Exciting Waves/Modes in Black-Hole Accretion Disks and other Rotating Astrophysical Flows

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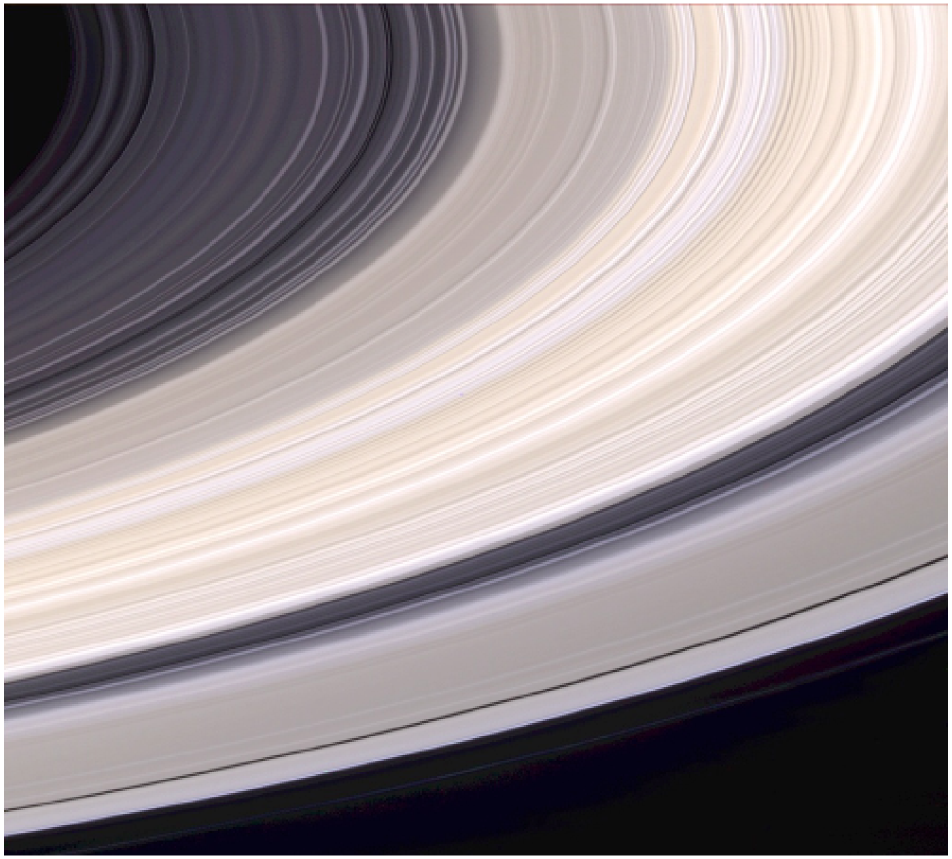
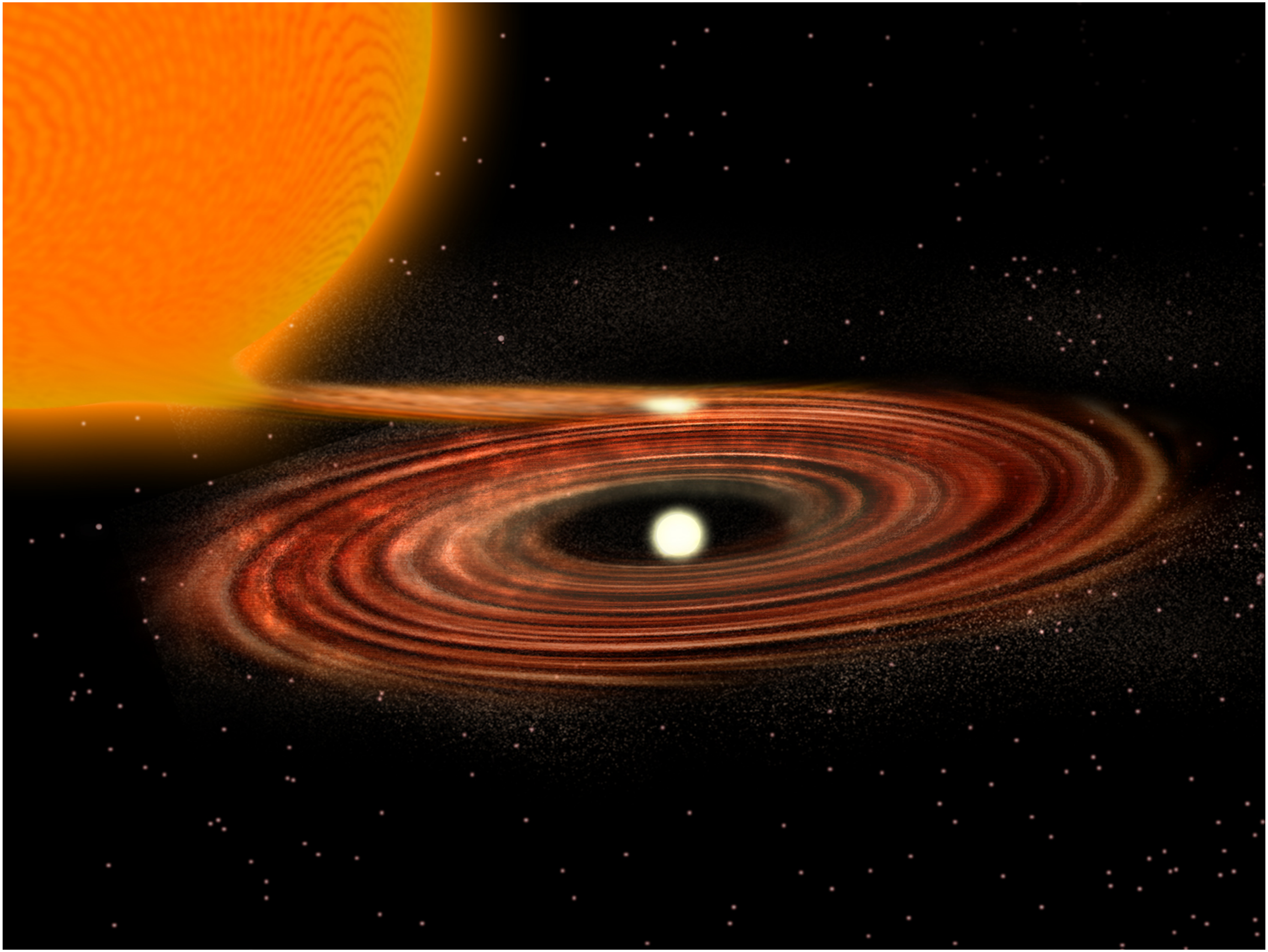
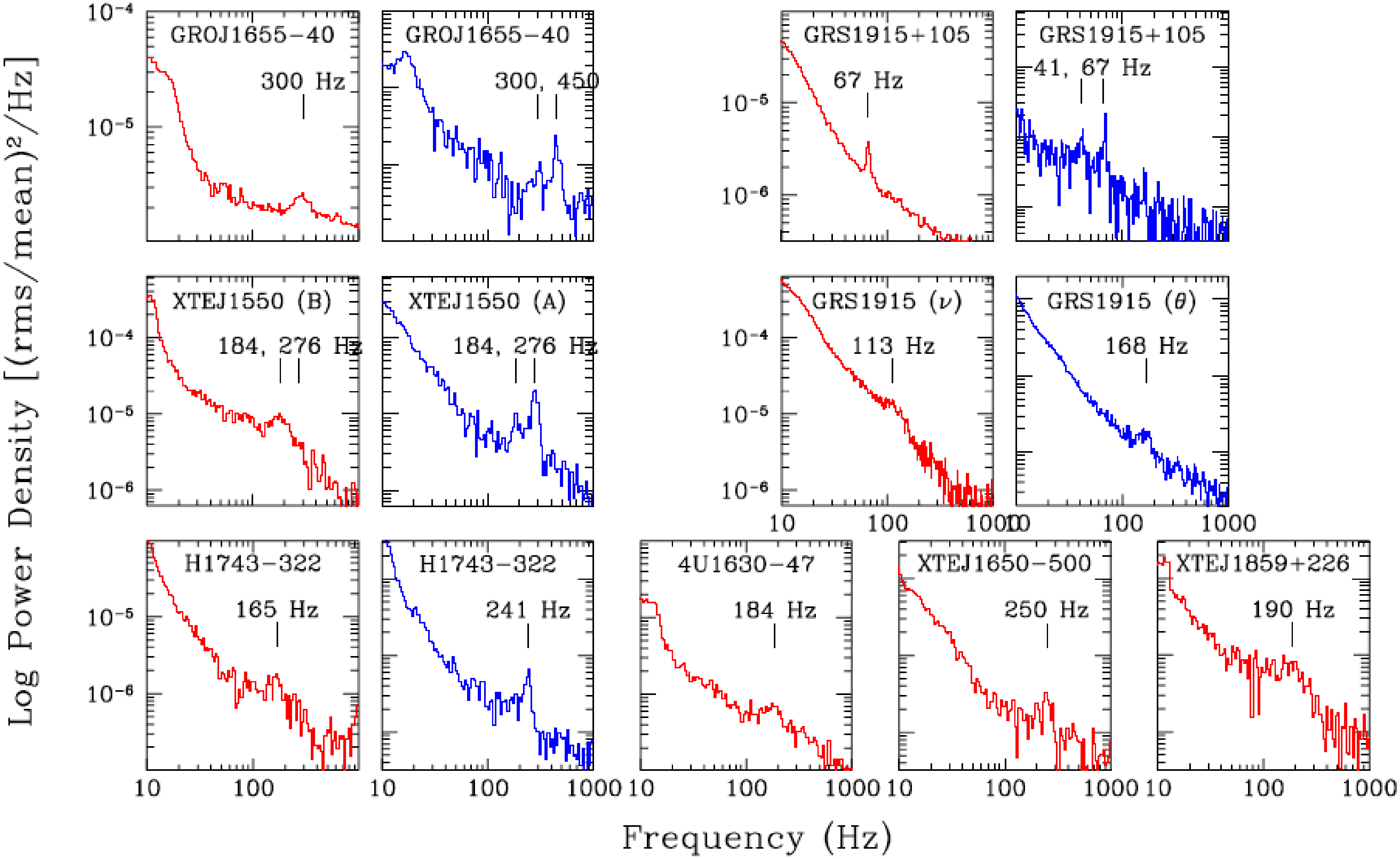


