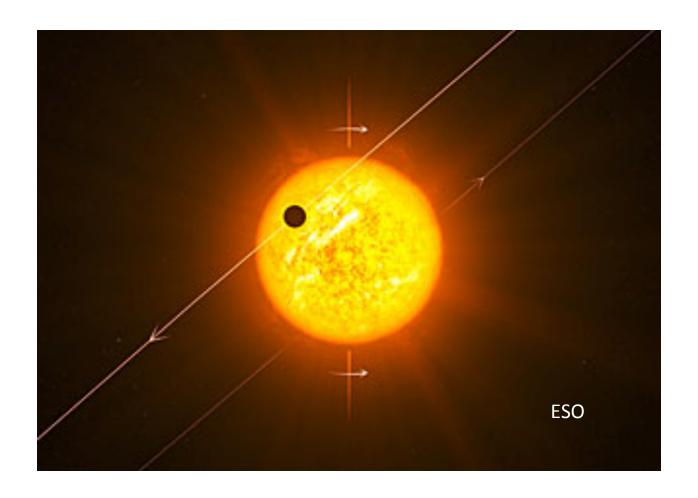
Misaligned Planets/Stars, Disks and Rings

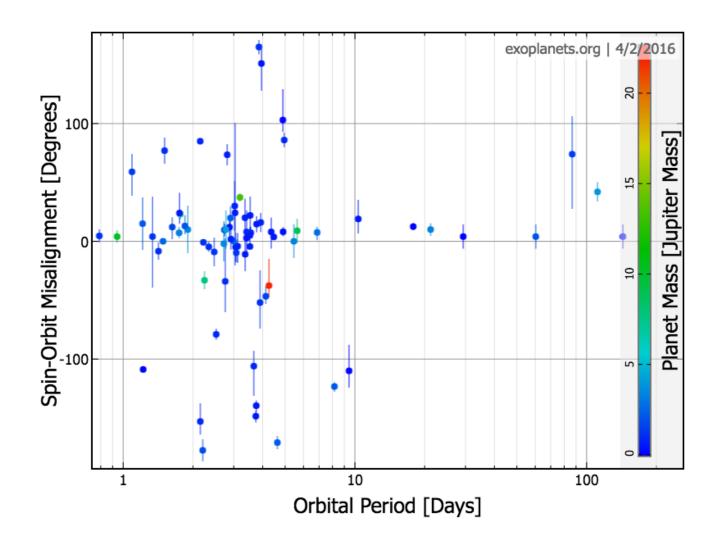
- (1) Binary-Disk-Star Interaction → Misaligned protoplanetary disk
- (2) Secular perturbation from binary on planet → Misaligned Hot Jupiters
- (3) Missing Kepler Multi's
- (4) Extended, misaligned ring/disk around planets/brown dwarfs
- (5) Planet IX and solar obliquity

Other topics:

- Circumbinary Disks (hydro) and circumbinary planets (dynamics)
- MMR capture and stability

Misaligned Planets





Also note: Misaligned multi-planet systems:

Kepler-56 (2 planets 10.5 & 21 days, 40-55 deg; Huber+2013); other candidates (?): Hirano+2014

How to produce misaligned planets (HJs)?

- -- "Primordial" misalignment between spin and disk
- -- "Lidov-Kozai migration" induced by a binary companion

-- Planet-Planet interactions:

dynamical scattering, secular chaos, coplanar high-e migration (Rasio & Ford 96; Chatterjee et al. 08; Juric & Tremaine 08; Nagasawa et al. 08; Wu & Lithwick 11; Beauge & Nesvorny 2012; Petrovich 15,16; Petrovich & Tremaine 16; Hamers et al.16)

Observational Hints:

Young HJs (Donati+16; David+16)

Ngo+15; Bryan+16 (External companions)

Huang+16 (neighbors of WJs/HJs)

Schlaufman & Winn 16 (companions inside ice line)

Ideas for Producing Primordial Misalignments

between Stellar Spin and Protoplanetary Disk

-- Star formation in turbulent medium

(Bate+2010; Fielding+2014)

-- Magnetic Star - Disk Interaction

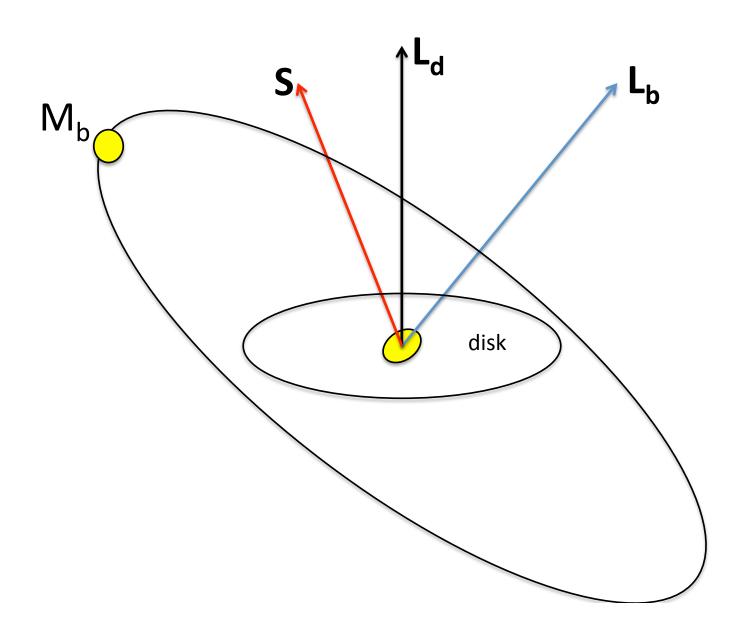
(Lai+2011, Foucart & Lai 2011)

-- Perturbation of Binary on Disk

(Batygin 2012; Batygin & Adams 2014; Spalding+2015; Lai 2014; Zanazzi & DL'17)

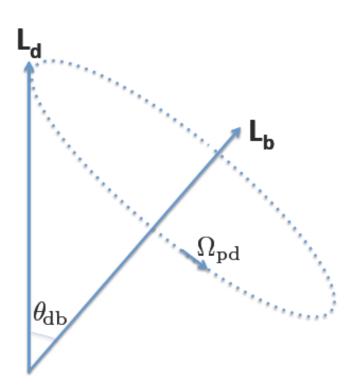
Star-Disk-Binary Interactions

DL2014



Companion makes disk precess

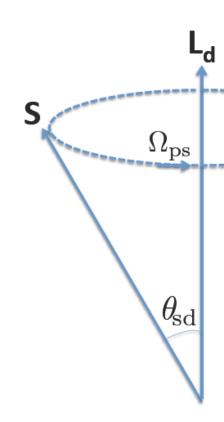
Disk behaves like a rigid body (bending waves, viscous stress, self-gravity) e.g. Foucart & Lai 2015; Caveat ? (J. Zanazzi & Lai, in prep)



$$\Omega_{\rm pd} \simeq -5 \times 10^{-6} \left(\frac{M_b}{M_{\star}}\right) \left(\frac{r_{\rm out}}{50 \,\text{AU}}\right)^{3/2} \left(\frac{a_b}{300 \,\text{AU}}\right)^{-3} \times \cos \theta_{\rm db} \left(\frac{2\pi}{\rm yr}\right)$$

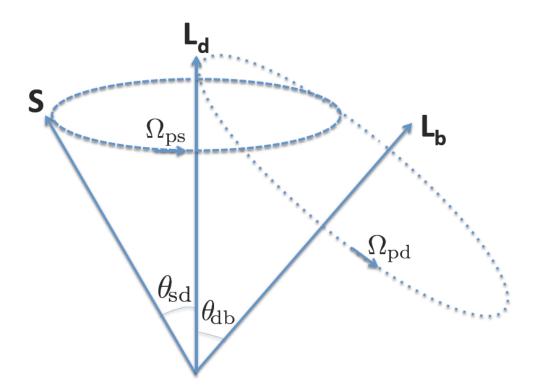
Disk makes the star precess

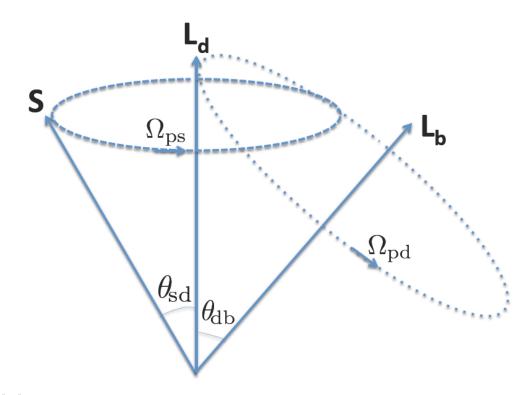
Due to gravitational torque from disk on rotating (oblate) star



$$\Omega_{\rm ps} \simeq -5 \times 10^{-5} \left(\frac{M_d}{0.1 M_{\star}}\right) \left(\frac{\bar{\Omega}_{\star}}{0.1}\right) \left(\frac{r_{\rm in}}{4 R_{\star}}\right)^{-2} \left(\frac{r_{\rm out}}{50 \,\text{AU}}\right)^{-1} \times \cos \theta_{\rm sd} \left(\frac{2\pi}{\rm yr}\right)$$

where
$$\Omega_{\star} = \left(\frac{2\pi}{3.3 \, \mathrm{days}}\right) \left(\frac{\bar{\Omega}_{\star}}{0.1}\right)$$

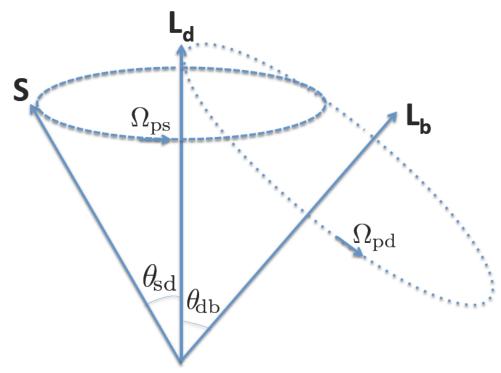




Two limiting cases:

(1)
$$|\Omega_{\rm ps}| \gg |\Omega_{\rm pd}|: \implies \theta_{\rm sd} \simeq {\rm constant}$$

(2)
$$|\Omega_{\rm ps}| \ll |\Omega_{\rm pd}|: \implies \theta_{\rm sb} \simeq {\rm constant}$$

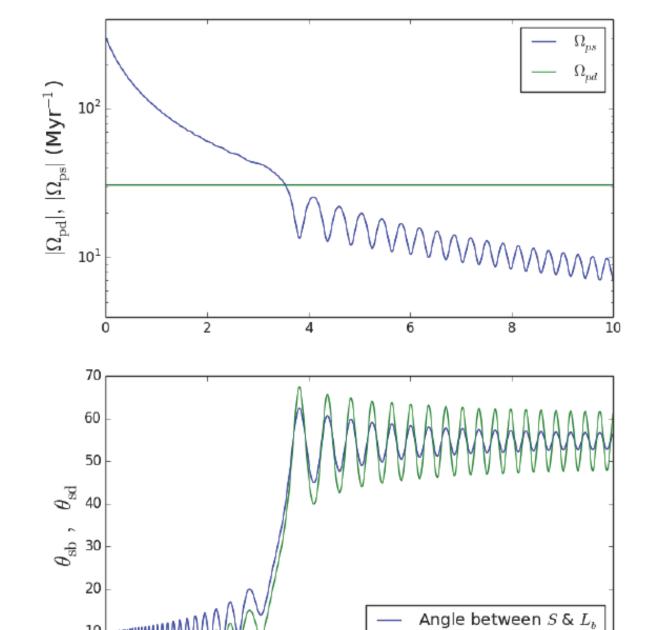


$$\Omega_{\rm pd} \simeq -5 \times 10^{-6} \left(\frac{M_b}{M_{\star}}\right) \left(\frac{r_{\rm out}}{50 \,\text{AU}}\right)^{3/2} \left(\frac{a_b}{300 \,\text{AU}}\right)^{-3}$$

$$\times \cos \theta_{\rm db} \left(\frac{2\pi}{\rm yr}\right)$$

$$\Omega_{\rm ps} \simeq -5 \times 10^{-5} \left(\frac{M_d}{0.1 M_{\star}} \right) \left(\frac{\bar{\Omega}_{\star}}{0.1} \right) \left(\frac{r_{\rm in}}{4 R_{\star}} \right)^{-2} \left(\frac{r_{\rm out}}{50 \,\text{AU}} \right)^{-1} \\
\times \cos \theta_{\rm sd} \left(\frac{2\pi}{\text{vr}} \right)$$

Initially $|\Omega_{\rm ps}|\gg |\Omega_{\rm pd}|$ M_d decreases over ~1 Myr



Initial:

$$\theta_{\rm db} = 5^{\circ}$$

$$\theta_{\rm sd} = 5^{\circ}$$

10

DL2014

10

Angle between $S \& L_d$

8

6

Time (Myrs)

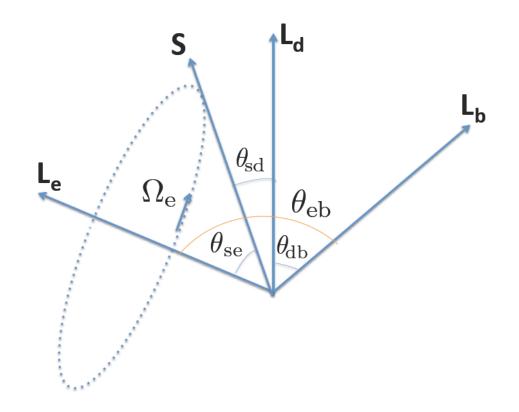
Resonance $\Omega_{ps} = \Omega_{pd}$

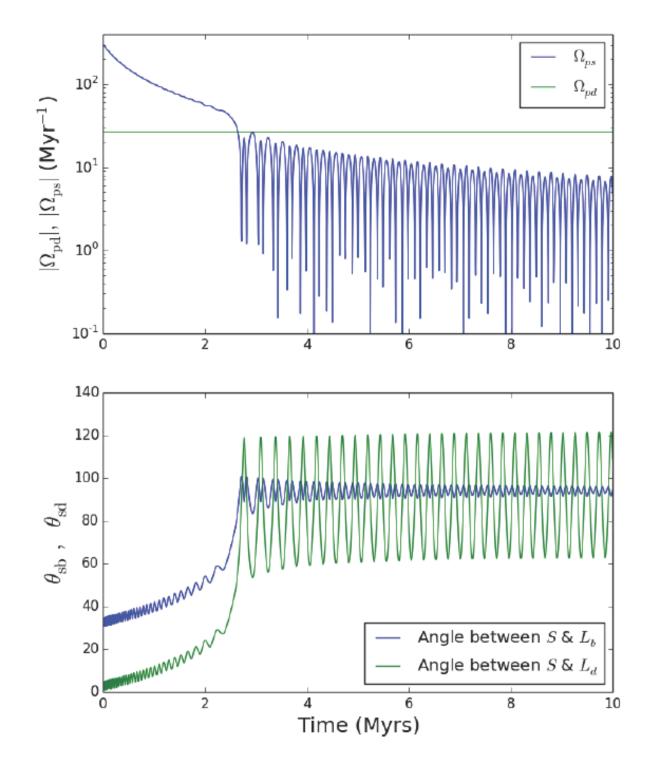
Geometric Interpretation

Lai 2014

$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{\mathrm{ps}}\hat{\mathbf{L}}_d \times \hat{\mathbf{S}}$$

