Dynamical Evolution of Compact Multi-Planet Systems: Effects of External Giant Planets

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Kepler: 4700 planets in 3600 systems
(mostly super-earths or sub-neptunes, <200 days)

Observed Transit Multiplicity Distribution $F(N_{\text{tran}})$

Number of Transiting planets

Number of systems
An Emerging Trend:

**Systems with smaller number of planets are dynamically hotter: higher eccentricities and mutual inclinations**  (Xie+16; Zhu+18)

Related Trends:
--- **Excess of Kepler Singles – Kepler dichotomy (?)**
   Models with single mutual inclination dispersion fall short to explain the observed number of Kepler singles (Lissauer+11; Johansen+12; Ballard & Johnson 16)

--- **Kepler Singles have higher stellar obliquities**  (Morton & Winn 14)
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Super-Earths and Cold Jupiter Connection

30-50% of inner super-Earths (P<200 d) have CJ companions (P>400 d);
These CJs are often eccentric, having undergone scatterings
(observations from many groups... see Lai & Pu 2017; Zhu & Wu 2018 for refs)
How Cold Jupiters (and scatterings) affect Inner planet system (& stellar obliquity)?

With Bonan (Michael) Pu
Kassandra Anderson

References:

Lai & Pu 2017 “Hiding Planets behind a Big Friends...”
Pu & Lai 2018 “Eccentricities and Inclinations of multiplanet systems....”
Pu & Lai 2018 “Scatterings of Cold Jupiters and its Influence on Inner Systems” (in prep)
Lai, Anderson & Pu 2018 “How do companions affect spin-orbit misalignments?”
Anderson & Lai 2018 “Teetering stars: Resonant excitation of stellar obliquities...”
N inner planets + one perturber (CJ)

**Perturber:** CJ on inclined and eccentric orbit

→ excites mutual inclinations and eccentricities in the inner system

**Numerical Simulations**

**Analytical theory/scalings...**

Lai & Pu 2017
Hansen 2017
Becker & Adams 2017
Read et al. 2017
Jontof-Hutter et al 2017
Pu & Lai 2018
Denham et al. 2018
Precession of $L_1$ around $L_p$:

$$\Omega_{1p} \sim \frac{m_p}{M_*} \left( \frac{a_1}{a_p} \right)^3 \frac{m_p}{a_3} a_1^{3/2}$$

Precession of $L_2$ around $L_p$:

$$\Omega_{2p} \propto \frac{m_p}{a_3^3} a_2^{3/2}$$
$L_1$ and $L_2$ precess around each other at rate $\omega_{12}, \omega_{21}$
Mutual inclination induced by perturber depends on **Coupling Parameter**

\[ \epsilon_{12} = \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \sim \frac{m_p}{m_{1,2}} \left( \frac{a_{1,2}}{a_p} \right)^3 \]
Maximum Mutual Inclination Induced by external perturber

\[ \epsilon_{12} = \frac{\Omega_{2p} - \Omega_{1p}}{\omega_{12} + \omega_{21}} \approx \frac{m_p}{m_{1,2}} \left( \frac{a_{1,2}}{a_p} \right)^3 \]

\[ (\theta_{12})_{\text{max}} \approx \epsilon_{12} \sin 2\theta_p \]
**Resonance Feature:**  \( \epsilon_{12} \sim 1 \)
exists when \( m_2 \gtrsim m_1 \)

Nodal Precession Resonance

In the \( m_2 \gg m_1 \) limit:

Resonance occurs at \( \Omega_{2p} = \Omega_{1p} + \omega_{12} \) or \( \epsilon_{12} = 1 \)
Resonance Feature: \( \epsilon_{12} \sim 1 \)

Can produce much larger mutual inclination than \( \theta_p \).
Eccentricity Excitation by External Perturber

Apsidal precession resonance

Pu & DL 2018
4 planet system with an external perturber

\[
\sigma_\theta [\text{Deg}] = \frac{\sin^2(2\theta_p) + \sin^4(\theta_p)}{\sqrt{2}}^{1/2} \quad \epsilon_3d = 1
\]

\[
\bar{\epsilon} = \epsilon_1d = 1 \quad \epsilon_2d = 1
\]

\[
\text{averaged coupling parameter } \propto m_p/a_p^3
\]

