# Dynamical Evolution of Compact Multi-Planet Systems: Effects of External Giant Planets 

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## Kepler: 4700 planets in 3600 systems

(mostly super-earths or sub-neptunes, <200 days)
Observed Transit Multiplicity Distribution $\mathrm{F}\left(\mathrm{N}_{\text {tran }}\right)$


## An Emerging Trend:

Systems with smaller number of planets are dynamically hotter: higher eccentricities and mutual inclinations (Xie+16; Zhu+18)

Related Trends:
--- Excess of Kepler Singles - Kepler dichotomy (?)
Models with single mutual inclination dispersion fall short to explain the observed number of Kepler singles (Lissauer+11; Johansen+12; Ballard \& Johnson 16)
-- Kepler Singles have higher stellar obliquities (Morton \& Winn 14)

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## Super-Earths and Cold Jupiter Connection

30-50\% of inner super-Earths ( $\mathrm{P}<200 \mathrm{~d}$ ) have CJ companions ( $\mathrm{P}>400 \mathrm{~d}$ );
These CJs are often eccentric, having undergone scatterings
(observations from many groups... see Lai \& Pu 2017; Zhu \& Wu 2018 for refs)

## How Cold Jupiters (and scatterings) affect Inner planet system (\& stellar obliquity)?

With Bonan (Michael) Pu Kassandra Anderson


## References:

Lai \& Pu 2017 "Hiding Planets behind a Big Friends..."
Pu \& Lai 2018 "Eccentricities and Inclinations of multiplanet systems...."
Pu \& Lai 2018 "Scatterings of Cold Jupiters and its Influence on Inner Systems" (in prep)
Lai, Anderson \& Pu 2018 "How do companions affect spin-orbit misalignments?"
Anderson \& Lai 2018 "Teetering stars: Resonant excitation of stellar obliquities..."

## N inner planets + one perturber (CJ)

Perturber: CJ on inclined and eccentric orbit
$\rightarrow$ excites mutual inclinations and eccentricities in the inner system



Precession of $L_{1}$ around $L_{p}$ :
$\Omega_{1 p} \sim \frac{m_{p}}{M_{\star}}\left(\frac{a_{1}}{a_{p}}\right)^{3} n_{1} \propto \frac{m_{p}}{a_{p}^{3}} a_{1}^{3 / 2}$
Precession of $L_{2}$ around $L_{p}$ :
$\Omega_{2 p} \propto \frac{m_{p}}{a_{p}^{3}} a_{2}^{3 / 2}$


$L_{1}$ and $L_{2}$ precess around each other at rate $\omega_{12}, \omega_{21}$
Mutual inclination induced by perturber depends on Coupling Parameter

$$
\epsilon_{12}=\frac{\Omega_{2 p}-\Omega_{1 p}}{\omega_{12}+\omega_{21}} \sim \frac{m_{p}}{m_{1,2}}\left(\frac{a_{1,2}}{a_{p}}\right)^{3}
$$

## Maximum Mutual Inclination Induced by external perturber



## Resonance Feature: $\epsilon_{12} \sim 1$

exists when $m_{2} \geq m_{1}$
Nodal Precession Resonance


In the $m_{2} \gg m_{1}$ limit:
Resonance occurs at $\Omega_{2 p}=\Omega_{1 p}+\omega_{12} \quad$ or $\quad \epsilon_{12}=1$

## Resonance Feature: $\epsilon_{12} \sim 1$

Can produce much larger mutual inclination than $\theta_{p}$


## Eccentricity Excitation by External Perturber



Pu \& DL 2018
Apsidal precession resonance

## 4 planet system with an external perturber



## N inner planets + 2 Cold Jupiters



$$
\begin{gathered}
0.1-0.5 \mathrm{AU} \\
\sim 3 \mathrm{M}_{\text {Earth }}
\end{gathered}
$$



$$
\begin{aligned}
& >3 \mathrm{AU} \\
& \sim 1 \mathrm{M}_{\mathrm{Jup}}
\end{aligned}
$$