In the frame rotating at rate $\Omega_{\rm pd}\hat{\mathbf{L}}_{\rm b}$





Initial:

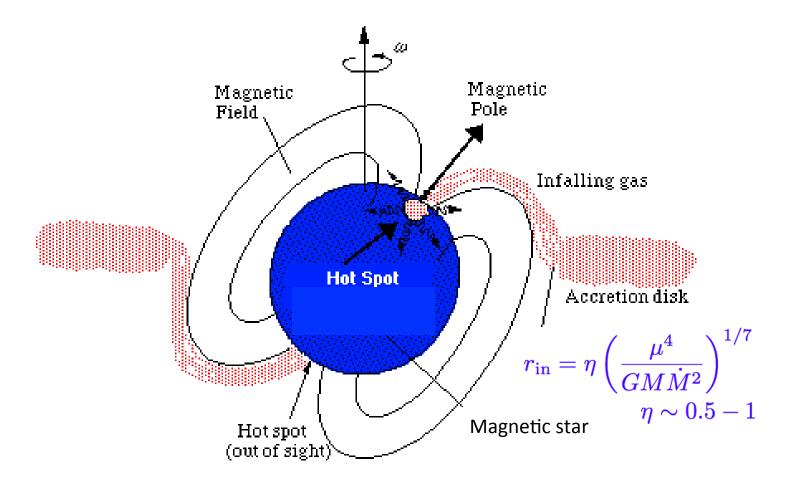
$$\theta_{\rm db} = 30^{\circ}$$

$$\theta_{\rm sd} = 5^{\circ}$$

Complications:

Accretion and magnetic interaction

Magnetic Star - Disk Interaction: Basic Picture



Accretion & Magnetic Torques (Order-of-Mag):

Accretion torque: $\mathcal{N}_{\rm acc} \simeq \dot{M} \sqrt{G M_{\star} r_{\rm in}}$

Magnetic (misalignment) torque: $N_{\rm mag} \sim \mu^2/r_{\rm in}^3$

Accretion & Magnetic Torques (Order-of-Mag):

Accretion torque:
$$\mathcal{N}_{\rm acc} \simeq \dot{M} \sqrt{G M_{\star} r_{\rm in}}$$

Magnetic (misalignment) torque: $N_{
m mag} \sim \mu^2/r_{
m in}^3$

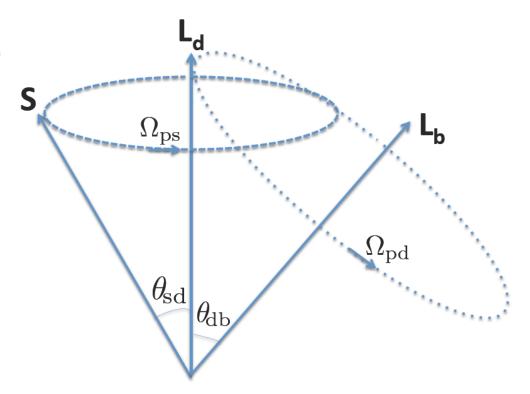
For
$$r_{\rm in} \sim \left(\frac{\mu^4}{GM_{\star}\dot{M}^2}\right)^{1/7}$$

$$lacksquare$$
 $\mathcal{N}_{
m acc} \sim \mathcal{N}_{
m mag}$

Note: Massive (>1.3 Sun) T Tauri stars have weaker dipole fields than less massive stars. Consequence on spin-orbit misalignment?

Star-Disk-Binary Interactions

Gravitational interactions...

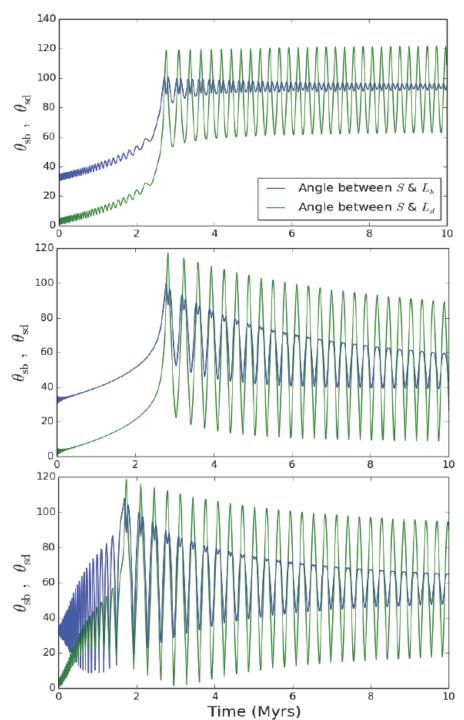


Now include Accretion and Magnetic Torques



Accretion/magnetic damps SL-angle

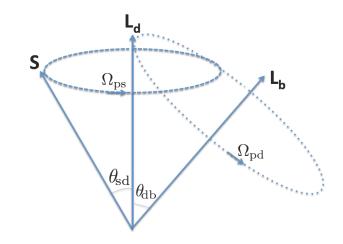
Accretion/magnetic increases SL-angle



DL2014

Summary (#1) Star-disk-binary interactions

- With a binary companion, spin-disk misalignment is "easily" generated
- The key is "resonance crossing"
- Accretion/magnetic torques affect it, but not diminish the effect



→ "primordial" misalignment of stellar spin and disk

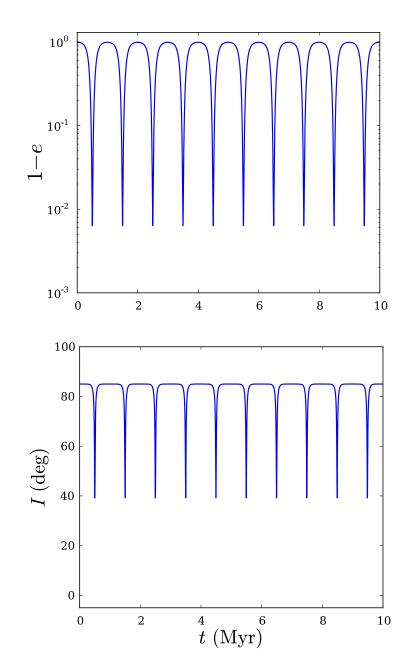
Too robust ??

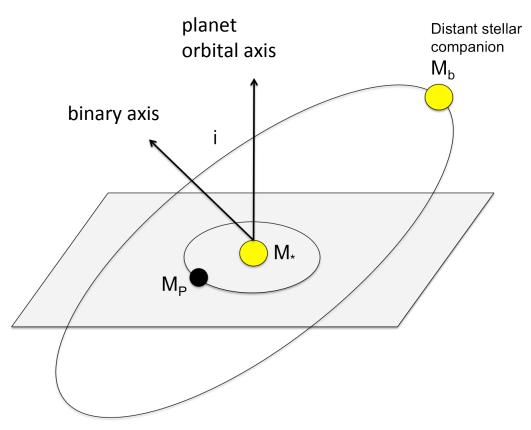
Realistic disk evolution, disk warping, planet-disk coupling (J.Zanazzi's Thesis)

Formation of Hot Jupiters: Lidov-Kozai Migration Driven by a Companion

Holman et al. 97; Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12; Petrovich 15,

Lidov-Kozai Oscillations

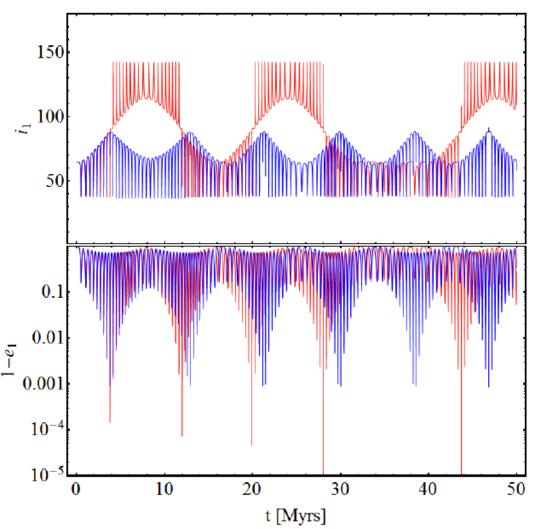




- Eccentricity and inclination oscillations induced if i > 40 degrees.
- If i large (85-90 degrees), get extremely large eccentricities (e > 0.99)

Lidov-Kozai Oscillations: Octupole Effect

e.g. Ford et al 2000; Naoz et al 11; Katz et al 11; Lithwick & Naoz 11; Li et al 14; Antognini 15



Octupole

- -- Very large *e* even for modest *i*
- -- L_p can flip wrt outer binary

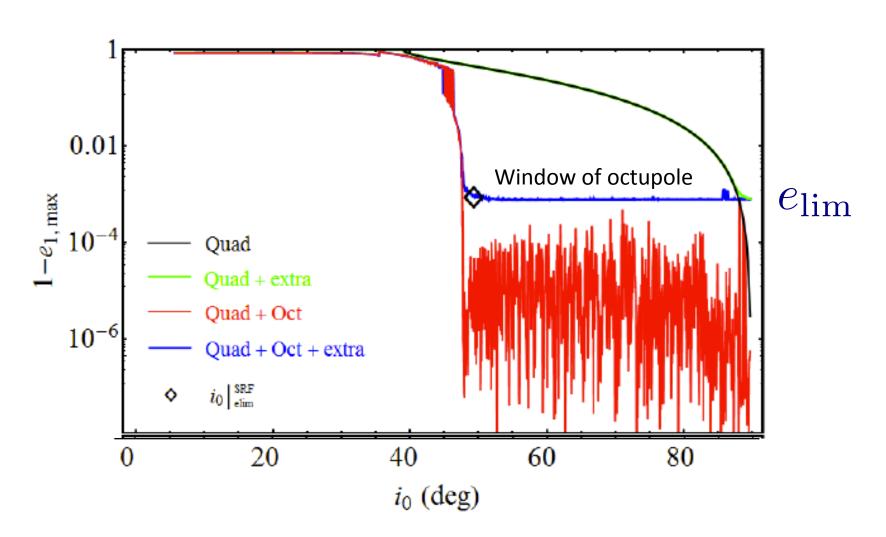
Including Short-Range Forces →

- -- e_{max} is reduced/limited
- -- flip is delayed/suppressed

B.Liu, D.Munoz & DL 2015

Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces

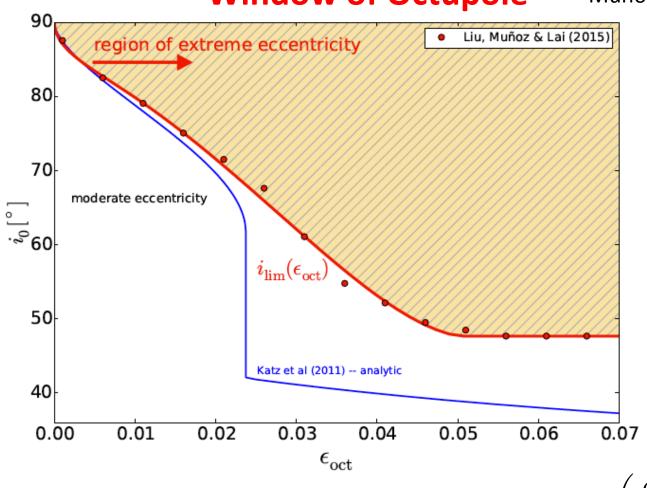
Bin Liu, D. Munoz, DL 2015



Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces

Window of Octupole

Munoz, DL & Liu 16

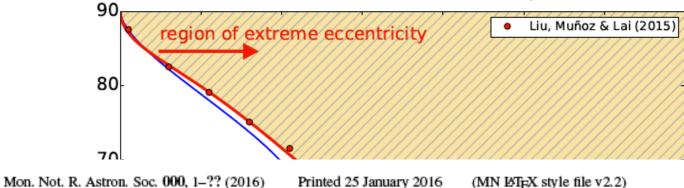


$$\epsilon_{\rm oct} = \left(\frac{a}{a_b}\right) \frac{e_b}{1 - e_b^2}$$

Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces



Munoz, DL & Liu 16



The formation efficiency of close-in planets via Lidov-Kozai migration: analytic calculations

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Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA

² Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

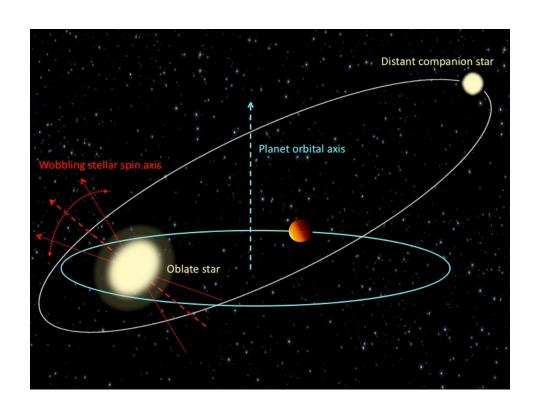
Hot Jupiter formation

Holman et al. 97; Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12, Katz et al.12; Petrovich 15

- Planet forms at ~ a few AU
- Companion star periodically pumps planet into high-e orbit (Lidov-Kozai)
- Tidal dissipation in planet during high-e phases causes orbital decay
- → Combined effects can result in planets in ~ few days orbit

Q: What is happening to the stellar spin axis?