DL & Pu 2017
N inner planets + 2 Cold Jupiters

$0.1 - 0.5 \text{ AU}$
$\sim 3 \, M_{\text{Earth}}$

$> 3 \text{ AU}$
$\sim 1 \, M_{\text{Jup}}$
N inner planets + 2 Cold Jupiters

- Gains eccentricity
- Gains inclination
- Moves inward

$m_a$, $m_b$, $m_1$, $m_2$

$0.1 - 0.5$ AU
$\sim 3 \, M_{\text{Earth}}$

M$_2$ ejected
What happen to inner planets during outer “violence”?

• Previous works have considered examples using N-body simulations (e.g. Carrera, Davies+16, Gratia & Fabrycky 17; Huang, Petrovich+16; Mustill, Davies+17)

• Issues of time-scales: Inner planets have $P \sim 10$ days, outer planets have $P > 5$ yrs; giant planet ejection timescale up to $t_{ej} \sim 10^9$ yrs, or $N_{ej} \sim 10^8$

outer orbits

➡ Difficult to gain understanding with only a smattering of N-body simulations
Hybrid N-Body-Secular Method

Pu & DL 2018 (in prep)

0.1 – 0.5 AU
\sim 3 \, M_{\text{Earth}}

> 3 \, \text{AU}
\sim 1 \, M_{\text{Jup}}
Giant planet scattering is a stochastic process...

Time evolution of $a$, $e$, $i$ resemble a random walk during scattering

This drives a corresponding random walk in $e$ and $i$ in the inner planets over secular timescales.
Giant planet scatterings:
Simulations + Statistical Theory (random walk in energy)
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- Planets can wander inward ➔ may directly “destroy” the inner system

Pu & Lai 2018 (in prep)
Giant planet scatterings:
Simulations + Statistical Theory (random walk in energy)

- Can have a wide range of ejection times...

\[ y \equiv \log N_{ej}, \quad b \equiv \frac{E_{2,i}}{\sqrt{\langle \delta E \rangle^2}} \]

\[ f(y) = \frac{b}{\sqrt{2\pi}} e^{-y/2} \exp \left( -\frac{1}{2} b^2 e^{-y} \right) \]

\[ \langle N_{ej} \rangle = e^{\langle y \rangle} \approx 3.56b^2. \]

\[ \sigma_y \equiv (\langle y^2 \rangle - \langle y \rangle^2)^{1/2} = \frac{\pi}{\sqrt{2}} \approx 2.22 \]

Pu & Lai 2018 (in prep)
“1 + 2” Problem (one planet + two unstable giants)
Expected Results for “N+2” Planet Systems

Depends crucially on $N_{ej}$

- Dynamical evolution of the inner planets determined by the ratio $N_{ej}/N_{sec}$

- For $N_{ej}/N_{sec} \gtrsim 1$, the eccentricity, inclination and mutual inclination scale proportionally with $\sqrt{N_{ej}}$.

- Scattering of CJs is highly destructive to inner systems. Inner SEs with semi-major axis within factor of $\sim 6$ are usually destroyed.

- Inner SEs that are farther away can have their eccentricities and inclinations excited beyond their naïve secular values, but multi-planet inner systems offer protection against this excitation.

Pu & Lai 2018, 2019 (in prep)
How external companion affects stellar obliquity?
Spin-orbit dynamics depend strongly on

$$\epsilon_{\star 1} \equiv \frac{\omega_{1p} - \omega_{\star p}}{\omega_{\star 1} + \omega_{1\star}}$$
See Boue & Fabrycky (2014) for general cases.
Resonant Excitation of stellar obliquity as star spins down

Kassandra Anderson & Lai 2018
Summary

• Orbital architecture of super-Earths can be strongly affected by Cold Jupiters. CJs can excite eccentricities and mutual inclinations, and may destroy SEs.

• “Simple” analytical/scaling relations for “N + 1” systems

• “N + 2” systems are subtle: Result depends on ejection timescale...
  (statistical theory for 2-planet scatterings)

• Stellar obliquities are also affected...

References:

Lai & Pu 2017 “Hiding Planets behind a Big Friends...”
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Thanks