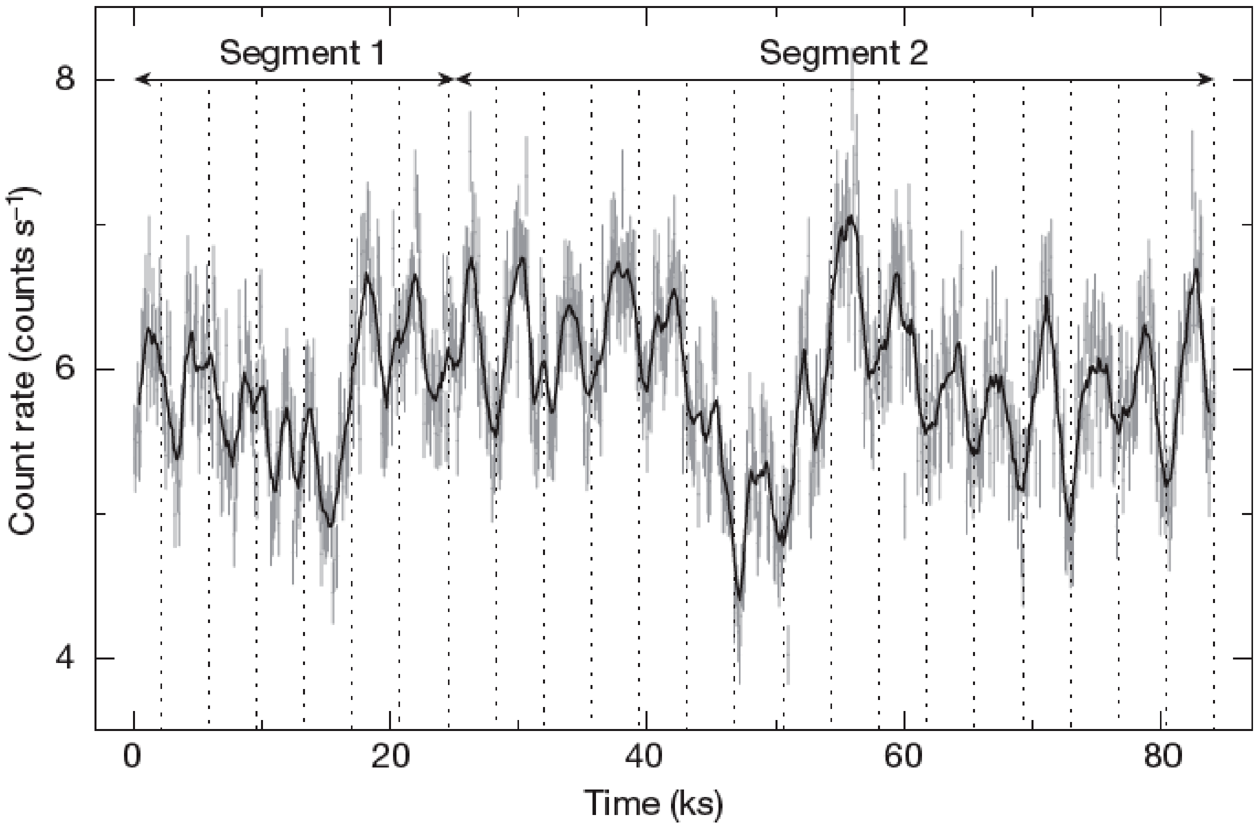


High-Frequency QPOs in BH X-Ray Binaries



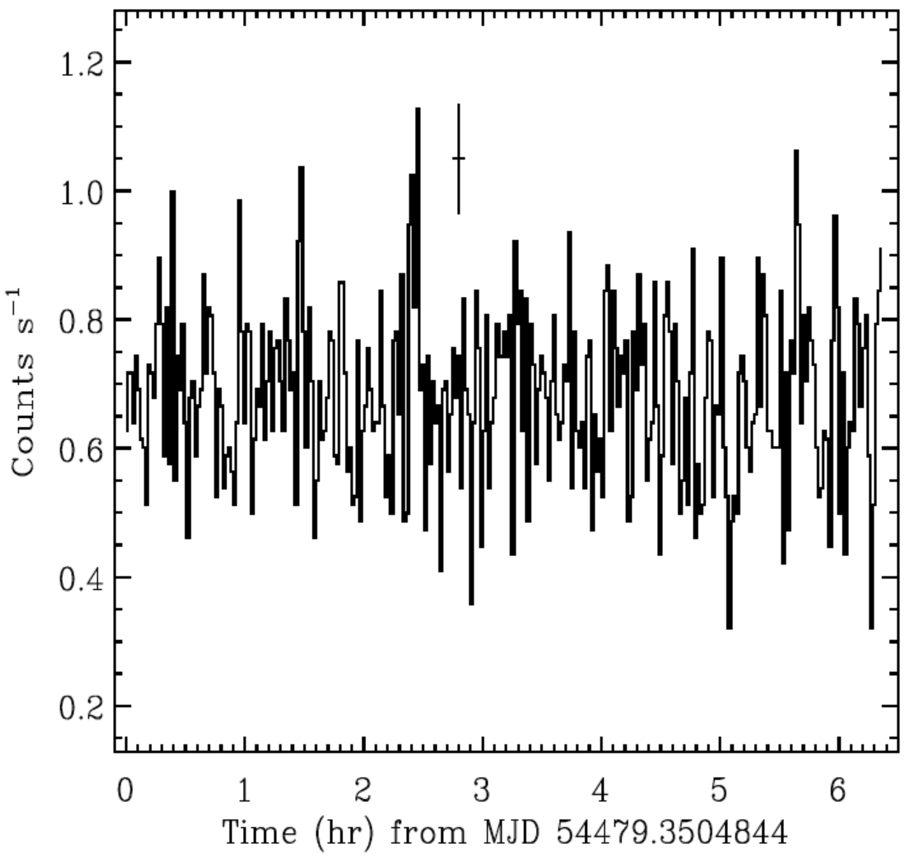
Remillard & McClintock 2006

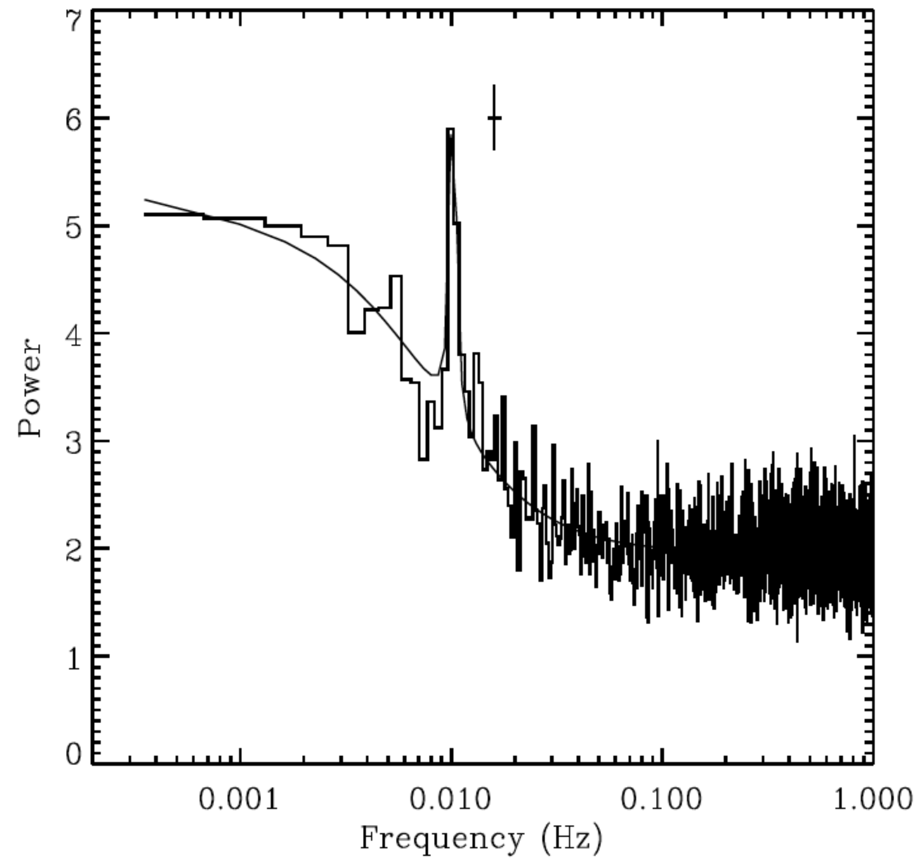
X-ray QPO (P ~ 1 hr) from active galaxy RE J1034+396



Gierlinski et al 2008, Nature

QPOs from Ultra-Luminous X-ray Source NGC 5408 X-1 (an Intermediate-mass BH?)





Strohmayer & Mushotzky 2009

Global wave modes and instabilities in accretion disks

Super-reflection

Role of corotation resonance

Trapped modes (p-modes): stability vs overstability

GR effects

Magnetic field effects