Chaotic Dynamics of Stellar Spin Driven by Planets Undergoing Lidov-Kozai Oscillations

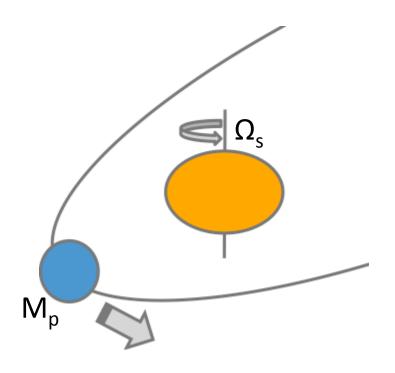


Natalia Storch
(Ph.D.15→ Caltech)

Kassandra Anderson

Storch, Anderson & DL 2014, Science Storch & DL 2015 MNRAS (Theory I) Anderson, Storch, DL 2016 (Pop Study) Storch, DL & Anderson2016 (Theory II)

Precession of Stellar Spin



Star is spinning (3-30 days)
 → oblate → will precess

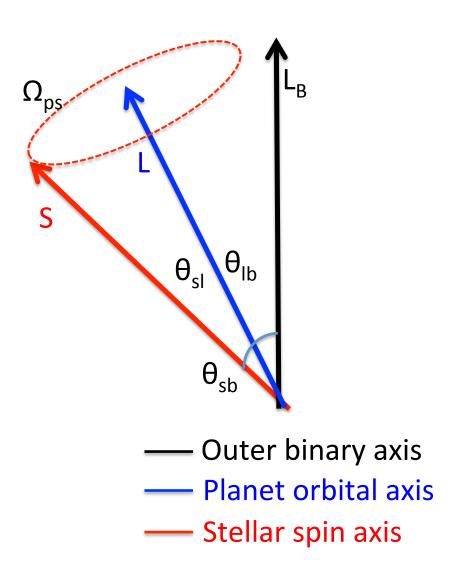
$$\frac{d\hat{\mathbf{S}}}{dt} = \Omega_{\rm ps}\hat{\mathbf{L}} \times \hat{\mathbf{S}}$$

$$\Omega_{\rm ps} = -\frac{3GM_p(I_3 - I_1)}{2a^3(1 - e^2)^{3/2}} \frac{\cos \theta_{\rm sl}}{S}$$

$$\propto \frac{\Omega_{\rm s}M_{\rm p}}{a^3(1 - e^2)^{3/2}}$$

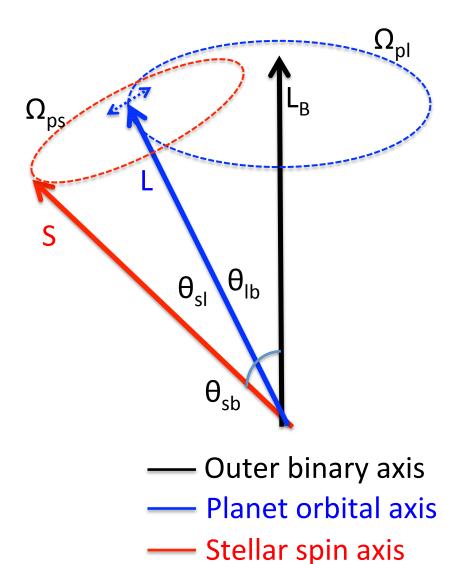
Spin Dynamics

 Stellar spin axis S wants to precess around planet orbital axis L.



Spin Dynamics

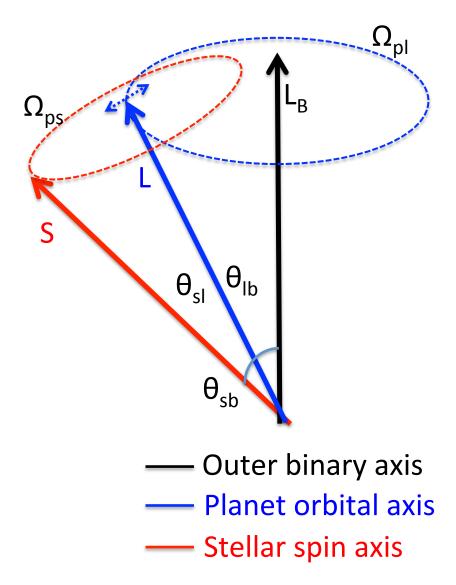
- Stellar spin axis S wants to precess around planet orbital axis L.
- But L itself is moving:
 - Nodal precession (L precesses around binary axis L_b)
 - Nutation (cyclic changes in inclination of L relative to L_b)



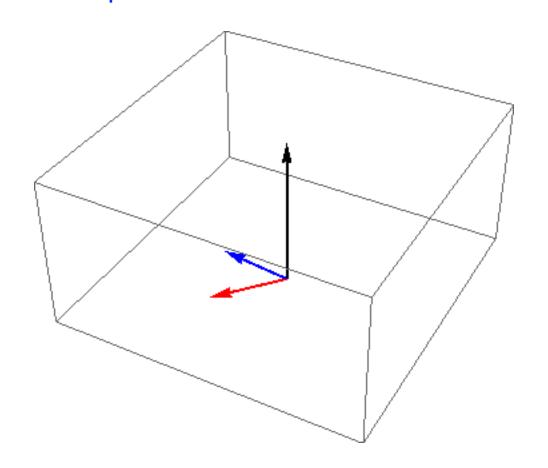
Spin Dynamics

- Q: Can S keep up with L?
- Answer depends on

$$\Omega_{\rm ps}$$
 vs $\Omega_{\rm pl}$



If $|\Omega_{ps}| >> |\Omega_{pl}|$: YES ("adiabatic")



 θ_{sl} = constant, i.e. initial spin-orbit misalignment is maintained for all time

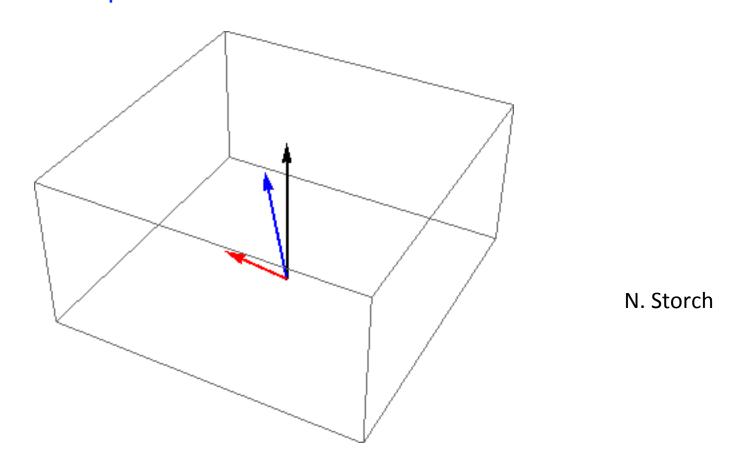
— Outer binary axis

— Planet orbital axis

N. Storch

— Stellar spin axis

If $|\Omega_{ps}| << |\Omega_{pl}|$: NO ("non-adiabatic")



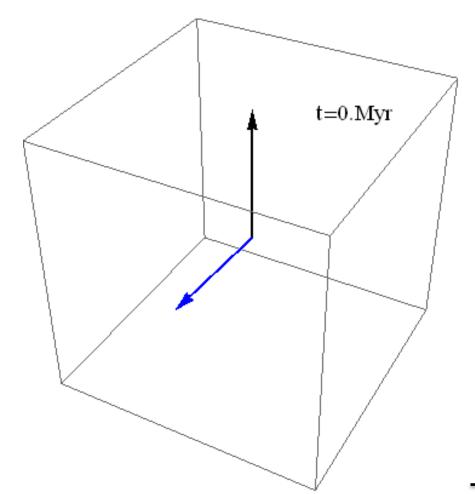
- Outer binary axis
- Planet orbital axis
- Stellar spin axis

If $|\Omega_{ps}|^{\sim} |\Omega_{pl}|$: "trans-adiabatic"



To answer, need to solve orbital evolution equations together with spin precession equation....

If $|\Omega_{ps}|^{\sim} |\Omega_{pl}|$: "trans-adiabatic"



N. Storch

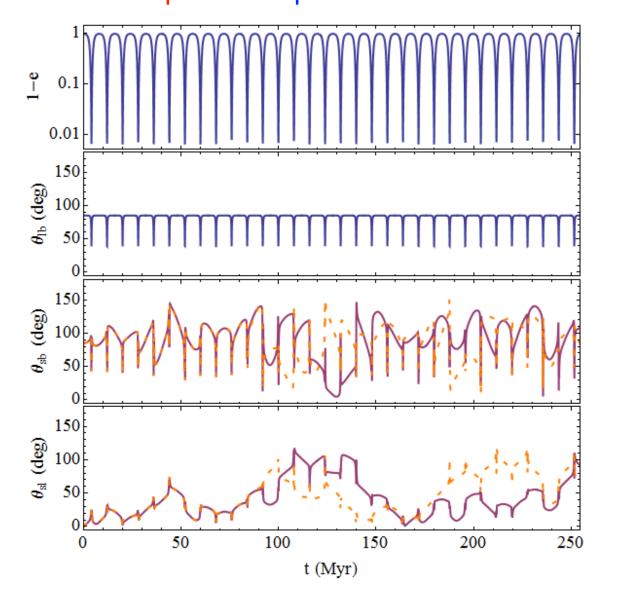
Q: Is it really chaotic?

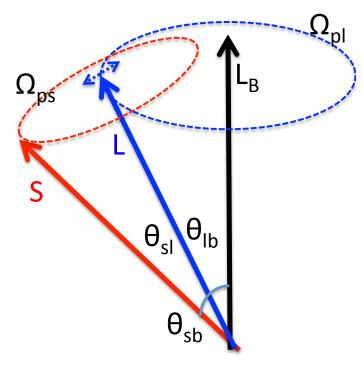
Outer binary axis

— Planet orbital axis

— Stellar spin axis

If $|\Omega_{ps}|^{\sim} |\Omega_{pl}|$: "trans-adiabatic"





Lyapunov time ~ 6 Myrs

Storch, Anderson & DL 14

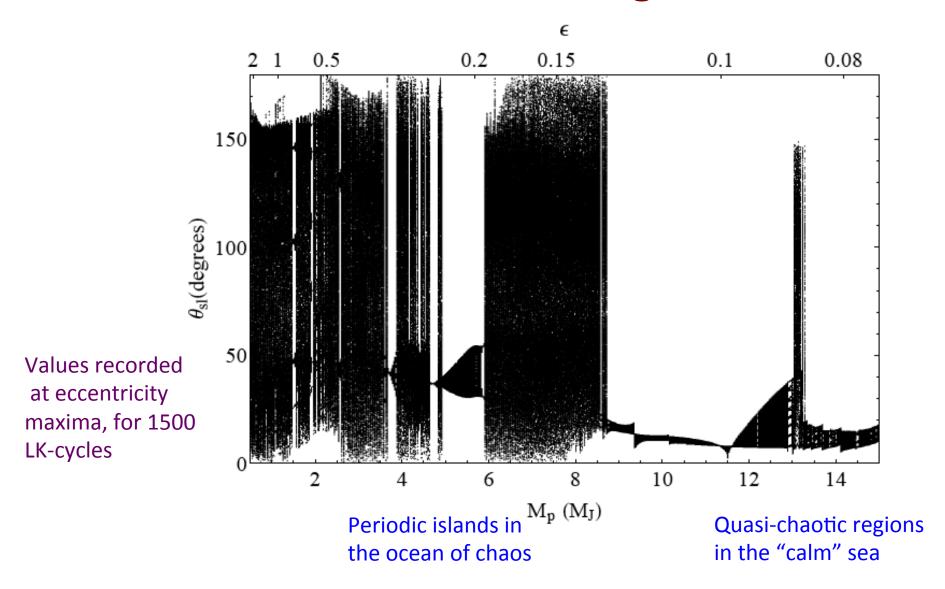
Complication & Richness

 Ω_{ps} & Ω_{pl} are strong functions of eccentricity (and time)

Key parameter:

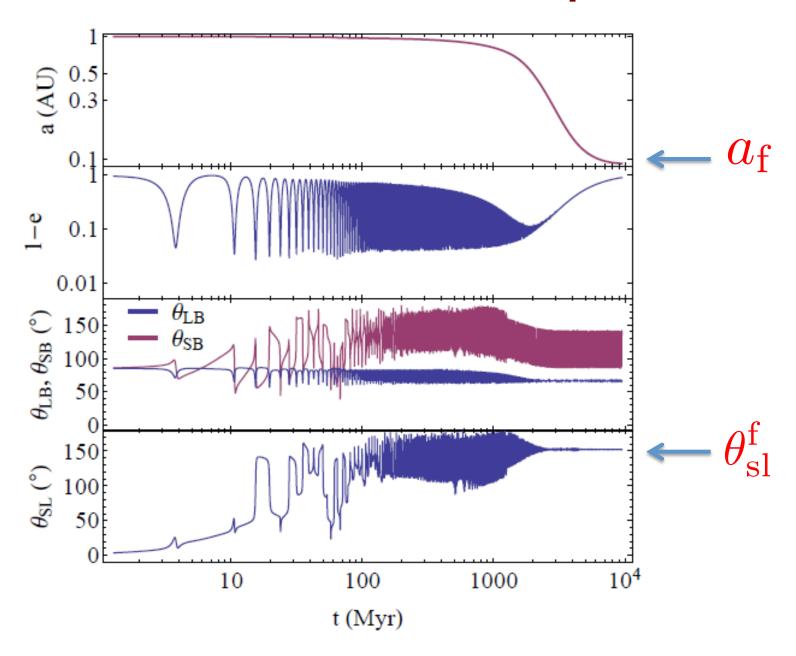
$$\epsilon=rac{\Omega_{
m pl0}}{\Omega_{
m ps0}}\propto rac{a^{9/2}}{M_p\Omega_s}$$
 The ratio of orbital precession frequency to spin precession frequency at zero eccentricity

Bifurcation Diagram



What about tidal dissipation?

