Misaligned Planets/Stars, Disks and Rings

Dong Lai

Cornell University

Nice Observatoire, 12/7/2016
Misaligned Planets/Stars, Disks and Rings

(1) Binary-Disk-Star Interaction ➞ Misaligned protoplanetary disk

(2) Secular perturbation from binary on planet ➞ Misaligned Hot Jupiters

(3) Missing Kepler Multi’s

(4) Extended, misaligned ring/disk around planets/brown dwarfs

(5) Planet IX and solar obliquity

Other topics:
• Circumbinary Disks (hydro) and circumbinary planets (dynamics)
• MMR capture and stability
Misaligned Planets
Also note: Misaligned multi-planet systems:
Kepler-56 (2 planets 10.5 & 21 days, 40-55 deg; Huber+2013); other candidates (?): Hirano+2014
How to produce misaligned planets (HJs)?

-- “Primordial” misalignment between spin and disk

-- “Lidov-Kozai migration” induced by a binary companion

-- Planet-Planet interactions:
  dynamical scattering, secular chaos, coplanar high-e migration
  (Rasio & Ford 96; Chatterjee et al. 08; Juric & Tremaine 08; Nagasawa et al. 08;
  Wu & Lithwick 11; Beauge & Nesvorny 2012; Petrovich 15,16; Petrovich & Tremaine 16;
  Hamers et al.16)

Observational Hints:
  Young HJs (Donati+16; David+16)
  Ngo+15; Bryan+16 (External companions)
  Huang+16 (neighbors of WJs/HJs)
  Schlaufman & Winn 16 (companions inside ice line)
Ideas for Producing Primordial Misalignments between Stellar Spin and Protoplanetary Disk

-- Star formation in turbulent medium
  (Bate+2010; Fielding+2014)

-- Magnetic Star – Disk Interaction
  (Lai+2011, Foucart & Lai 2011)

-- Perturbation of Binary on Disk
  (Batygin 2012; Batygin & Adams 2014; Spalding+2015; Lai 2014; Zanazzi & DL’17)
Star-Disk-Binary Interactions

$M_b$, $S$, $L_d$, $L_b$, disk

DL2014
Companion makes disk precess

Disk behaves like a rigid body
(bending waves, viscous stress, self-gravity)
e.g. Foucart & Lai 2015;
Caveat ? (J. Zanazzi & Lai, in prep)

$$\Omega_{pd} \simeq -5 \times 10^{-6} \left( \frac{M_b}{M_*} \right) \left( \frac{r_{out}}{50 \text{ AU}} \right)^{3/2} \left( \frac{a_b}{300 \text{ AU}} \right)^{-3} \times \cos \theta_{db} \left( \frac{2\pi}{\text{yr}} \right)$$
Disk makes the star precess

Due to gravitational torque from disk on rotating (oblate) star

\[ \Omega_{ps} \simeq -5 \times 10^{-5} \left( \frac{M_d}{0.1 M_*} \right) \left( \frac{\bar{\Omega}_*}{0.1} \right) \left( \frac{r_{in}}{4 R_*} \right)^{-2} \left( \frac{r_{out}}{50 \text{ AU}} \right)^{-1} \]

\[ \times \cos \theta_{sd} \left( \frac{2\pi}{\text{yr}} \right) \]

where \( \bar{\Omega}_* = \left( \frac{2\pi}{3.3 \text{ days}} \right) \left( \frac{\bar{\Omega}_*}{0.1} \right) \).
Two limiting cases:

1. \(|\Omega_{ps}| \gg |\Omega_{pd}| : \implies \theta_{sd} \simeq \text{constant}

2. \(|\Omega_{ps}| \ll |\Omega_{pd}| : \implies \theta_{sb} \simeq \text{constant}
\[ \Omega_{pd} \simeq -5 \times 10^{-6} \left( \frac{M_b}{M_\star} \right) \left( \frac{r_{out}}{50 \text{ AU}} \right)^{3/2} \left( \frac{a_b}{300 \text{ AU}} \right)^{-3} \times \cos \theta_{db} \left( \frac{2\pi}{\text{yr}} \right) \]

\[ \Omega_{ps} \simeq -5 \times 10^{-5} \left( \frac{M_d}{0.1M_\star} \right) \left( \frac{\tilde{\Omega}_*}{0.1} \right) \left( \frac{r_{in}}{4R_\star} \right)^{-2} \left( \frac{r_{out}}{50 \text{ AU}} \right)^{-1} \times \cos \theta_{sd} \left( \frac{2\pi}{\text{yr}} \right) \]

Initially \( |\Omega_{ps}| \gg |\Omega_{pd}| \)