Other diskoseismic modes in BH accretion disks

QPOs in black-hole X-ray binaries

Other rotating astrophysical flows

with David Tsang (Cornell Ph.D. 09 --> Caltech)
Wen Fu (Cornell Ph.D. student)

Free waves in 2D disks (Spiral density waves):

$$\delta v, \delta P \propto \exp(ik_r r + im\varphi - i\omega t)$$

Dispersion relation:

$$\tilde{\omega}^2 = \kappa^2 + k_r^2 c_s^2$$

$$\tilde{\omega} \equiv \omega - m\Omega$$

$$\kappa^2 = \frac{2\Omega}{r}\frac{d}{dr}(r^2\Omega)$$

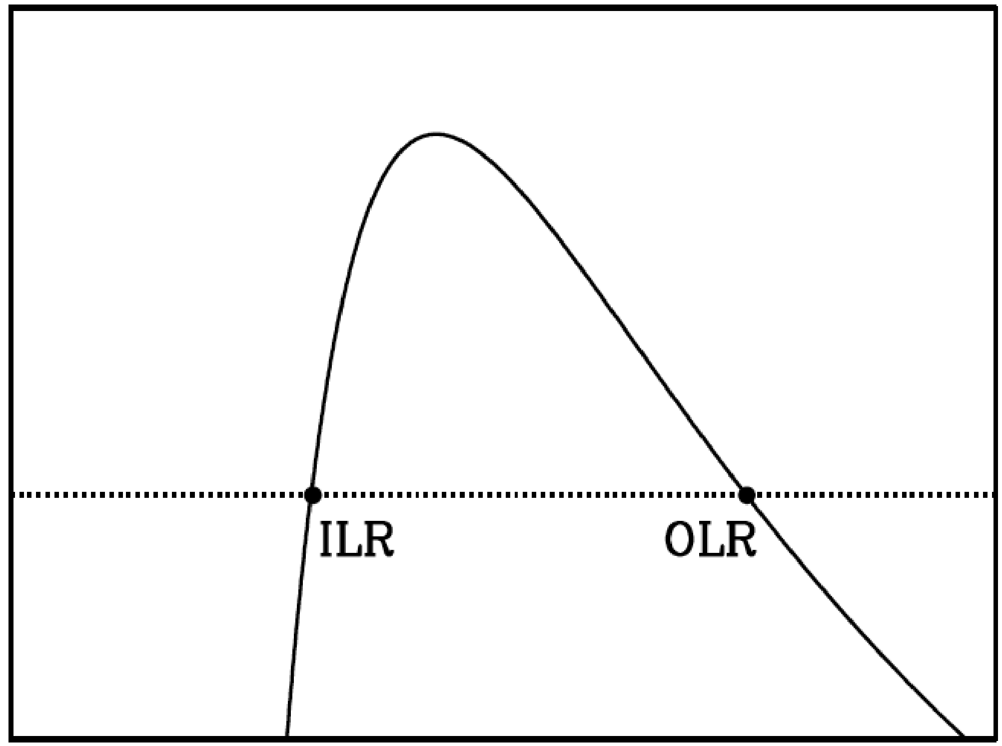
Lindblad Resonance: $\tilde{\omega} = \pm\kappa$

Wave can propagates in the region:

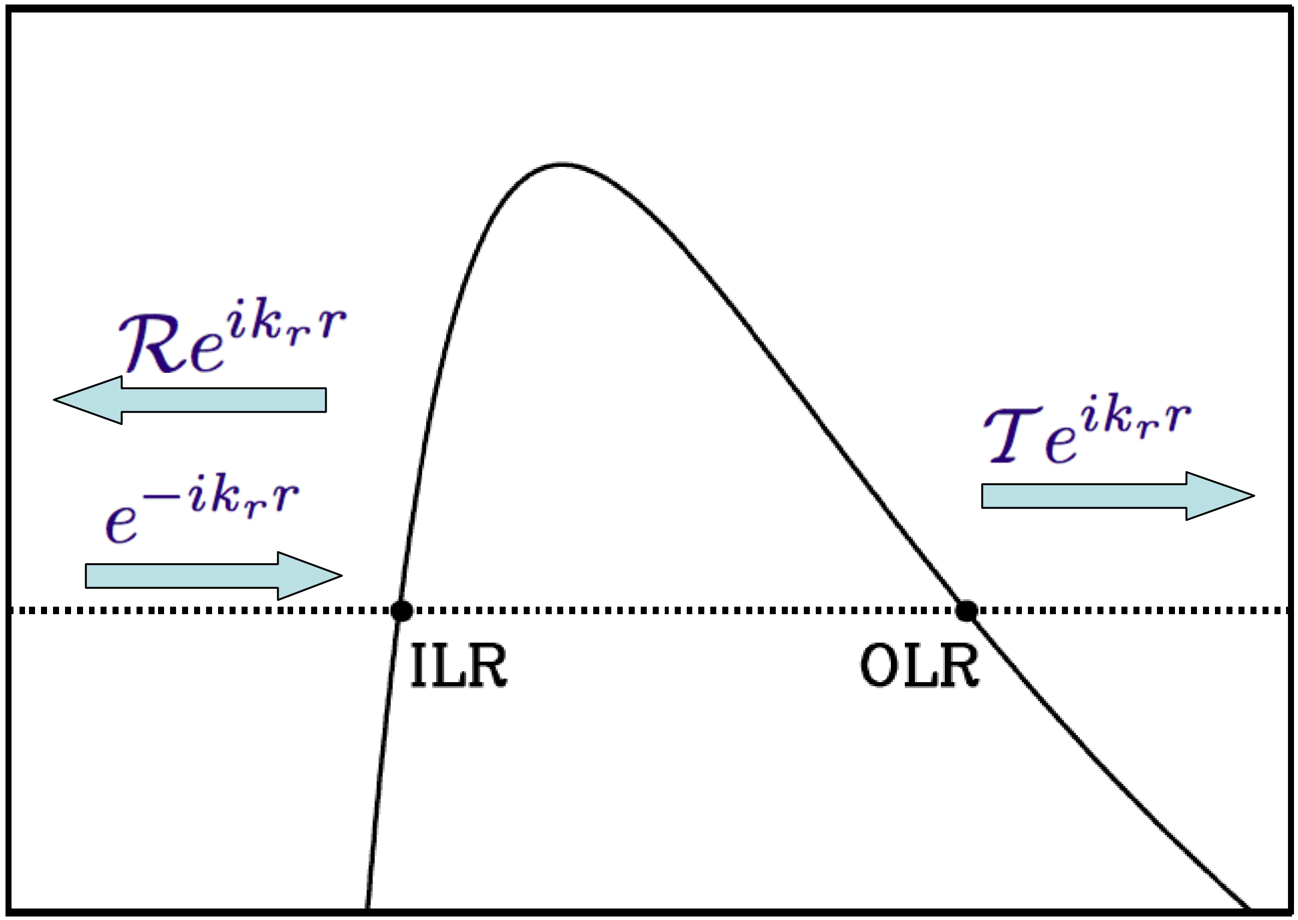
$$r < r_{\rm ILR} \quad \text{or} \quad r > r_{\rm OLR}$$