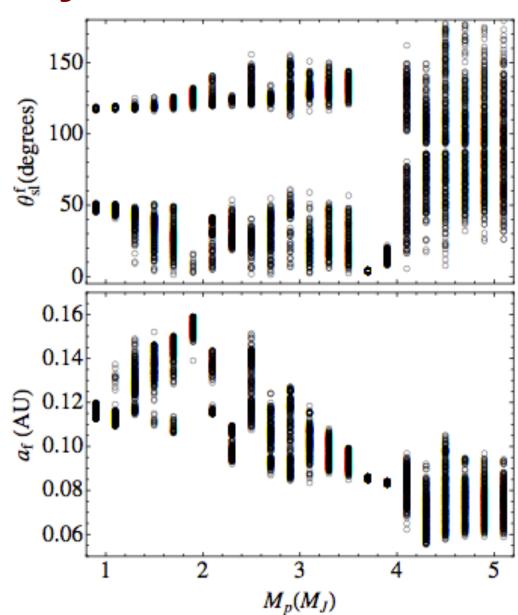
Lidov-Kozai + Tidal Dissipation



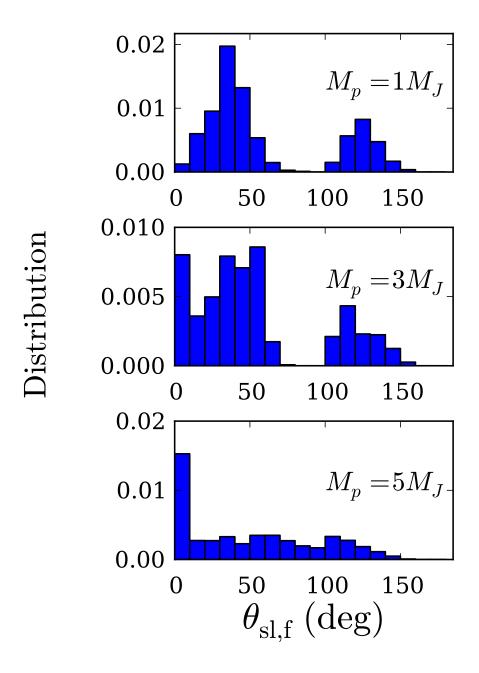
Memory of Chaos

A tiny spread in initial conditions can lead to a large spread in the final spin-orbit misalignment

Initial orbital inclination θ_{lb} =85± 0.05°, with spindown



Distributions of the final spin-orbit angle



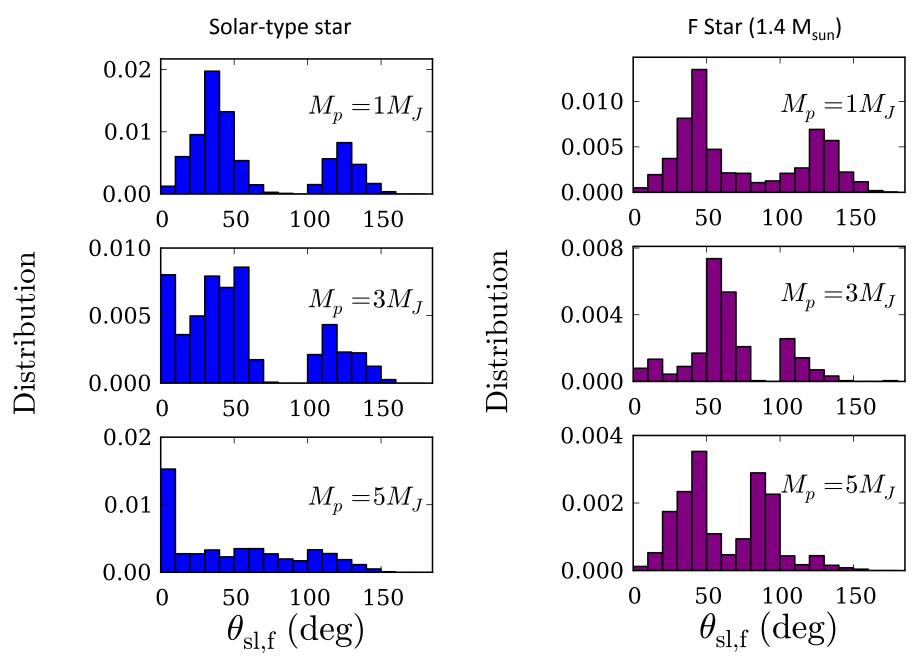
Parameters:

$$a_b = 200 \text{ AU}$$

 $M_* = M_b = 1 M_{sun}$

Stellar spin-down calibrated such that spin period = 27 days at 5 Gyr (Skumanich law)

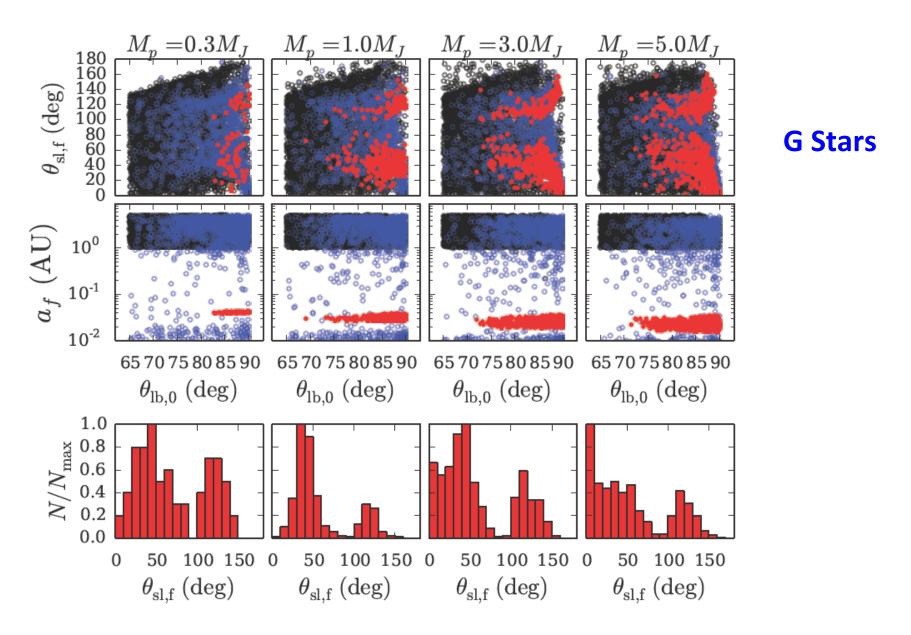
Uniform distribution of initial semi-major axes (a = 1.5 - 3.5 AU)



Anderson, Storch & DL 16

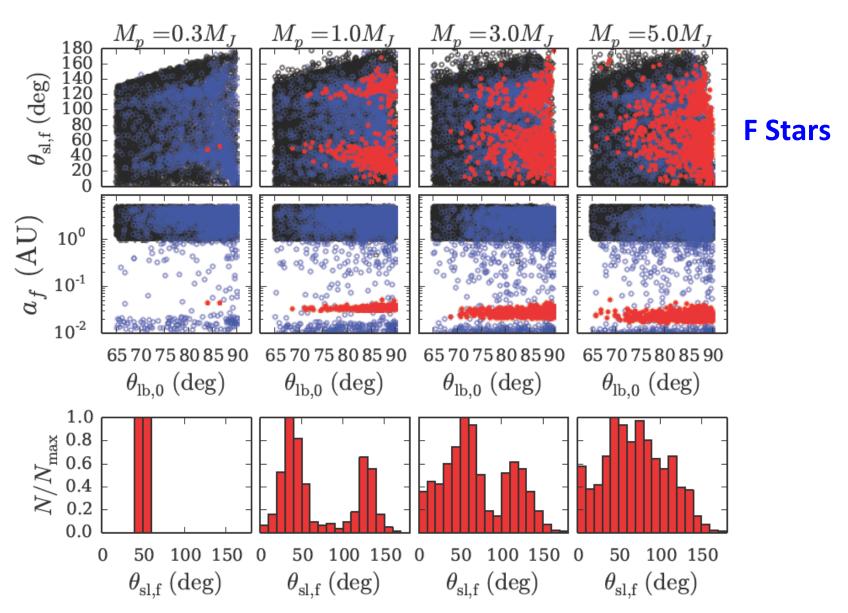
Formation of Hot Jupiters in Stellar Binaries

Anderson, Storch & DL 2016

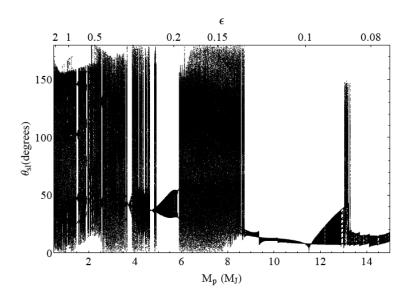


Formation in of Hot Jupiters in Stellar Binaries

Anderson, Storch & DL 2016



Theory of Spin Chaos/Evolution

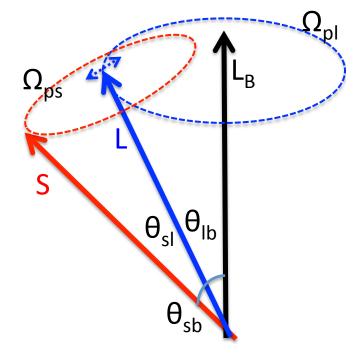


Storch & DL 15 Storch, DL & Anderson 16

In Hamiltonian system, Chaos arises from **overlapping resonances** (Chirikov criterion; 1979)

 Ω_{ps} & Ω_{pl} are strong functions of e and t (with period P_{e})

What resonances??



Hamiltonian Perturbation Theory

$$H = -\frac{1}{2}\alpha(t)\cos^2\theta_{\rm sl} - \Omega_{\rm pl}(t)\left[\cos\theta_{\rm lb}(t)\cos\theta_{\rm sl} - \sin\theta_{\rm lb}(t)\sin\theta_{\rm sl}\cos\phi\right] - \dot{\theta}_{\rm lb}(t)\sin\theta_{\rm sl}\sin\phi,$$
 Spin precession Orbit's nodal precession Orbit's nutation

$$H' = \frac{\bar{\alpha}}{n_e} \left(-\frac{1}{2} p^2 + \psi(\tau) p - \sqrt{1 - p^2} \left[\beta(\tau) \cos \phi + \gamma(\tau) \sin \phi \right] \right)$$

$$\beta(\tau) = -\frac{\Omega_{\rm pl}(\tau)}{\alpha(\tau)} \sin \theta_{\rm lb}(\tau),$$

$$\gamma(\tau) = \frac{\dot{\theta}_{\rm lb}(\tau)}{\alpha(\tau)},$$

$$\psi(\tau) = -\frac{\Omega_{\rm pl}(\tau)}{\alpha(\tau)} \cos \theta_{\rm lb}(\tau).$$

$$H' = \frac{\bar{\alpha}}{n_e} \left(-\frac{1}{2} p^2 + \epsilon \, \psi_0 \, p + \epsilon \, p \sum_{M=1}^{\infty} \psi_M \cos M t \right)$$
$$-\frac{\epsilon}{2} \sqrt{1 - p^2} \sum_{M=0}^{\infty} \left[(\beta_M + \gamma_M) \cos(\phi - M\tau) + (\beta_M - \gamma_M) \cos(\phi + M\tau) \right]$$

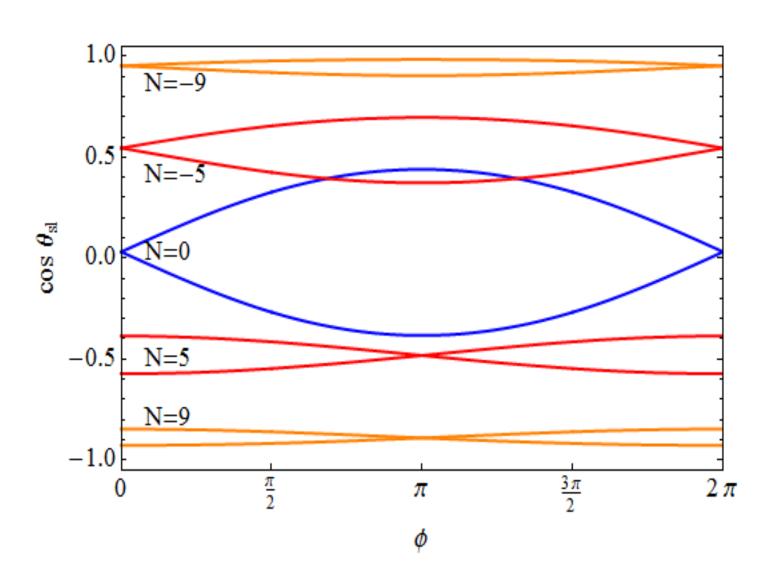
Spin-Orbit Resonances

$$\bar{\Omega}_{\rm ps} = -\bar{\alpha}\cos\theta_{\rm sl} = N\,\frac{2\pi}{P_e}$$

Average spin precession frequency

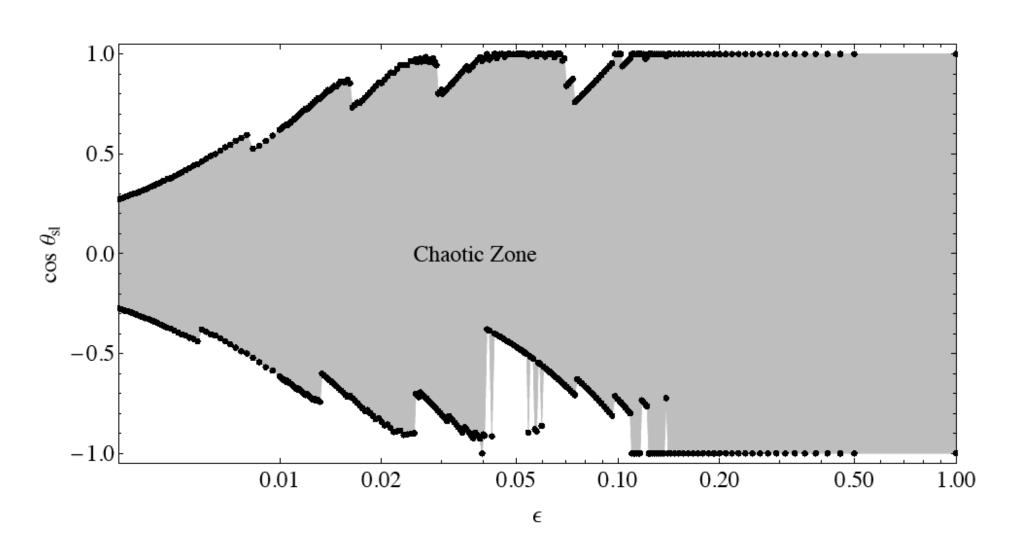
Integer multipleof mean Lidov-Kozai frequency