\( M_d \) decreases over \( \sim 1 \) Myr
Initial:
\[ \theta_{db} = 5^\circ \]
\[ \theta_{sd} = 5^\circ \]
Resonance $\Omega_{ps} = \Omega_{pd}$

$$\frac{d\hat{S}}{dt} = \Omega_{ps}\hat{L}_d \times \hat{S}$$

In the frame rotating at rate $\Omega_{pd}\hat{L}_b$

$$\left(\frac{d\hat{S}}{dt}\right)_{rot} = \Omega_{ps}\hat{L}_d \times \hat{S} - \Omega_{pd}\hat{L}_b \times \hat{S}$$

$$= \left(\Omega_{ps}\hat{L}_d - \Omega_{pd}\hat{L}_b\right) \times \hat{S}$$

$\Omega_e\hat{L}_e$
Initial:
\[ \theta_{db} = 30^\circ \]
\[ \theta_{sd} = 5^\circ \]
Complications:

Accretion and magnetic interaction
Magnetic Star - Disk Interaction: Basic Picture

\[ r_{in} = \eta \left( \frac{\mu^4}{GMM^2} \right)^{1/7} \]

\( \eta \sim 0.5 - 1 \)
Accretion & Magnetic Torques (Order-of-Mag):

Accretion torque: \( \mathcal{N}_{\text{acc}} \simeq \dot{M} \sqrt{GM_\star r_{\text{in}}} \)

Magnetic (misalignment) torque: \( \mathcal{N}_{\text{mag}} \sim \mu^2/r_{\text{in}}^3 \)
**Accretion & Magnetic Torques (Order-of-Mag):**

Accretion torque: \[ N_{\text{acc}} \sim \dot{M} \sqrt{G M_* r_{\text{in}}} \]

Magnetic (misalignment) torque: \[ N_{\text{mag}} \sim \frac{\mu^2}{r_{\text{in}}^3} \]

For \[ r_{\text{in}} \sim \left( \frac{\mu^4}{G M_* \dot{M}^2} \right)^{1/7} \]

\[ \Rightarrow \quad N_{\text{acc}} \sim N_{\text{mag}} \]

Note: Massive (>1.3 Sun) T Tauri stars have weaker dipole fields than less massive stars. Consequence on spin-orbit misalignment?
Star-Disk-Binary Interactions

Gravitational interactions...

Now include Accretion and Magnetic Torques
No accretion/magnetic

Accretion/magnetic damps SL-angle

Accretion/magnetic increases SL-angle

DL2014
Summary (#1)
Star-disk-binary interactions

• With a binary companion, spin-disk misalignment is “easily” generated
• The key is “resonance crossing”
• Accretion/magnetic torques affect it, but not diminish the effect

“primordial” misalignment of stellar spin and disk

Too robust ??
Realistic disk evolution, disk warping, planet-disk coupling
(J.Zanazzi’s Thesis)
Formation of Hot Jupiters: Lidov-Kozai Migration Driven by a Companion

Holman et al. 97; Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12; Petrovich 15, ....
Lidov-Kozai Oscillations

- Eccentricity and inclination oscillations induced if $i > 40$ degrees.
- If $i$ large (85-90 degrees), get extremely large eccentricities ($e > 0.99$)
Lidov-Kozai Oscillations: Octupole Effect

e.g. Ford et al 2000; Naoz et al 11; Katz et al 11; Lithwick & Naoz 11; Li et al 14; Antognini 15

Octupole

-- Very large $e$ even for modest $i$
-- $L_p$ can flip wrt outer binary

Including Short-Range Forces

-- $e_{\text{max}}$ is reduced/limited
-- flip is delayed/suppressed

B.Liu, D.Munoz & DL 2015
Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces

Bin Liu, D. Munoz, DL 2015
Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces

Window of Octupole

\[ \varepsilon_{\text{oct}} = \left( \frac{a}{a_b} \right) \frac{e_b}{1 - e_b^2} \]
Lidov-Kozai Oscillations: Octupole Effect + Short-Range Forces

Window of Octupole

The formation efficiency of close-in planets via Lidov-Kozai migration: analytic calculations

Diego J. Muñoz\textsuperscript{1*}, Dong Lai\textsuperscript{1} and Bin Liu\textsuperscript{2}

\textsuperscript{1} Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA
\textsuperscript{2} Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China
Hot Jupiter formation

Holman et al. 97; Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12, Katz et al.12; Petrovich 15

- Planet forms at ~ a few AU
- Companion star periodically pumps planet into high-e orbit (Lidov-Kozai)
- Tidal dissipation in planet during high-e phases causes orbital decay