## N inner planets + 2 Cold Jupiters

- Gains eccentricity
- Gains inclination
- Moves inward


$$
\begin{gathered}
0.1-0.5 \mathrm{AU} \\
\sim 3 \mathrm{M}_{\text {Earth }}
\end{gathered}
$$



## What happen to inner planets during outer "violence"?

- Previous works have considered examples using N-body simulations (e.g. Carrera, Davies+16, Gratia \& Fabrycky 17; Huang, Petrovich+16; Mustill, Davies+17)
- Issues of time-scales: Inner planets have $P$ ~ 10 days, outer planets have $P>5$ yrs; giant planet ejection timescale up to $\mathrm{t}_{\mathrm{ej}} \sim 10^{9} \mathrm{yrs}$, or $\mathrm{N}_{\mathrm{ej}} \sim 10^{8}$ outer orbits
$\rightarrow$ Difficult to gain understanding with only a smattering of N -body simulations


## Hybrid N-Body-Secular Method

Pu \& DL 2018 (in prep)


## Giant planet scattering is a stochastic process...




Time evolution of $a, e, i$ resemble a random walk during scattering

This drives a corresponding random walk in $e$ and $i$ in the inner planets over secular timescales

## Giant planet scatterings:

Simulations + Statistical Theory (random walk in energy)

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## Simulations + Statistical Theory (random walk in energy)

- Planets can wander inward $\rightarrow$ may directly "destroy" the inner system


Pu \& Lai 2018
(in prep)

## Giant planet scatterings:

Simulations + Statistical Theory (random walk in energy)

- Can have a wide range of ejection times...


$$
\begin{aligned}
& y \equiv \log N_{e j}, \quad \mathrm{~b} \equiv \frac{E_{2, i}}{\sqrt{\langle\delta E\rangle^{2}}} \\
& f(y)=\frac{b}{\sqrt{2 \pi}} e^{-y / 2} \exp \left(-\frac{1}{2} b^{2} e^{-y}\right) \\
& \left\langle N_{e j}\right\rangle=e^{\langle y\rangle} \simeq 3.56 b^{2} \\
& \sigma_{y} \equiv\left(\left\langle y^{2}\right\rangle-\langle y\rangle^{2}\right)^{1 / 2}=\pi / \sqrt{2} \approx 2.22
\end{aligned}
$$

Pu \& Lai 2018 (in prep)

## "1 + 2" Problem (one planet + two unstable giants)






## Expected Results for "N+2" Planet Systems Depends crucially on $\mathrm{N}_{\mathrm{ej}}$

- Dynamical evolution of the inner planets determined by the ratio $N_{e j} / N_{\text {sec }}$
- For $N_{e j} / N_{\text {sec }} \gtrsim 1$, the eccentricity, inclination and mutual inclination scale protionally with $\sqrt{N_{e j}}$.
- Scattering of CJs is highly destructive to inner systems. Inner SEs with semi-major axis within factor of $\sim 6$ are usually destroyed.
- Inner SEs that are farther away can have their eccentricities and inclinations excited beyond their naïve secular values, but multi-planet inner systems offer protection against this excitation.


## How external companion affects stellar obliquity?



Spin-orbit dynamics depend strongly on

$$
\epsilon_{\star 1} \equiv \frac{\omega_{1 p}-\omega_{\star p}}{\omega_{\star 1}+\omega_{1 \star}}
$$




Lai, Anderson \& Pu (2018)
See Boue \& Fabrycky (2014) for general cases

## Resonant Excitation of stellar obliquity as star spins down

Kassandra Anderson \& Lai 2018


## Summary

- Orbital architecture of super-Earths can be strongly affected by Cold Jupiters. CJs can excite eccentricities and mutual inclinations, and may destroy SEs.
- "Simple" analytical/scaling relations for " $\mathrm{N}+1$ " systems
- " $\mathrm{N}+2$ " systems are subtle: Result depends on ejection timescale... (statistical theory for 2-planet scatterings)
- Stellar obliquities are also affected...

References:

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## Thanks