Individual Resonance



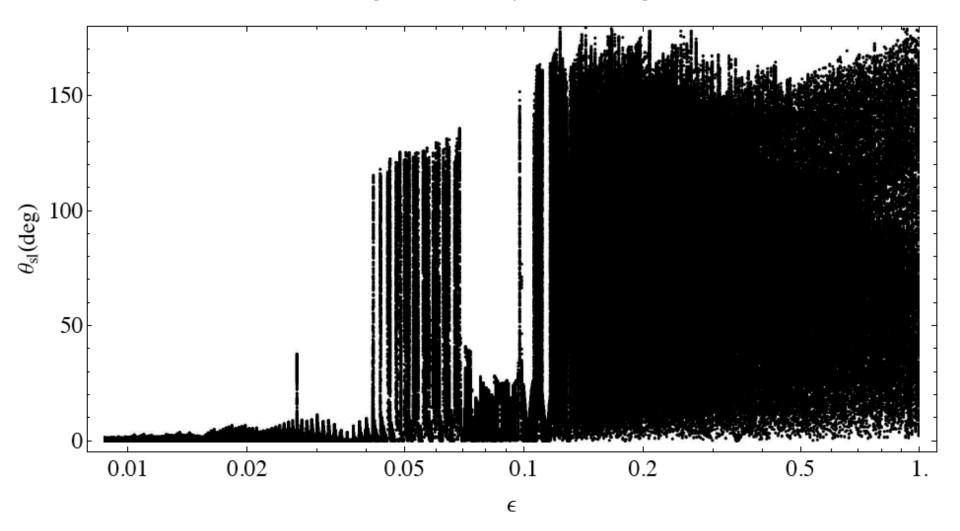
Boundary of Chaotic Zone

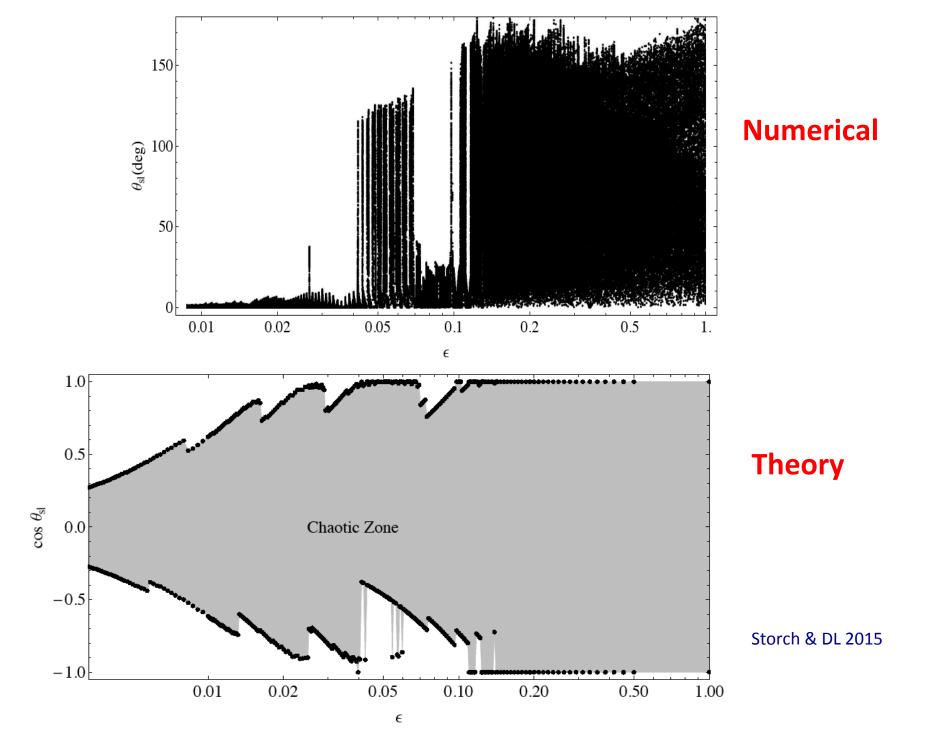
Analytical theory: From outmost overlapping resonances



Numerical Bifurcation Diagram

Starting from zero spin-orbit angle

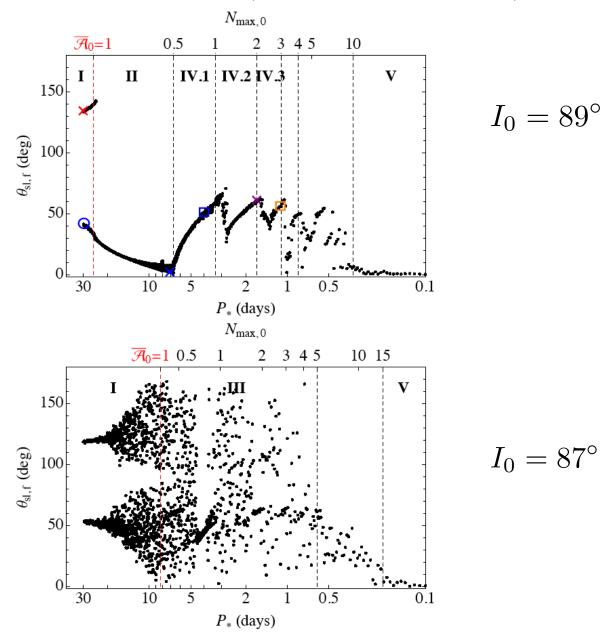




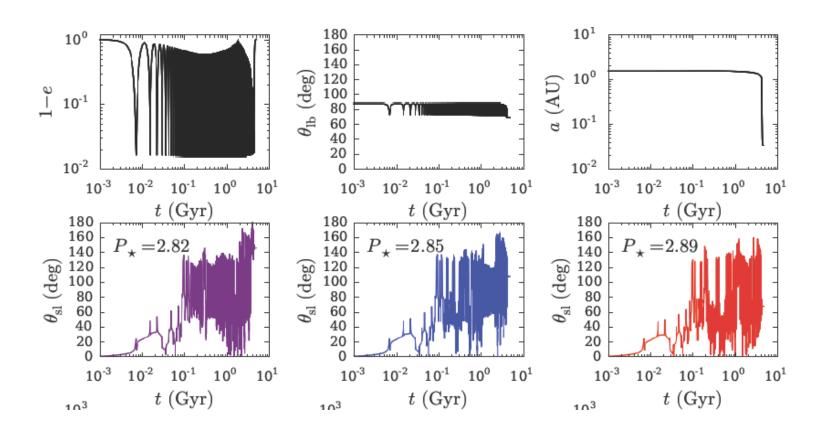
Tidal Dissipation: Paths to Final Misalignments

Storch, DL, Anderson 2016

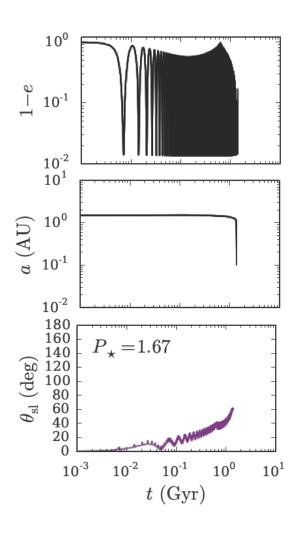
Experiments with given $a_0 = 1.5$ au, $a_b = 300$ au, etc varying stellar rotation period (S-L coupling strength)

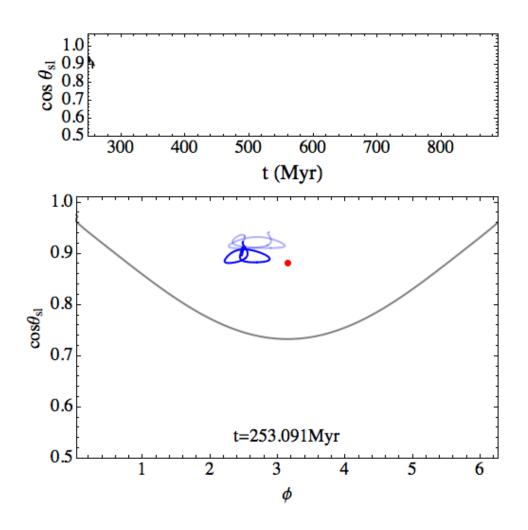


Tidal Dissipation: Paths to Final Misalignments CHAOTIC



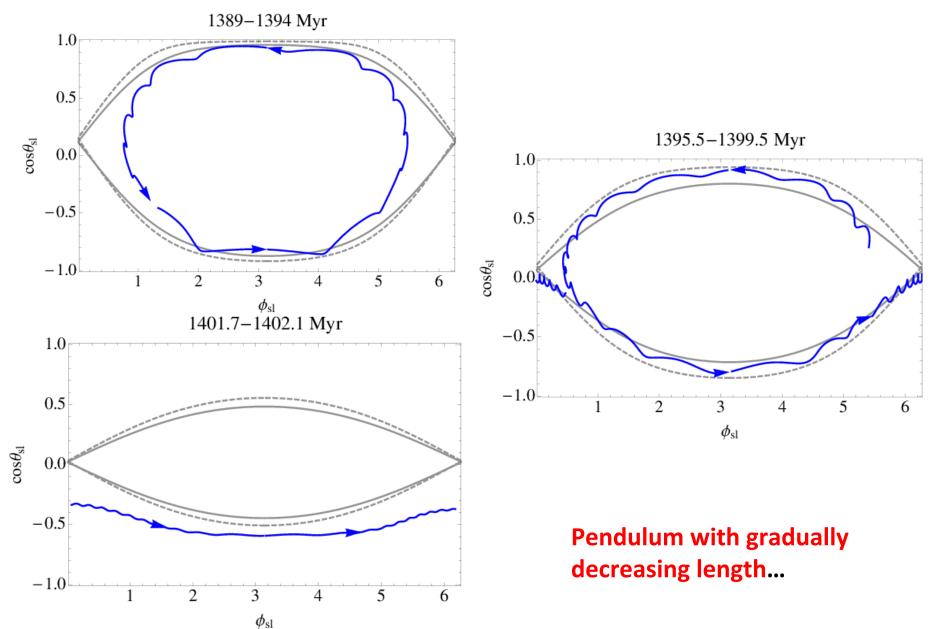
Adiabatic Resonance Advection





N. Storch

Bifurcation Bimodal Distribution Misalignment



Summary (# 2) Chaotic Stellar Spin and Formation of Hot Jupiters

- Dynamics of stellar spin is important for the formation of hot Jupiters, e.g. affects the observed spin-orbit misalignments (dependence on planet mass, stellar rotation/history etc)
- Spin dynamics can be chaotic
- Spin dynamics/evolution can be understood from resonance theory
- Migration fractions can be calculated analytically

Missing Kepler Multi's?

Multi-Planet System with an External Companion

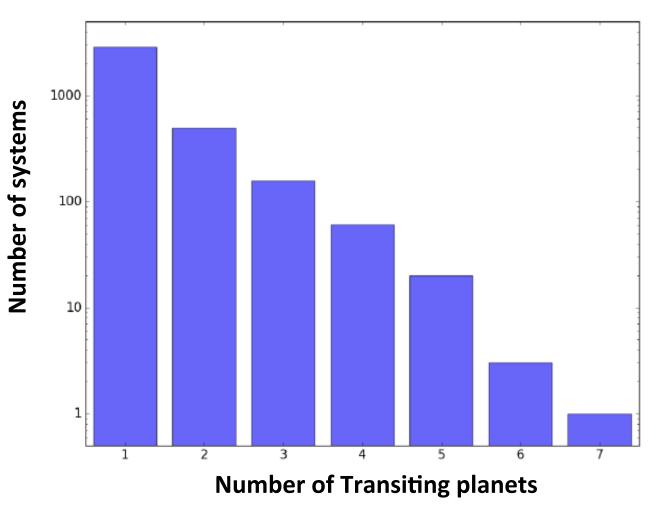
DL & Bonan Pu

arXiv:1606.08855

Kepler: 4700 planets in 3600 systems

(mostly super-earth or sub-neptunes, <200 days)

Observed Transit Multiplicity Distribution F (N_{tran})



F (N_{tran}) ===> the underlying multiplicity distribution (& mutual inclinations)

Degeneracy can be broken by RV data (Tremaine & Dong 12)

Transit duration ratio (Fabrycky+14)



Kepler compact systems are flat, with mutual inclination dispersion < 2 degrees

Lissauer+11, Tremaine & Dong 12, Figueira+12, Fabrycky+14; Fang & Margot 12

Excess of Kepler Singles (?)

Models with single mutual inclination dispersion (in Rayleigh) fall short to explain the observed number of Kepler singles (Lissauer+11; Johansen+12; Ballard & Johnson 16) (depends somewhat on the assumed multiplicity function)

Some Kepler singles might be multi-planet systems with higher mutual inclinations...