→ Combined effects can result in planets in ~ few days orbit

Q: What is happening to the stellar spin axis?
Chaotic Dynamics of Stellar Spin Driven by Planets Undergoing Lidov-Kozai Oscillations

With
Natalia Storch
(Ph.D.15 → Caltech)
Kassandra Anderson

Storch, Anderson & DL 2014, Science
Storch & DL 2015 MNRAS (Theory I)
Anderson, Storch, DL 2016 (Pop Study)
Storch, DL & Anderson 2016 (Theory II)
Precession of Stellar Spin

- Star is spinning (3-30 days) → oblate → will precess

\[ \frac{d\hat{S}}{dt} = \Omega_{ps} \hat{L} \times \hat{S} \]

\[ \Omega_{ps} = -\frac{3GM_p(I_3 - I_1) \cos \theta_{sl}}{2a^3(1 - e^2)^{3/2}} \frac{S}{S} \]

\[ \propto \frac{\Omega_s M_p}{a^3(1 - e^2)^{3/2}} \]
Spin Dynamics

- Stellar spin axis $S$ wants to precess around planet orbital axis $L$. 

![Diagram]

- $\Omega_{ps}$
- $L$
- $S$
- $\theta_{sl}$
- $\theta_{lb}$
- $\theta_{sb}$

- Outer binary axis
- Planet orbital axis
- Stellar spin axis
Spin Dynamics

- Stellar spin axis $S$ wants to precess around planet orbital axis $L$.
- But $L$ itself is moving:
  - Nodal precession ($L$ precesses around binary axis $L_b$)
  - Nutation (cyclic changes in inclination of $L$ relative to $L_b$)
Spin Dynamics

• **Q:** Can S keep up with L?

• Answer depends on

\[ \Omega_{ps} \text{ vs } \Omega_{pl} \]
If $|\Omega_{ps}| \gg |\Omega_{pl}|$: YES ("adiabatic")

$\theta_{sl} = \text{constant}$, i.e. initial spin-orbit misalignment is maintained for all time.

N. Storch
If $|\Omega_{ps}| \ll |\Omega_{pl}|$: NO ("non-adiabatic")
If $|\Omega_{ps}| \sim |\Omega_{pl}|$: “trans-adiabatic”

To answer, need to solve orbital evolution equations together with spin precession equation....
If $|\Omega_{ps}| \sim |\Omega_{pl}|$: “trans-adiabatic”

Q: Is it really chaotic?
If $|\Omega_{ps}| \sim |\Omega_{pl}|$: “trans-adiabatic”

Lyapunov time $\sim 6$ Myrs

Storch, Anderson & DL 14
Complication & Richness

$\Omega_{ps}$ & $\Omega_{pl}$ are strong functions of eccentricity (and time)

**Key parameter:**

$$\epsilon = \frac{\Omega_{pl0}}{\Omega_{ps0}} \propto \frac{a^{9/2}}{M_p \Omega_s}$$

The ratio of orbital precession frequency to spin precession frequency at zero eccentricity
Values recorded at eccentricity maxima, for 1500 LK-cycles

Periodic islands in the ocean of chaos

Quasi-chaotic regions in the “calm” sea

Storch, Anderson & DL 14; Storch & DL 15
What about tidal dissipation?
Lidov-Kozai + Tidal Dissipation

\[
\begin{align*}
\alpha_f \\
1-e \\
\theta_{LB}, \theta_{SB} \\
\theta_{SL} \quad (^\circ)
\end{align*}
\]
A tiny spread in initial conditions can lead to a large spread in the final spin-orbit misalignment.

Initial orbital inclination $\theta_{ib}=85\pm0.05^\circ$, with spindown.
Distributions of the final spin-orbit angle

Parameters:

\[ a_b = 200 \text{ AU} \]
\[ M_* = M_b = 1 \text{ M}_{\odot} \]

Stellar spin-down calibrated such that spin period = 27 days at 5 Gyr (Skumanich law)

Uniform distribution of initial semi-major axes (\( a = 1.5 - 3.5 \text{ AU} \))

Anderson, Storch & DL 16
Solar-type star

\[ M_p = 1 M_J \]

\[ M_p = 3 M_J \]

\[ M_p = 5 M_J \]

F Star (1.4 \( M_{\text{sun}} \))

\[ M_p = 1 M_J \]

\[ M_p = 3 M_J \]

\[ M_p = 5 M_J \]

\( \theta_{\text{sl,f}} \) (deg)

Anderson, Storch & DL 16
Formation of Hot Jupiters in Stellar Binaries

Anderson, Storch & DL 2016

G Stars
Formation in of Hot Jupiters in Stellar Binaries

Anderson, Storch & DL 2016
Theory of Spin Chaos/Evolution

In Hamiltonian system, Chaos arises from **overlapping resonances** (Chirikov criterion; 1979)

$$\Omega_{ps} \ & \ \Omega_{pl} \text{ are strong functions of } e \text{ and } t \ \ (\text{with period } P_e)$$

What resonances??