Effective potential

$$V_{\text{eff}}(r) = \frac{\kappa^2 - \tilde{\omega}^2}{c_s^2}$$



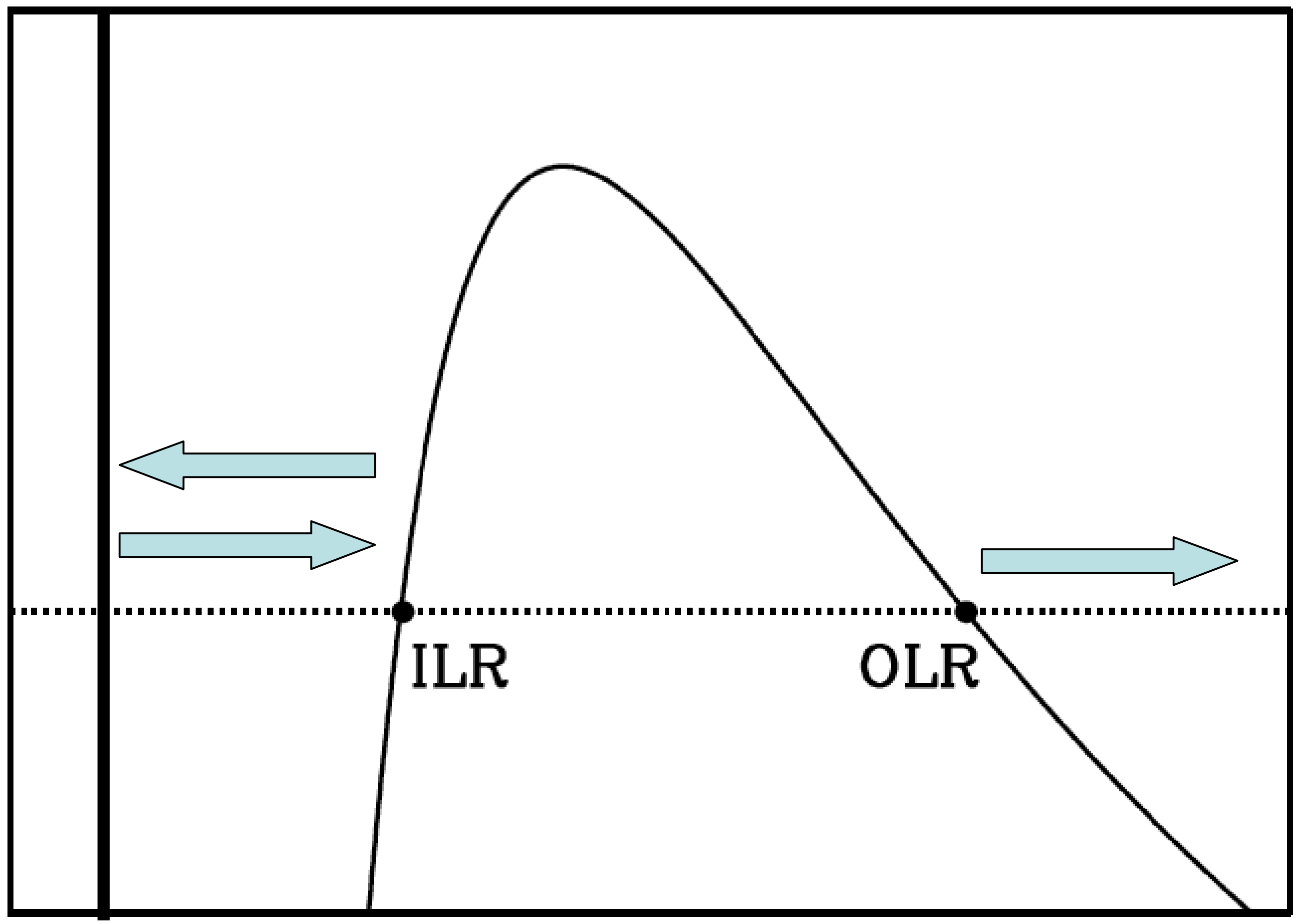
$$\text{wave at } r > r_{\text{OLR}} : \quad \omega/m > \Omega \;\Rightarrow\; \text{positive energy}$$
$$\text{wave at } r < r_{\text{ILR}} : \quad \omega/m < \Omega \;\Rightarrow\; \text{negative energy}$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2$$

$$\Rightarrow \ |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 > 1$$

Super-reflection



Trapped mode between $r_{\rm in}$ and $r_{\rm ILR}$: overstable

This is not the whole story:

So far we have neglected the corotation resonance, where

$$\omega/m = \Omega$$

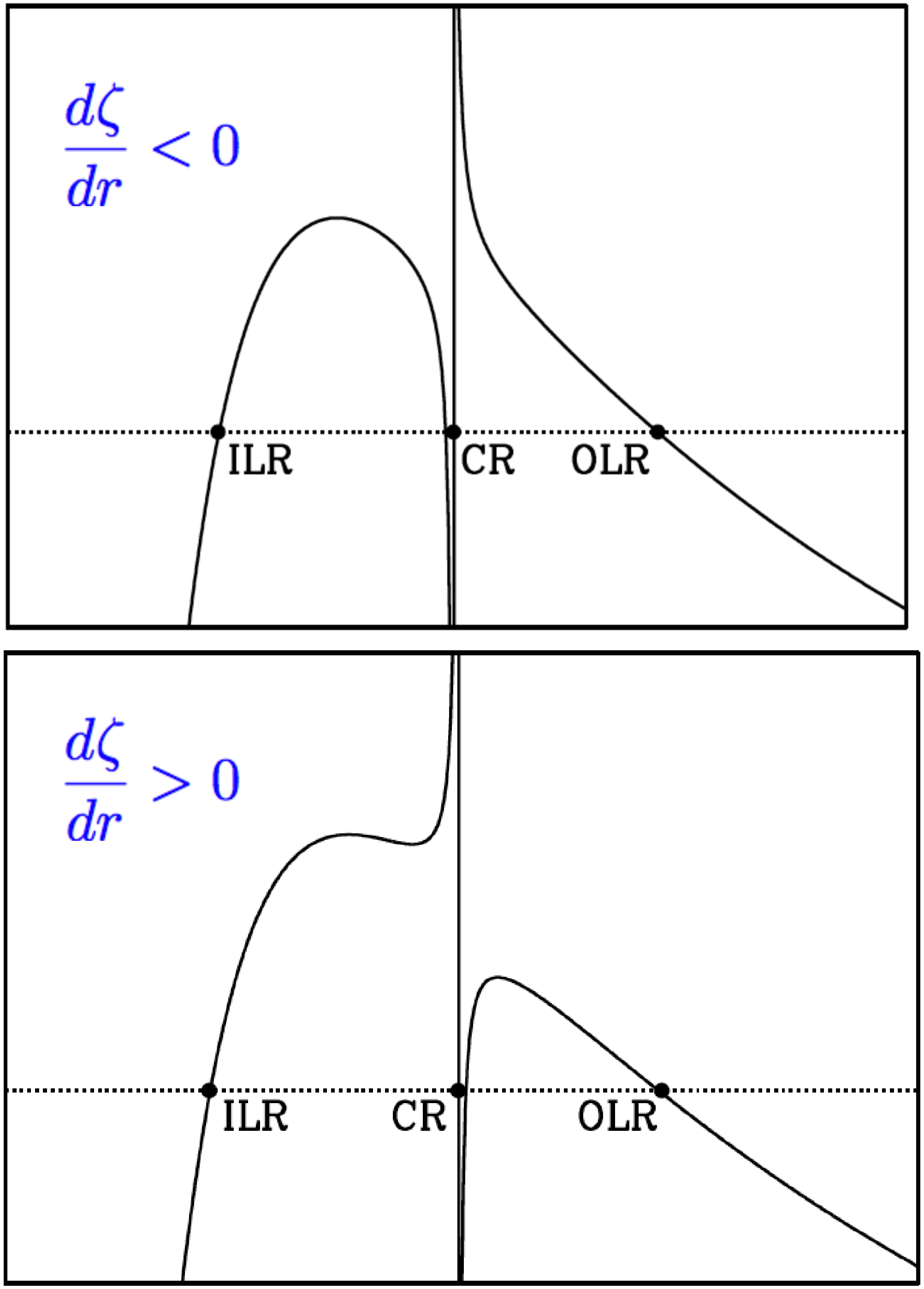
$$\text{or} \quad \tilde{\omega} = \omega - m\Omega = 0$$