Other evidence that (some) Kepler singles are "special":

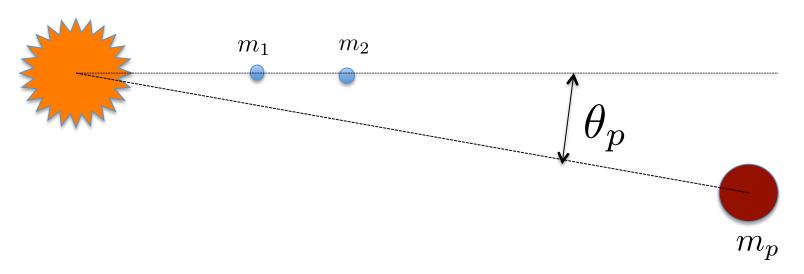
- (1) "singles" have higher stellar obliquities (Morton & Winn 14)
- (2) Multi's have more detectable TTVs than "singles" (Xie, Wu & Lithwick 14)
- (3) 10-30% of singles have higher e's (Xie, S.Dong et al 2016)
- (4) Single "Hot Earth" excess (Jason Staffen)

Effect of External Perturber On Compact Multi-Planet System

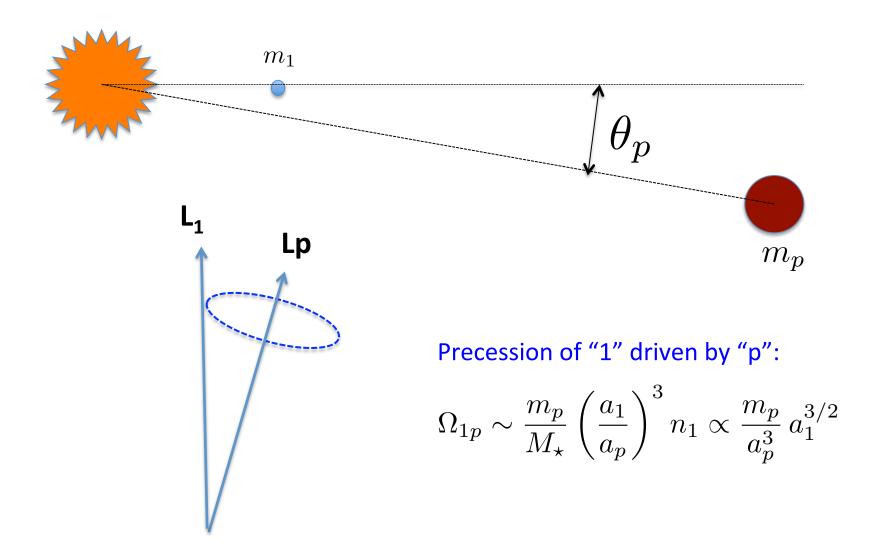
Perturber: Giant planet (~AUs) or Companion Star (~10's AUs)

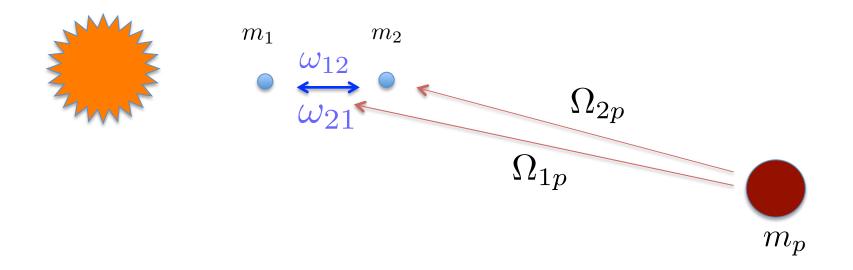
Two-planet system with an external inclined perturber

m₁, m₂ initially co-planar...



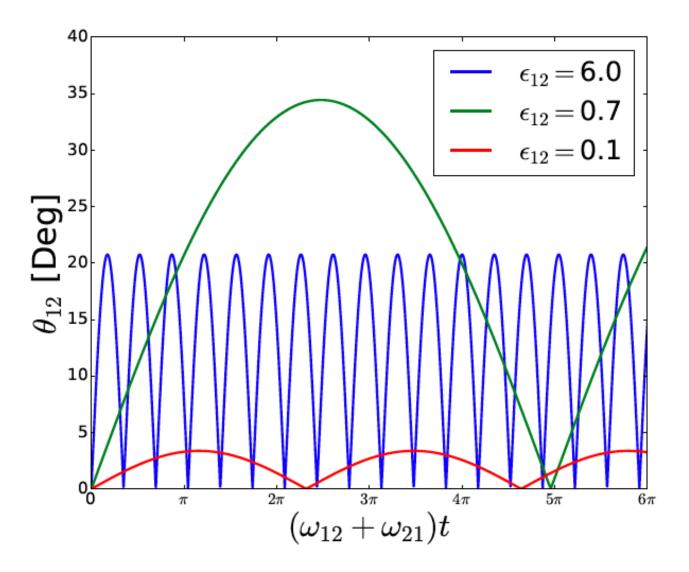
companion star or "cold Jupiter" (e.g. produced by planet-planet scatterings)



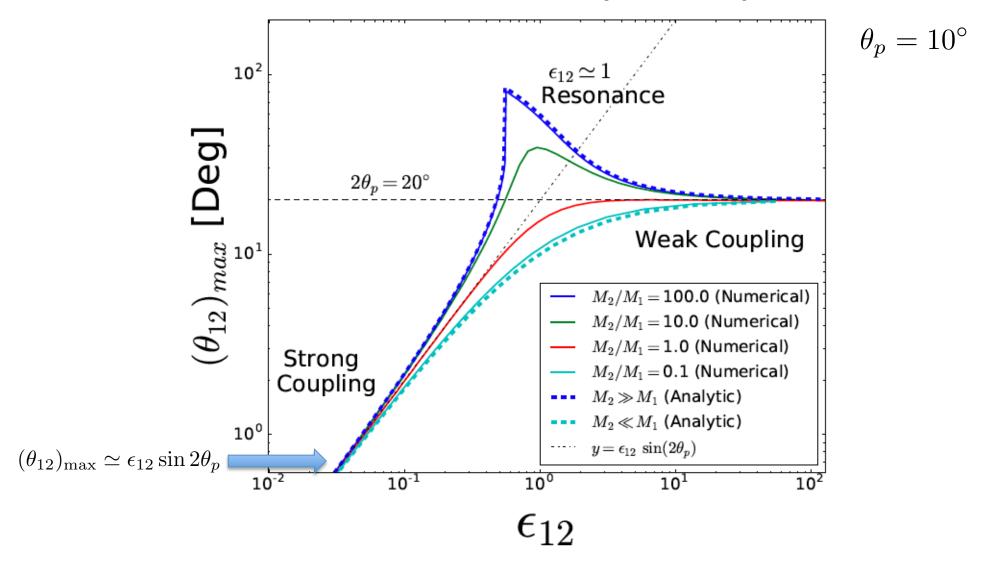


Mutual inclination induced by perturber depends on Coupling Parameter

$$\epsilon_{12} \equiv \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}}$$



Maximum Mutual Inclination Induced by external perturber

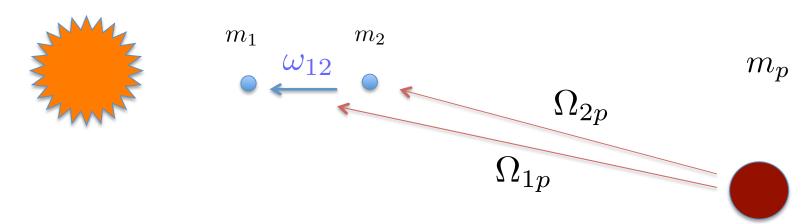


$$\epsilon_{12} \equiv \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \propto \frac{m_p}{a_p^3}$$

Resonance Feature: $\epsilon_{12} \sim 1$

exists when $m_2 \gtrsim m_1$

Nodal Precession Resonance



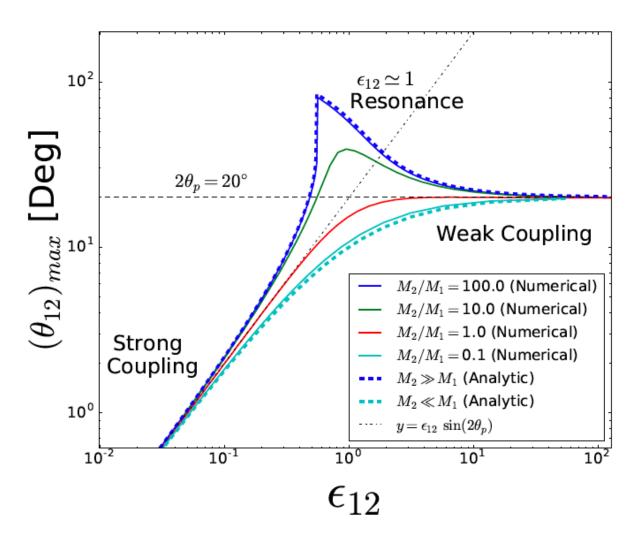
In the $m_2 \gg m_1$ limit:

Resonance occurs at $\Omega_{2p} = \Omega_{1p} + \omega_{12}$ or $\epsilon_{12} = 1$

Resonance Feature: $\epsilon_{12} \sim 1$



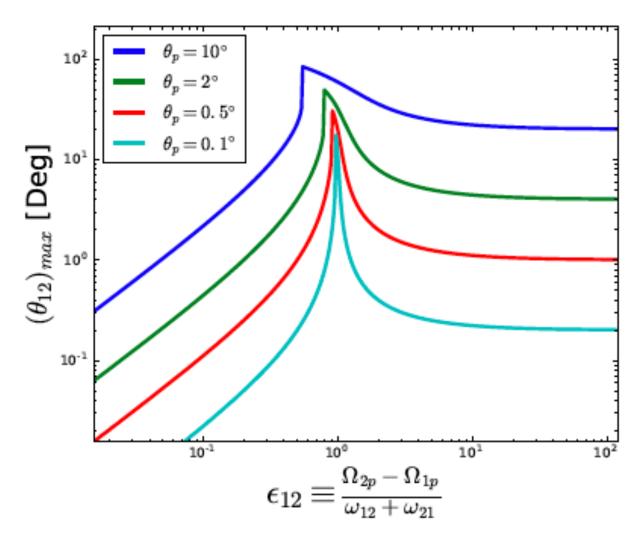
Can produce much larger mutual inclination than θ_p



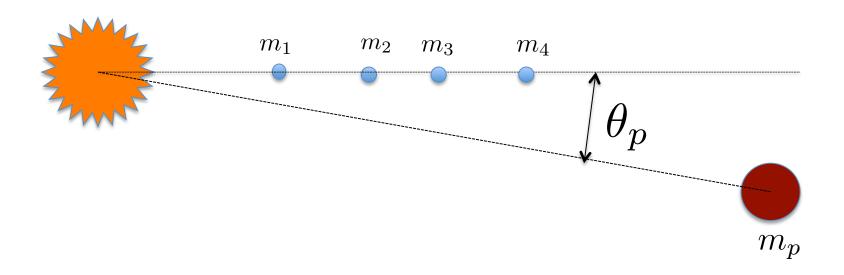
Resonance Feature: $\epsilon_{12} \sim 1$



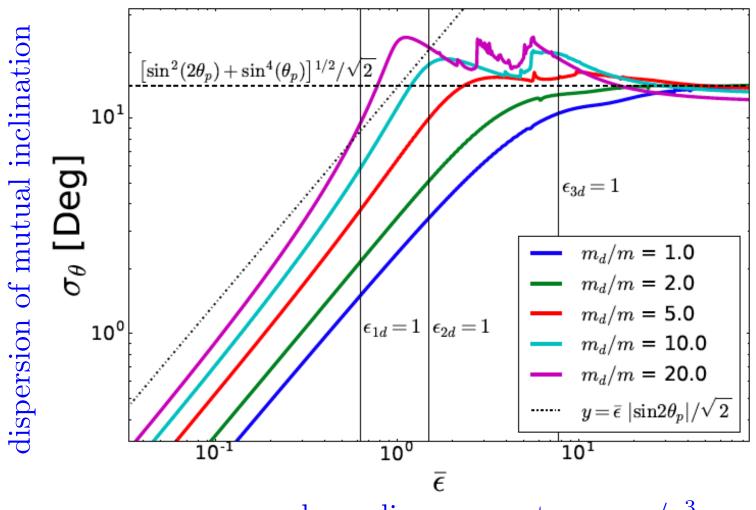
Can produce much larger mutual inclination than θ_p



Multi-planet system with an external inclined perturber

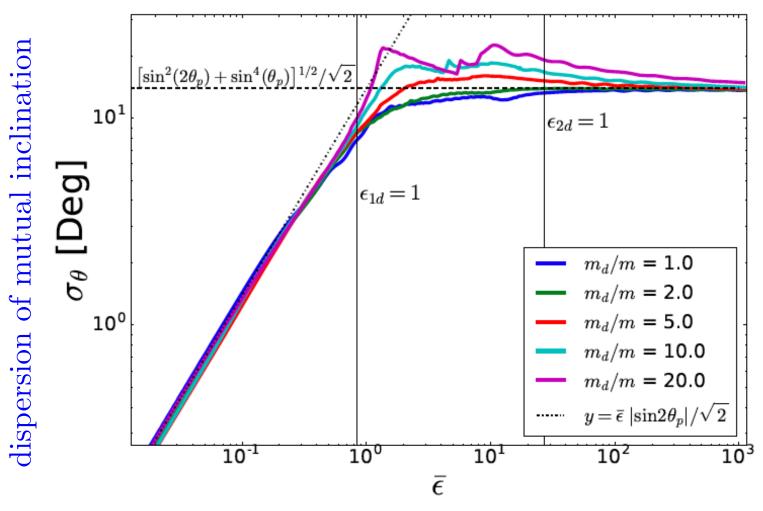


4 planet system with an external perturber



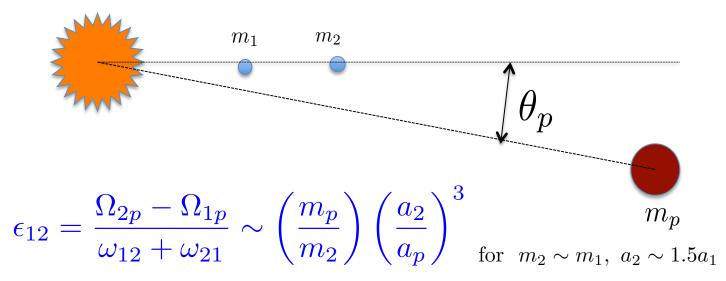
averaged coupling parameter $\propto m_p/a_p^3$

4 planet system with an external perturber



averaged coupling parameter $\propto m_p/a_p^3$

Recap: An understanding and **semi-analytic expression** for the mutual inclination of multi-planets induced by distant perturber



e.g., for $m_2 \sim m_{\oplus}$ at $a_2 \sim 0.3$ au $1M_J$ perturber at 3 au gives $\epsilon_{12} \sim 0.3$

For $\epsilon_{12} \gtrsim 2$: $\theta_{12} \sim 2\theta_p$

For $\epsilon_{12} \lesssim 0.3$: $\theta_{12} \sim \epsilon_{12} \sin 2\theta_p$

Application:

"Evaluate"/constrain external perturber of observed systems

Examples: (see DL & Pu for more)

Kepler-68: Two super-Earths at 0.06,0.09 au, 1 M₁ planet at 1.4 au is weak

Kepler-56: ...

WASP-47: ...

Kepler-48: Three transiting planets (0.012,0.046,0.015M₁) at 0.053,0.085,0.23 au,

>2M₁ companion (Kepler-48e) at 1.85 au (Marcy+14)

→ Kepler-48e must be aligned <2 degree

Kepler-454: one super-Earth at 0.1 au with a >5M₁ at 524 days (Gettel+16)

→ any neighbor to the inner planet could be strongly misaligned by the giant (This could of multi-planet system that has been turned into Kepler single

by an external giant planet)

Confront with Observations?