Storch & DL 15
Storch, DL & Anderson 16
Hamiltonian Perturbation Theory

\[ H = -\frac{1}{2} \alpha(t) \cos^2 \theta_{sl} - \Omega_{pl}(t) \left[ \cos \theta_{lb}(t) \cos \theta_{sl} - \sin \theta_{lb}(t) \sin \theta_{sl} \cos \phi \right] - \dot{\theta}_{lb}(t) \sin \theta_{sl} \sin \phi, \]

Spin precession | Orbit’s nodal precession | Orbit’s nutation

\[ H' = \frac{\bar{\alpha}}{n_e} \left( -\frac{1}{2} p^2 + \psi(\tau)p - \sqrt{1 - p^2} [\beta(\tau) \cos \phi + \gamma(\tau) \sin \phi] \right) \]

\[ \beta(\tau) = -\frac{\Omega_{pl}(\tau)}{\alpha(\tau)} \sin \theta_{lb}(\tau), \]
\[ \gamma(\tau) = \frac{\dot{\theta}_{lb}(\tau)}{\alpha(\tau)}, \]
\[ \psi(\tau) = -\frac{\Omega_{pl}(\tau)}{\alpha(\tau)} \cos \theta_{lb}(\tau). \]

\[ H' = \frac{\bar{\alpha}}{n_e} \left( -\frac{1}{2} p^2 + \epsilon \psi_0 p + \epsilon \sum_{M=1}^{\infty} \psi_M \cos Mt \right. \]
\[ \left. -\frac{\epsilon}{2} \sqrt{1 - p^2} \sum_{M=0}^{\infty} \left[ (\beta_M + \gamma_M) \cos(\phi - M\tau) + (\beta_M - \gamma_M) \cos(\phi + M\tau) \right] \right) \]
Spin-Orbit Resonances

\[ \bar{\Omega}_{ps} = -\bar{\alpha} \cos \theta_{sl} = N \frac{2\pi}{P_e} \]

Average spin precession frequency = Integer multiple of mean Lidov-Kozai frequency
Individual Resonance

\[
\cos \theta^\bullet_{\phi} \quad \phi
\]

- \( N = -9 \)
- \( N = -5 \)
- \( N = 0 \)
- \( N = 5 \)
- \( N = 9 \)
Boundary of Chaotic Zone

Analytical theory: From outmost overlapping resonances
Numerical Bifurcation Diagram

Starting from zero spin-orbit angle
Numerical Theory

Storch & DL 2015
Tidal Dissipation: Paths to Final Misalignments

Storch, DL, Anderson 2016
Experiments with given $a_0 = 1.5$ au, $a_b = 300$ au, etc varying stellar rotation period (S-L coupling strength)

$I_0 = 89^\circ$

$I_0 = 87^\circ$
Tidal Dissipation: Paths to Final Misalignments

CHAOTIC
Adiabatic Resonance Advection

\[ 1 - e \]

\[ a \text{ (AU)} \]

\[ \theta_{sl} \text{ (deg)} \]

\[ P_\ast = 1.67 \]

\[ t \text{ (Gyr)} \]

\[ \cos \theta_{sl} \]

\[ t = 253.091 \text{ Myr} \]

N. Storch
Bifurcation ➔ Bimodal Distribution Misalignment

Pendulum with gradually decreasing length...
Summary (# 2)
Chaotic Stellar Spin and Formation of Hot Jupiters

- Dynamics of stellar spin is important for the formation of hot Jupiters, e.g. affects the observed spin-orbit misalignments (dependence on planet mass, stellar rotation/history etc)
- Spin dynamics can be chaotic
- Spin dynamics/evolution can be understood from resonance theory
- Migration fractions can be calculated analytically
Missing Kepler Multi’s?

Multi-Planet System with an External Companion

DL & Bonan Pu

arXiv:1606.08855
**Kepler:** 4700 planets in 3600 systems
(mostly super-earth or sub-neptunes, <200 days)

**Observed Transit Multiplicity Distribution** $F(N_{\text{tran}})$

Number of systems

Number of Transiting planets
\begin{align*}
F(N_{\text{tran}}) & \implies \\
\text{the underlying multiplicity distribution (\& mutual inclinations)}
\end{align*}

Degeneracy can be broken by RV data (Tremaine \& Dong 12) 
Transit duration ratio (Fabrycky+14)

\textbf{Kepler compact systems are flat, with mutual inclination dispersion < 2 degrees}

\text{Lissauer+11, Tremaine \& Dong 12, Figueira+12, Fabrycky+14; Fang \& Margot 12}
Excess of Kepler Singles (?)