Fluid equations for $\delta P, \delta\Sigma, \delta\mathbf{v} \propto \exp(im\phi - i\omega t)$
reduce to (for barotropic flow):

$$\left[\frac{d^2}{dr^2} - V_{\rm eff}(r)\right]\eta(r) = 0, \qquad \eta \propto \frac{\delta P}{\Sigma}$$

$$V_{\rm eff}(r) \simeq \frac{\kappa^2 - \tilde{\omega}^2}{c_s^2} + \frac{m^2}{r^2} - \frac{2m\Omega}{r\tilde{\omega}}\left(\frac{d}{dr}\ln\zeta\right)$$

$$\zeta \equiv \frac{\kappa^2}{2\Omega\Sigma} = \frac{(\nabla \times \mathbf{v_0})\cdot\hat{z}}{\Sigma}$$ (vortensity)



$$V_{\text{eff}}(r) \simeq \frac{\kappa^2 - \tilde{\omega}^2}{c_s^2} + \frac{m^2}{r^2} - \frac{2m\Omega}{r\tilde{\omega}}\left(\frac{d}{dr}\ln\zeta\right)$$

$$\zeta = \frac{\kappa^2}{2\Omega\Sigma}$$

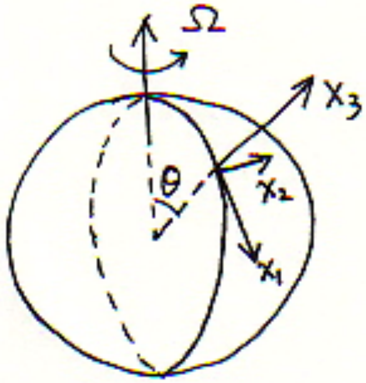
Local dispersion relation:

$$-k_r^2 = V_{\text{eff}}(r)$$

Corotation resonance/singularity:

$$\tilde{\omega} = \omega - m\Omega = 0$$

Rossby Waves in Planetary atmosphere



$$v_3 \ll |\vec{v}_h|$$

$$\frac{\partial}{\partial t}\vec{v}_h = -\frac{1}{\rho}\nabla_h P - 2\left(\vec{\Omega}\times\vec{v}\right)_h$$

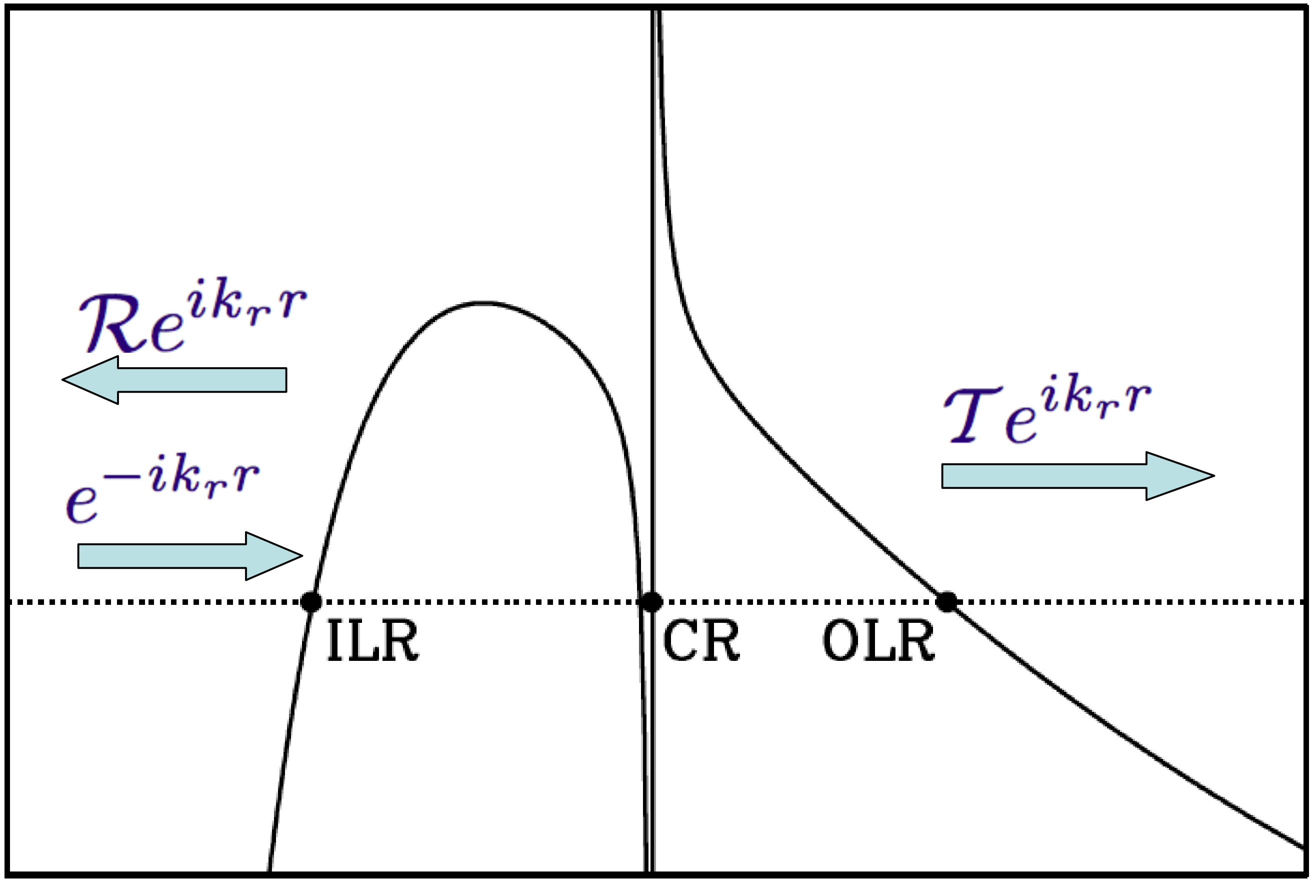
$$\Rightarrow \begin{cases} \frac{\partial}{\partial t}\delta v_1 = -\frac{1}{\rho}\frac{\partial}{\partial x_1}\delta P + 2\Omega_3\,\delta v_2 \\ \frac{\partial}{\partial t}\delta v_2 = -\frac{1}{\rho}\frac{\partial}{\partial x_2}\delta P - 2\Omega_3\,\delta v_1 \end{cases}$$