Some/significant (?) fraction of Kepler singles (N_{tran}=1) are/were multi-planet systems that have been misaligned (in mutual inclinations) or disrupted by external perturbers (cold giant planets or stellar companions)

Cold Giant Planets:

Some have been found, but general census not clear?

(50% of HJs/WJs have 1-20M₁ companion at 5-20 au; Bryan et al 2016)

Stellar Companions:

J.Wang et al (2015): ~5% of Kepler multis have stellar companion 1-100 au (4x lower than field stars)

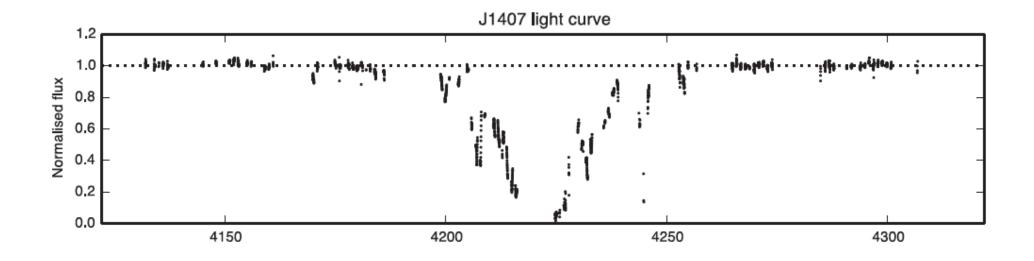
More studies are needed.

Extended & Inclinded Disks/Rings around Planets/Brown Dwarfs

J.J. Zanazzi & DL 2016

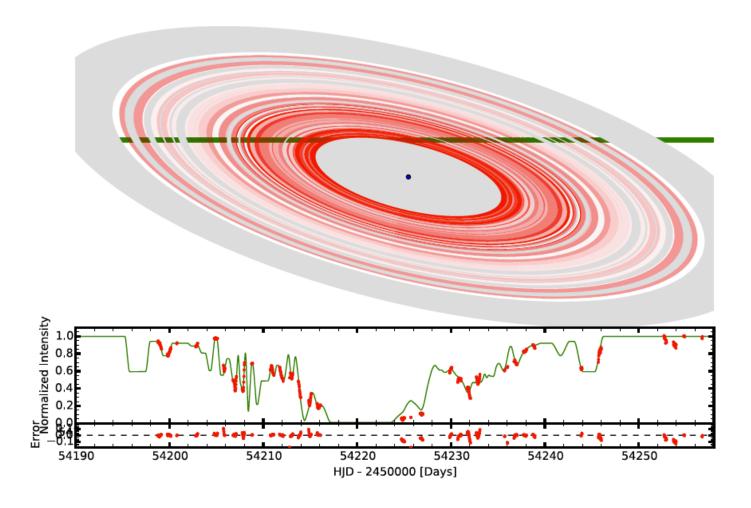
Lightcurve of 1SWASP J1407 (~16 Myr, 0.9Msun)

A series "eclipses" lasting 56 days around April 2007: big eclipse (3.5 mag) with symmetric pairs of smaller (~mag) eclipses



→ Large ring/disk system around unseen substellar companion (?)

Mamajek et al. 2012 van Werkhoven et al. 2014 Kenworthy et al.2015



Kenworthy & Mamajek 2015

Claimed best fit model:

Ring size ~0.6 AU, with gaps (moons?), total mass ~ 1 M_earth Orbital period 10-30 years (eccentric)

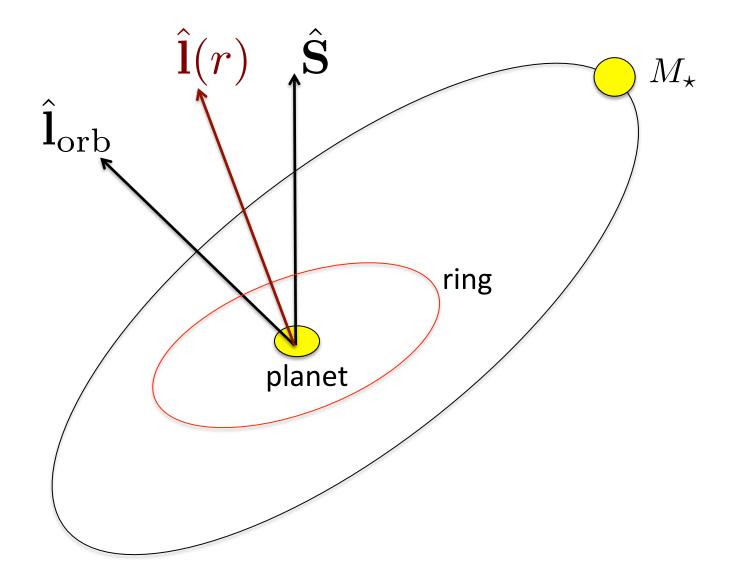
Question:

Under what conditions can an extended, inclined ring/disk exist around a planet/BD?

• Outer radius of ring $r_{
m out} \lesssim 0.3 r_H$ $r_H = a \left(\frac{M_p}{3 M_{\star}} \right)^{1/3}$

$$r_H = a \left(\frac{M_p}{3M_{\star}}\right)^{1/3}$$

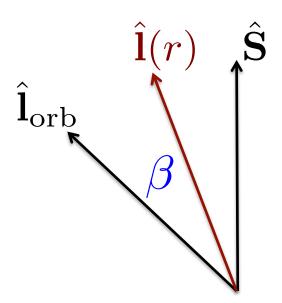
How to produce/maintain inclination?



Ring at radius r experiences two torques

From star :
$$T_\star \sim \frac{G M_\star r^2}{a^3}$$

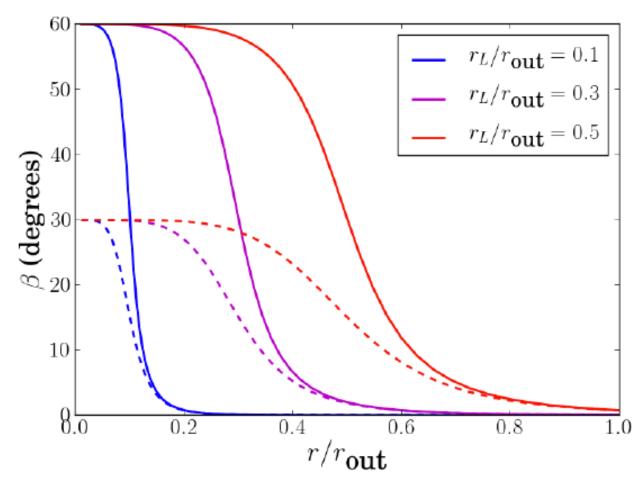
From oblate planet: $T_p \sim \frac{G(J_2 M_p R_p^2)}{r^3}$

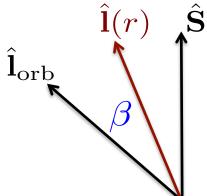


 $T_{\star} = T_p$ at Laplace radius:

$$r_L \sim \left(J_2 \frac{M_p}{M_{\star}} R_p^2 a^3\right)^{1/5} \sim (3J_2 R_p^2 r_H^3)^{1/5}$$

Equilibrium State: Laplace Surface



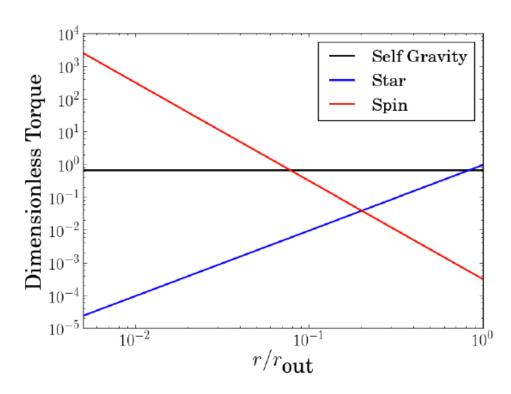


How to make the disk sufficiently "rigid" to have appreciable $\beta(r_{\mathrm{out}})$?

Internal stresses (pressure, viscosity): Not effective for thin rings

• Self-gravity:
$$T_{
m sg} \sim \pi G r \Sigma$$

Compare to
$$T_{\star} \sim \frac{G M_{\star} r^2}{a^3}$$
 $T_p \sim \frac{G (J_2 M_p R_p^2)}{r^3}$



How to make the disk sufficiently "rigid" to have appreciable $\beta(r_{\mathrm{out}})$?

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• Self-gravity:
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m sg}\sim \pi Gr \Sigma$$
 Compare to $T_{\star}\sim rac{GM_{\star}r^2}{a^3}$ $T_p\sim rac{G(J_2M_pR_p^2)}{r^3}$

$$T_{\rm sg}(r_{
m out}) \sim rac{GM_d}{r_{
m out}} \gtrsim T_{\star}(r_{
m out})$$

$$\sigma \equiv rac{M_d}{M_{\star}} \left(rac{a}{r_{
m out}}
ight)^3 = rac{3M_d}{M_p} \left(rac{r_H}{r_{
m out}}
ight)^3 \gtrsim 1$$

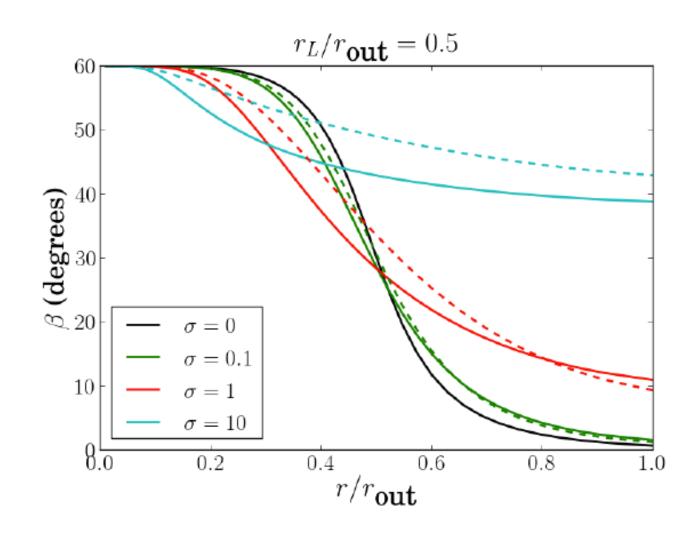
Equilibrium State with Self-Gravity

$$\mathbf{T}_{\star} + \mathbf{T}_{p} + \mathbf{T}_{sg} = 0$$

$$\mathbf{T}_{sg} = \frac{\pi G}{2} \int_{r_{in}}^{r_{out}} \frac{r' \Sigma(r') dr'}{\max(r, r')} \chi b_{3/2}^{(1)}(\chi) \left[\mathbf{l}(r) \cdot \mathbf{l}(r') \right] \left[\mathbf{l}(r) \times \mathbf{l}(r') \right]$$

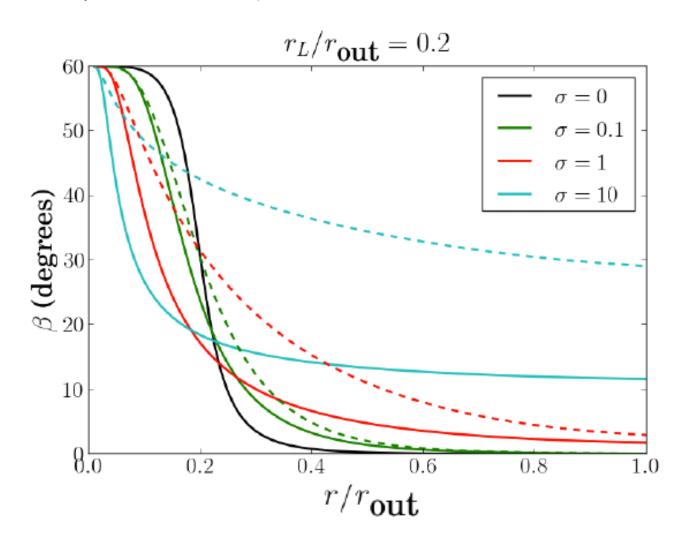
Equilibrium State with Self-Gravity

(modified Laplace Surface)



Equilibrium State with Self-Gravity

(modified Laplace Surface)



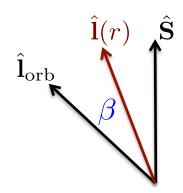
What if disk not in equilibrium?

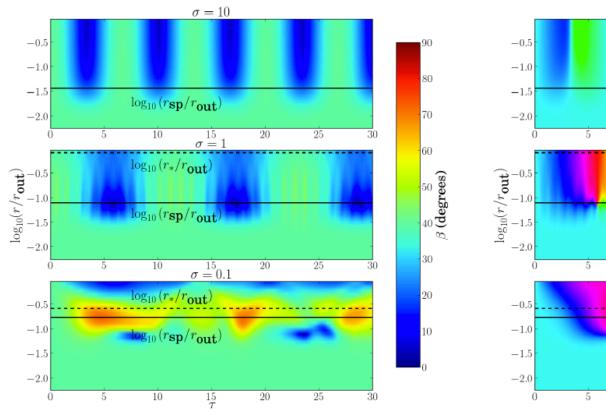
Can disk "precess" coherently?