Models with single mutual inclination dispersion (in Rayleigh) fall short to explain the observed number of Kepler singles (Lissauer+11; Johansen+12; Ballard & Johnson 16) (depends somewhat on the assumed multiplicity function)

Some Kepler singles might be multi-planet systems with higher mutual inclinations...

Other evidence that (some) Kepler singles are “special”:
(1) “singles” have higher stellar obliquities (Morton & Winn 14)
(2) Multi’s have more detectable TTVs than “singles” (Xie, Wu & Lithwick 14)
(3) 10-30% of singles have higher e’s (Xie, S.Dong et al 2016)
(4) Single “Hot Earth” excess (Jason Staffen)
Effect of External Perturber
On Compact Multi-Planet System

**Perturber:** Giant planet (~AUs) or Companion Star (~10’s AUs)
Two-planet system with an external inclined perturber

$m_1, m_2$ initially co-planar...

companion star or "cold Jupiter" (e.g. produced by planet-planet scatterings)
Precession of “1” driven by “p”:

$$\Omega_{1p} \sim \frac{m_p}{M_\star} \left(\frac{a_1}{a_p}\right)^3 n_1 \propto \frac{m_p}{a_p^3} a_1^{3/2}$$
Mutual inclination induced by perturber depends on Coupling Parameter

\[ \epsilon_{12} \equiv \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \]
Maximum Mutual Inclination Induced by external perturber

$\theta_p = 10^\circ$

$(\theta_{12})_{\text{max}} \approx \epsilon_{12} \sin 2\theta_p$

$\epsilon_{12} \equiv \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \propto \frac{m_p}{a_p^3}$
Resonance Feature: $\epsilon_{12} \sim 1$
exists when $m_2 \gtrsim m_1$

Nodal Precession Resonance

In the $m_2 \gg m_1$ limit:
Resonance occurs at $\Omega_{2p} = \Omega_{1p} + \omega_{12}$ or $\epsilon_{12} = 1$
Resonance Feature: $\epsilon_{12} \sim 1$

Can produce much larger mutual inclination than $\theta_p$.
Resonance Feature: $\epsilon_{12} \sim 1$

Can produce much larger mutual inclination than $\theta_p$
Multi-planet system with an external inclined perturber
4 planet system with an external perturber

\[ \left[ \sin^2(2\theta_p) + \sin^4(\theta_p) \right]^{1/2} / \sqrt{2} \]

\( \sigma_\theta \) [Deg]

\( \bar{\epsilon} \)  

\( \epsilon_{1d} = 1 \)

\( \epsilon_{2d} = 1 \)

\( \epsilon_{3d} = 1 \)

\( \epsilon \)

averaged coupling parameter \( \propto m_p / a_p^3 \)

graphical representation with curves for different values of \( m_d/m \)
4 planet system with an external perturber

The averaged coupling parameter is related to the dispersion of mutual inclination by:

$$\sigma_\theta \propto \frac{m_p}{a_p^3} \left[ \sin^2(2\theta_p) + \sin^4(\theta_p) \right]^{1/2} \frac{1}{\sqrt{2}}$$

where $\epsilon_{1d} = 1$, $\epsilon_{2d} = 1$, and $y = \bar{\epsilon} \left| \sin 2\theta_p \right| / \sqrt{2}$.
Recap: An understanding and semi-analytic expression for the mutual inclination of multi-planets induced by distant perturber

\[ \epsilon_{12} = \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \sim \left( \frac{m_p}{m_2} \right) \left( \frac{a_2}{a_p} \right)^3 \]

for \( m_2 \sim m_1, a_2 \sim 1.5a_1 \)

e.g., for \( m_2 \sim m_\oplus \) at \( a_2 \sim 0.3 \) au

\( 1M_J \) perturber at 3 au gives \( \epsilon_{12} \sim 0.3 \)

For \( \epsilon_{12} \gtrsim 2 \) : \( \theta_{12} \sim 2\theta_p \)

For \( \epsilon_{12} \lesssim 0.3 \) : \( \theta_{12} \sim \epsilon_{12} \sin 2\theta_p \)
Application:
“Evaluate”/constrain external perturber of observed systems

Examples:  (see DL & Pu for more)

Kepler-68:  Two super-Earths at 0.06,0.09 au, 1 M_J planet at 1.4 au is weak
Kepler-56:  ...
WASP-47:    ...

Kepler-48:  Three transiting planets (0.012,0.046,0.015M_J) at 0.053,0.085,0.23 au,
            >2M_J companion (Kepler-48e) at 1.85 au (Marcy+14)
            ➔ Kepler-48e must be aligned <2 degree

Kepler-454: one super-Earth at 0.1 au with a >5M_J at 524 days (Gettel+16)
            ➔ any neighbor to the inner planet could be strongly misaligned by the giant
            (This could of multi-planet system that has been turned into Kepler single
            by an external giant planet)
Confront with Observations?

Some/significant (?) fraction of Kepler singles ($N_{\text{tran}}=1$) are/were multi-planet systems that have been misaligned (in mutual inclinations) or disrupted by external perturbers (cold giant planets or stellar companions)

**Cold Giant Planets:**

Some have been found, but general census not clear?

(50% of HJs/WJs have 1-20M$_J$ companion at 5-20 au; Bryan et al 2016)

**Stellar Companions:**

J. Wang et al (2015): ~5% of Kepler multis have stellar companion 1-100 au

(4x lower than field stars)

More studies are needed.
Extended & Inclinded Disks/Rings around Planets/Brown Dwarfs

J.J. Zanazzi & DL 2016
Lightcurve of 1SWASP J1407 (~16 Myr, 0.9Msun)
A series “eclipses” lasting 56 days around April 2007:
big eclipse (3.5 mag) with symmetric pairs of smaller (~mag) eclipses

→ Large ring/disk system around unseen substellar companion (?)

Mamajek et al. 2012
van Werkhoven et al. 2014
Kenworthy et al. 2015
Claimed best fit model:

Ring size $\sim 0.6$ AU, with gaps (moons ?), total mass $\sim 1 \text{ M}_\text{earth}$

Orbital period 10-30 years (eccentric)
Question:
Under what conditions can an extended, inclined ring/disk exist around a planet/BD?

- Outer radius of ring \( r_{\text{out}} \lesssim 0.3r_H \)

\[
r_H = a \left( \frac{M_p}{3M_*} \right)^{1/3}
\]

- How to produce/maintain inclination?
Ring at radius $r$ experiences two torques

From star:
$$T_* \sim \frac{G M_* r^2}{a^3}$$

From oblate planet:
$$T_p \sim \frac{G (J_2 M_p R_p^2)}{r^3}$$

$$T_* = T_p \text{ at Laplace radius :}$$
$$r_L \sim \left( J_2 \frac{M_p}{M_*} R_p^2 a^3 \right)^{1/5} \sim (3 J_2 R_p^2 r_H^3)^{1/5}$$
Equilibrium State: Laplace Surface

\[ \hat{\mathbf{l}}(r) \]
\[ \hat{s} \]
\[ \hat{\mathbf{i}}_{\text{orb}} \]

\[ r_L/r_{\text{out}} = 0.1 \]
\[ r_L/r_{\text{out}} = 0.3 \]
\[ r_L/r_{\text{out}} = 0.5 \]

\[ \beta \text{ (degrees)} \]

\[ r/r_{\text{out}} \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]
How to make the disk sufficiently “rigid” to have appreciable $\beta(r_{\text{out}})$?

- **Internal stresses (pressure, viscosity):** Not effective for thin rings

- **Self-gravity:**
  \[
  T_{\text{sg}} \sim \pi Gr \Sigma
  \]

Compare to
\[
T_* \sim \frac{GM_* r^2}{a^3} \quad T_p \sim \frac{G(J_2 M_p R_p^2)}{r^3}
\]
How to make the disk sufficiently “rigid” to have appreciable $\beta(r_{out})$?

- **Internal stresses (pressure, viscosity):** Not effective for thin rings

- **Self-gravity:**
  \[ T_{sg} \sim \pi Gr \Sigma \]
  \[ T_* \sim \frac{GM_* r^2}{a^3} \]
  \[ T_p \sim \frac{G(J_2 M_p R_p^2)}{r^3} \]

Compare to

\[ T_{sg}(r_{out}) \sim \frac{GM_d}{r_{out}} \geq T_*(r_{out}) \]

\[ \sigma \equiv \frac{M_d}{M_*} \left( \frac{a}{r_{out}} \right)^3 = \frac{3M_d}{M_p} \left( \frac{r_H}{r_{out}} \right)^3 \geq 1 \]
Equilibrium State with Self-Gravity

\[ T_\ast + T_p + T_{sg} = 0 \]

\[ T_{sg} = \frac{\pi G}{2} \int_{r_{in}}^{r_{out}} \frac{r' \Sigma(r') \mathrm{d}r'}{\max(r, r')} \chi b^{(1)}_{3/2}(\chi) [ \mathbf{1}(r) \cdot \mathbf{1}(r') ] [ \mathbf{1}(r) \times \mathbf{1}(r') ] \]
Equilibrium State with Self-Gravity
(modified Laplace Surface)
Equilibrium State with Self-Gravity
(modified Laplace Surface)
What if disk not in equilibrium?
Can disk “precess” coherently?
Time evolution of disk

\[ r^2 \Omega \frac{\partial \hat{I}(r, t)}{\partial t} = T_* + T_p + T_{sg} \]