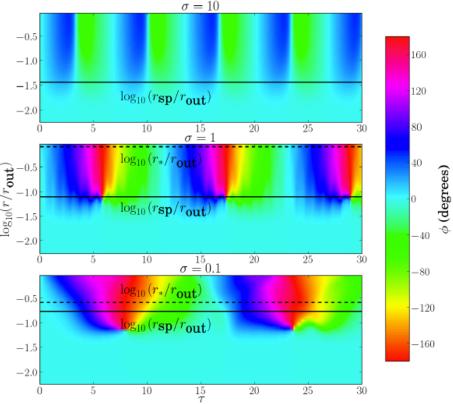
Time evolution of disk

$$r^2 \Omega \frac{\partial \hat{\mathbf{l}}(r,t)}{\partial t} = \mathbf{T}_{\star} + \mathbf{T}_p + \mathbf{T}_{\mathrm{sg}}$$

Initial $\hat{\mathbf{l}}(r,0)$ aligned with $\hat{\mathbf{S}}$ (at $\beta=40^{\circ}$)



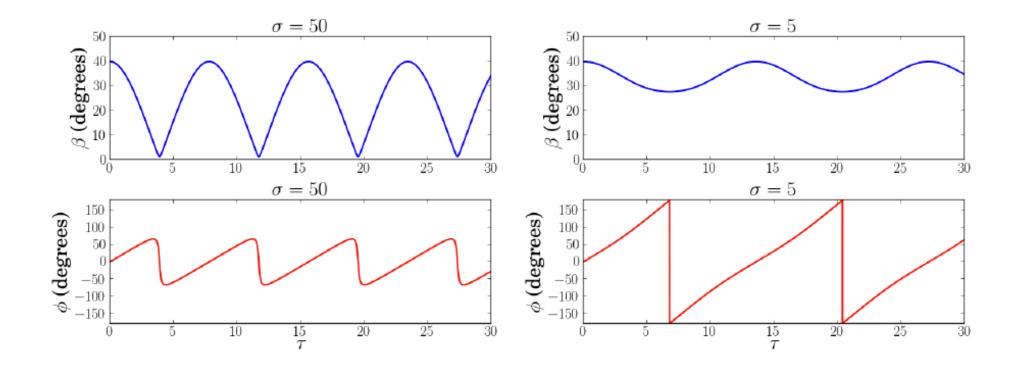




Time evolution of disk

Inner disk $(T_p \gtrsim T_{\rm sg}, T_{\star})$: stays aligned with $\hat{\mathbf{S}}$

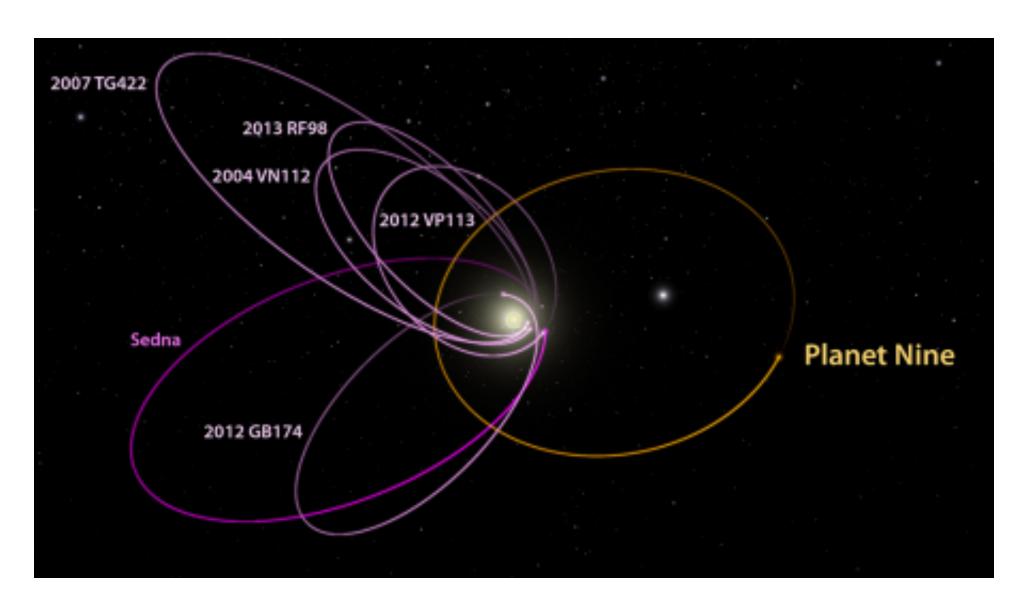
Outer disk $(T_{\rm sg} \gtrsim T_p)$: evolves coherently if $T_{\rm sg}(r) \gtrsim T_{\star}(r) \iff \sigma \gtrsim 1$ nutation in β , precession (smaller σ)/libration (larger σ) in ϕ



Summary (# 4) Extended, Inclined Disk/Ring Around Planets/BDs

- Intriguing "eclipses" of J1407 due to extended disk/ring?
- r_H and r_L
- If $r_{out} < r_L$, disk is inclined wrt orbit (provide S inclined wrt orbit)
- If r_{out} > r_L, require sufficient disk mass for self-gravity to maintain disk coherence and out disk inclination
 - -- generalized Laplace surface (with self-gravity)
 - -- precessing/librating/nutating self-gravitating disk

$$\frac{M_d}{M_p} \gtrsim \frac{1}{3} \left(\frac{r_{
m out}}{r_H}\right)^3$$



~10 Earth mass, a~700 AU, e=0.6 (distance 300-1000 AU, period 20,000 years) inclined by 10-30 degrees

SOLAR OBLIQUITY INDUCED BY PLANET NINE

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Draft version July 21, 2016

The numerical evaluation of the system's evolution can be robustly carried out after transforming the Hamiltonian to nonsingular Poincaré Cartesian coordinates

$$x = \sqrt{2Z}\cos(z)$$

$$y = \sqrt{2Z}\sin(z)$$

$$x_9 = \sqrt{2Z_9}\cos(z_9)$$

$$y_9 = \sqrt{2Z_9}\sin(z_9).$$

Then, the truncated and expanded Hamiltonian (4) becomes

$$\mathcal{H} = -\frac{1}{4} \frac{\mathcal{G} m m_9}{a_9} \left(\frac{a}{a_9}\right)^2 \frac{1}{\varepsilon_9^3} \left[\frac{1}{4} \left(2 - \frac{6}{\Gamma} \left(\frac{x^2 + y^2}{2} \right) \right) \left(3 \left(1 - \frac{1}{\Gamma_9} \left(\frac{x_9^2 + y_9^2}{2} \right) \right)^2 - 1 \right) + 3 \left(1 - \frac{1}{\Gamma_9} \left(\frac{x_9^2 + y_9^2}{2} \right) \right) \sqrt{1 - \frac{1}{2\Gamma_9} \left(\frac{x_9^2 + y_9^2}{2} \right)} \sqrt{\frac{1}{\Gamma\Gamma_9}} \left(x x_9 + y y_9 \right) \right].$$

Explicitly, Hamilton's equations $dx/dt = -\partial \mathcal{H}/\partial y$, $dy/dt = \partial \mathcal{H}/\partial x$ take the form:

$$\frac{dx}{dt} = \frac{a^2 \mathcal{G} m m_9}{4 a_9^3 \epsilon_9^3} \left(\frac{3y_9 (2\Gamma_9 - x_9^2 - y_9^2)}{4\Gamma_9} \sqrt{\frac{4\Gamma_9 - x_9^2 - y_9^2}{\Gamma\Gamma_9^2}} + \frac{3y}{2\Gamma} \left(1 - \frac{3(2\Gamma_9 - x_9^2 - y_9^2)^2}{4\Gamma_9^2} \right) \right)
\frac{\partial y}{\partial t} = \frac{3 a^2 \mathcal{G} m m_9}{32 a_9^3 \Gamma \Gamma_9^2 \epsilon_9^3} \left(2x_9 \sqrt{\Gamma (4\Gamma_9 - x_9^2 - y_9^2)} (x_9^2 + y_9^2 - 2\Gamma_9) + x \left(8\Gamma_9^2 + 3x_9^4 - 12\Gamma_9 y_9^2 + 3y_9^4 + 6x_9^2 \left(y_9^2 - 2\Gamma_9 \right) \right) \right)
\frac{\partial x_9}{\partial t} = \frac{3 a^2 \mathcal{G} m m_9}{16 a_9^3 \Gamma_9^2 \epsilon_9^3} \left(-2y_9 \left(xx_9 + yy_9 \right) \sqrt{\frac{4\Gamma_9 - x_9^2 - y_9^2}{\Gamma}} + y \left(2\Gamma_9 - x_9^2 - y_9^2 \right) \sqrt{\frac{4\Gamma_9 - x_9^2 - y_9^2}{\Gamma}} \right)
+ \frac{1}{\Gamma} y_9 \left(2\Gamma - 3x^2 - 3y^2 \right) \left(x_9^2 + y_9^2 - 2\Gamma_9 \right) - y_9 \left(xx_9 + yy_9 \right) \frac{2\Gamma_9 - x_9^2 - y_9^2}{\Gamma} \right)$$

The inclination of the planetary system relative to the solar equator may be explained by the presence of Planet 9

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SOLAR OBLIQUITY INDUCED BY PLANET NINE: SIMPLE CALCULATION

Dong Lai

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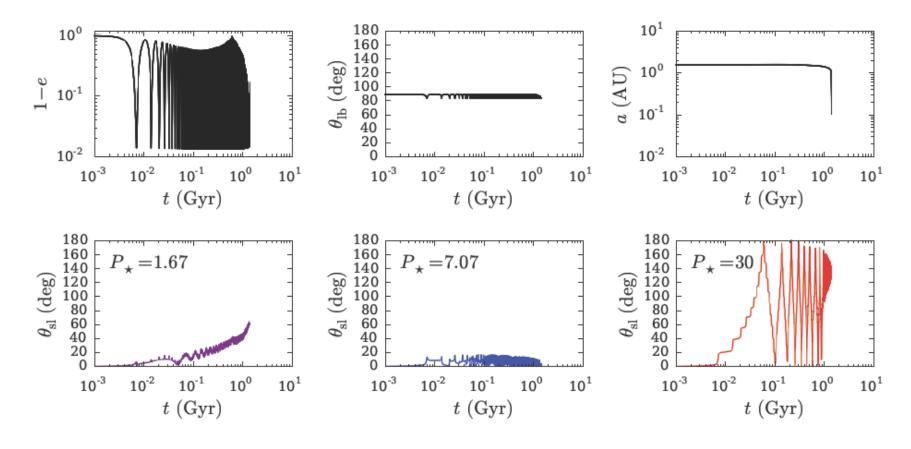
ABSTRACT

Bailey et al. (2016) and Gomes et al. (2016) recently suggested that the 6 degree misalignment between the Sun's rotational equator and the orbital plane of the major planets may be produced by the forcing from the hypothetical Planet Nine on an inclined orbit. Here we present a simple but accurate calculation of the effect, which provides a clear description of how the Sun's spin orientation depends on the property of Planet Nine in this scenario.

Thanks

Tidal Dissipation: Paths to Final Misalignments

"Regular"

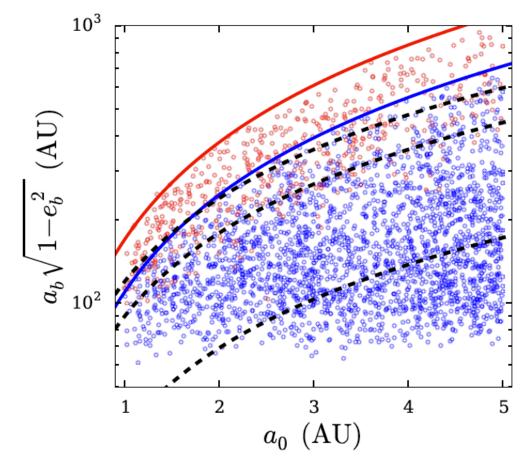


Condition for LK Migration, Fractions...

For given initial a of planet, e_{max} must be sufficiently large so that a $(1-e_{max})$ is small enough for efficient tidal dissipation

$$e_{\max} = e_{\max}(a, a_b, M_{\star}, M_b, M_p, R_p, \dots; i_0)$$

(1) $e_{\lim}(a, a_b, M_{\star}, M_b, M_p, R_p, \cdots)$ large enough



Anderson, Storch & DL 16

Condition for LK Migration, Fractions...

For given initial a of planet, e_{max} must be sufficiently large so that a $(1 - e_{max})$ is small enough for efficient tidal dissipation

$$e_{\max} = e_{\max}(a, a_b, M_{\star}, M_b, M_p, R_p, \dots; i_0)$$

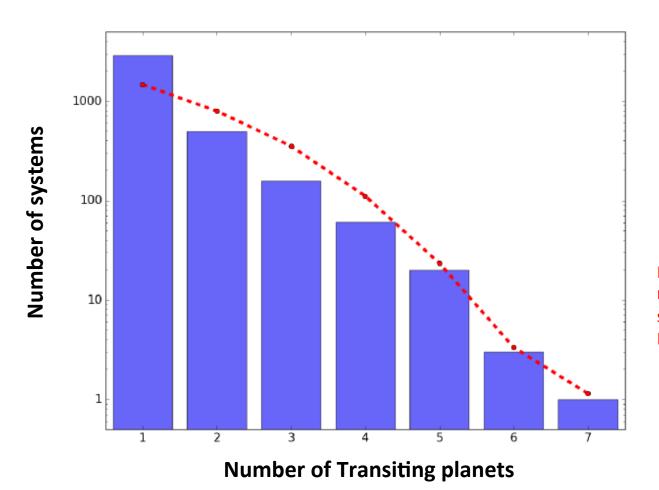
- (1) $e_{\lim}(a, a_b, M_{\star}, M_b, M_p, R_p, \cdots)$ large enough
- (2) i_0 large enough (Octupole window)



Analytic calcuations of migration/disruption fractions for all types of planets (Munoz, DL & Liu 2016)

Models with single inclination dispersion (e.g. in Rayleigh) do not fit well: Under-predict Kepler singles by a factor of > 2

Lissauer+11, Johansen, Davies +12, Weissbein+12, Ballard & Johnson+16



Red: best-fit to mutiple-transit planet systems with single Inclination dispersion

"Kepler Dichotomy"

Kepler systems consist of at least two underlying populations:

- (1) Systems N > 6 planets with small mutual inclinations (2 degrees): Account for most of Kepler Multi's (N_{tran} >1)
- (2) Systems with fewer planets or with higher mutual inclinations: Account for a (large) fraction Kepler singles (N_{tran}=1)

Origin of Kepler Dichotomy

-- Primordial

in-situ assembly of planetesimal disks with different mass & density profile (Mariarty & Ballard 15)

-- Dynamical instability

tightly packed system → unstable → collision/consolidation (Volk & Gladman 15; Pu & Wu 15)

-- External Perturber (Giant planet or companion star)

General Comment: Influences of External Perturbers on (Inner) Planetary Systems

- --- Mutual inclinations (DL & Pu 2016)
- --- Formation of Hot Jupiters and Warm Jupiters (high-e migration) (many papers...)
 - e.g. High-e migration induced by stellar companion contrinutes ~10-15% of HJs (Petrovich 15; Anderson, Storch & DL 16; Munoz, DL+16)
- --- Evection resonance (Touma & Sridhar 15; Xu & DL 16)