Initial \( \hat{I}(r, 0) \) aligned with \( \hat{S} \) (at \( \beta = 40^\circ \))
**Time evolution of disk**

Inner disk ($T_p \gtrsim T_{sg}, T_\star$): stays aligned with $\hat{S}$

Outer disk ($T_{sg} \gtrsim T_p$): evolves coherently if $T_{sg}(r) \gtrsim T_\star(r) \iff \sigma \gtrsim 1$

nutation in $\beta$, precession (smaller $\sigma$)/libration (larger $\sigma$) in $\phi$
Summary (# 4)
Extended, Inclined Disk/Ring Around Planets/BDs

- Intriguing “eclipses” of J1407 due to extended disk/ring?
- $r_H$ and $r_L$
- If $r_{\text{out}} < r_L$, disk is inclined wrt orbit (provide $S$ inclined wrt orbit)
- If $r_{\text{out}} > r_L$, require sufficient disk mass for self-gravity to maintain disk coherence and out disk inclination
  -- generalized Laplace surface (with self-gravity)
  -- precessing/librating/nutating self-gravitating disk

$$\frac{M_d}{M_p} \gtrsim \frac{1}{3} \left( \frac{r_{\text{out}}}{r_H} \right)^3$$
~10 Earth mass, a~700 AU, e=0.6  (distance 300-1000 AU, period 20,000 years)  
inclined by 10-30 degrees
SOLAR OBLIQUITY INDUCED BY PLANET NINE

ELIZABETH BAILEY, KONSTANTIN BATYGIN, MICHAEL E. BROWN
Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125

Draft version July 21, 2016

The numerical evaluation of the system’s evolution can be robustly carried out after transforming the Hamiltonian to nonsingular Poincaré Cartesian coordinates

\[ x = \sqrt{2 Z} \cos(z) \quad y = \sqrt{2 Z} \sin(z) \]
\[ x_0 = \sqrt{2 Z_0} \cos(z_0) \quad y_0 = \sqrt{2 Z_0} \sin(z_0). \]

Then, the truncated and expanded Hamiltonian (4) becomes

\[
\mathcal{H} = -\frac{1}{4} \frac{\mathcal{G} m m_0}{a_9} \left(\frac{a}{a_9}\right)^2 \frac{1}{z_0^2} \left[\frac{1}{2} \left(2 - \frac{6}{1} \left(\frac{x^2 + y^2}{2}\right) \right) \left(3 \left(1 - \frac{1}{1} \left(\frac{x_0^2 + y_0^2}{2}\right) \right)^2 - 1\right) \right.

+ 3 \left(1 - \frac{1}{1} \left(\frac{x_0^2 + y_0^2}{2}\right) \right) \sqrt{1 - \frac{1}{2} \left(\frac{x_0^2 + y_0^2}{2}\right) \sqrt{\frac{1}{1} \left(\frac{x_0 + y_0}{2}\right)}}].
\]

Explicitly, Hamilton’s equations \(dx/\delta t = -\partial \mathcal{H}/\partial y, dy/\delta t = \partial \mathcal{H}/\partial x\) take the form:

\[
\frac{dx}{\delta t} = \frac{a^2 \mathcal{G} m m_0}{4 a_9^3 c_9^3} \left(3 g_9 (2 \Gamma_9 - x_0^2 - y_0^2) \sqrt{4 \Gamma_9 - x_0^2 - y_0^2} + 3 y \left(1 - \frac{3(2 \Gamma_9 - x_0^2 - y_0^2)}{4 \Gamma_9}\right) \right)
\]

\[
\frac{\partial y}{\partial t} = \frac{3 a^2 \mathcal{G} m m_0}{32 a_9^3 \Gamma_9^3 c_9^3} \left(2 x_0 \sqrt{4 (4 \Gamma_9 - x_0^2 - y_0^2)(x_0^2 + y_0^2 - 2 \Gamma_9) + x \left(8 \Gamma_9^3 + 3 x_0^4 - 12 \Gamma_9 y_0^2 + 3 y_0^4 + 6 \mu_9 (y_0^2 - 2 \Gamma_9)\right)}\right)
\]

\[
\frac{\partial x_0}{\partial t} = \frac{3 a^2 \mathcal{G} m m_0}{16 a_9^3 \Gamma_9^3 c_9^3} \left(-2 y_0 (x_0 + y_0) \sqrt{4 \Gamma_9 - x_0^2 - y_0^2} + y \left(2 \Gamma_9 - x_0^2 - y_0^2\right) \sqrt{4 \Gamma_9 - x_0^2 - y_0^2} \right)
\]

\[
+ \frac{1}{c_9} g_9 \left(2 \Gamma - 3 x^2 - 3 y^2\right) \left(x_0^2 + y_0^2 - 2 \Gamma_9\right) - y_0 (x_0 + y_0) \frac{2 \Gamma_9 - x_0^2 - y_0^2}{\Gamma}\right).\]
The inclination of the planetary system relative to the solar equator may be explained by the presence of Planet 9

Rodney Gomes
Observatório Nacional
Rua General José Cristino 77, CEP 20921-400, Rio de Janeiro, Brazil
rodney@on.br

Rogerio Deienno
Laboratoire Lagrange, UCA, OCA, CNRS, Nice, France
Instituto Nacional de Pesquisas Espaciais, São José dos Campos, SP, Brazil

Alessandro Morbidelli
Laboratoire Lagrange, UMR7293, Université Côte d'Azur, CNRS, Observatoire de la Côte d'Azur
SOLAR OBLIQUITY INDUCED BY PLANET NINE: SIMPLE CALCULATION

DONG LAI
Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853

ABSTRACT

Bailey et al. (2016) and Gomes et al. (2016) recently suggested that the 6 degree misalignment between the Sun’s rotational equator and the orbital plane of the major planets may be produced by the forcing from the hypothetical Planet Nine on an inclined orbit. Here we present a simple but accurate calculation of the effect, which provides a clear description of how the Sun’s spin orientation depends on the property of Planet Nine in this scenario.
Thanks
Tidal Dissipation: Paths to Final Misalignments

“Regular”
Condition for LK Migration, Fractions...

For given initial $a$ of planet, $e_{\text{max}}$ must be sufficiently large so that $a (1 - e_{\text{max}})$ is small enough for efficient tidal dissipation

$$e_{\text{max}} = e_{\text{max}}(a, a_b, M_*, M_b, M_p, R_p, \cdots ; i_0)$$

(1) $e_{\text{lim}}(a, a_b, M_*, M_b, M_p, R_p, \cdots )$ large enough

Anderson, Storch & DL 16
Condition for LK Migration, Fractions...

For given initial $a$ of planet, $e_{\text{max}}$ must be sufficiently large so that $a (1 - e_{\text{max}})$ is small enough for efficient tidal dissipation

$$e_{\text{max}} = e_{\text{max}}(a, a_b, M_*, M_b, M_p, R_p, \cdot \cdot \cdot ; i_0)$$

(1) $e_{\text{lim}}(a, a_b, M_*, M_b, M_p, R_p, \cdot \cdot \cdot)$ large enough

(2) $i_0$ large enough (Octupole window)

Analytic calculations of migration/disruption fractions for all types of planets  (Munoz, DL & Liu 2016)
Models with single inclination dispersion (e.g. in Rayleigh) do not fit well: Under-predict Kepler singles by a factor of > 2

Lissauer+11, Johansen, Davies +12, Weissbein+12, Ballard & Johnson+16
“Kepler Dichotomy”

Kepler systems consist of at least two underlying populations:

(1) Systems $N > \sim 6$ planets with small mutual inclinations ($\sim 2$ degrees):
    Account for most of Kepler Multi’s ($N_{\text{tran}} > 1$)

(2) Systems with fewer planets or with higher mutual inclinations:
    Account for a (large) fraction Kepler singles ($N_{\text{tran}} = 1$)
Origin of Kepler Dichotomy

-- **Primordial**
   in-situ assembly of planetesimal disks with different mass & density profile
   (Mariarty & Ballard 15)

-- **Dynamical instability**
   tightly packed system → unstable → collision/consolidation
   (Volk & Gladman 15; Pu & Wu 15)

-- **External Perturber (Giant planet or companion star)**
General Comment: 
Influences of External Perturbers on (Inner) Planetary Systems

--- Mutual inclinations (DL & Pu 2016)

--- Formation of Hot Jupiters and Warm Jupiters (high-e migration) 
    (many papers...)
    e.g. High-e migration induced by stellar companion contributes 
    ~10-15% of HJs (Petrovich 15; Anderson, Storch & DL 16; Munoz, DL+16)

--- Evection resonance (Touma & Sridhar 15; Xu & DL 16)