where $\Omega_3 \equiv \Omega\cos\theta$

$$\nabla_h \cdot \delta\vec{v}_h = 0$$

for $\delta\vec{v}_h \propto e^{ik_1x_1 + ik_2x_2 - i\omega t}$

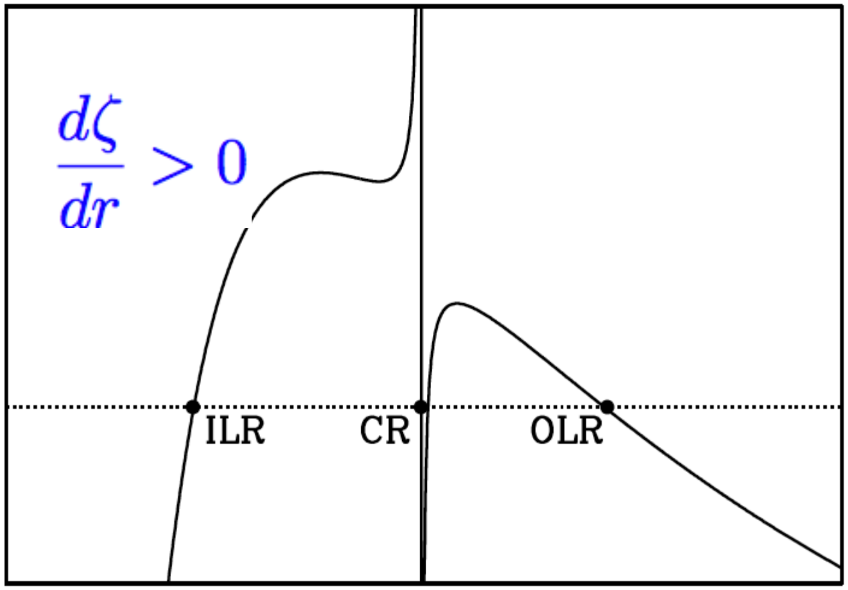
$$\Rightarrow \quad \omega = \frac{2k_2}{k_1^2+k_2^2}\left(\frac{\partial\Omega_3}{\partial x_1}\right)$$



$$(-1) = (-1)|\mathcal{R}|^2 + |\mathcal{T}|^2 + \mathcal{D}_c$$

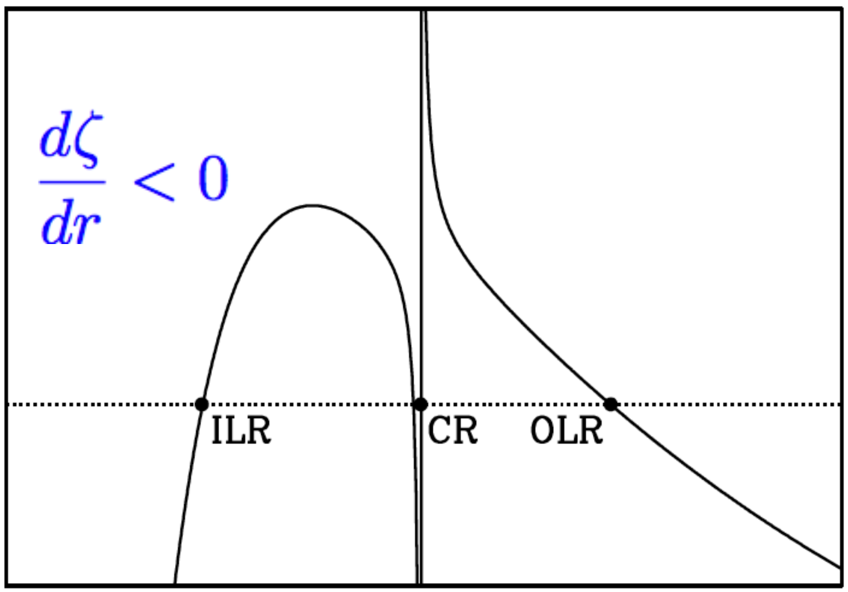
Wave absorption at corotation

$$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c$$



$\Rightarrow \mathcal{D}_c > 0$

$\Rightarrow |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c > 1$

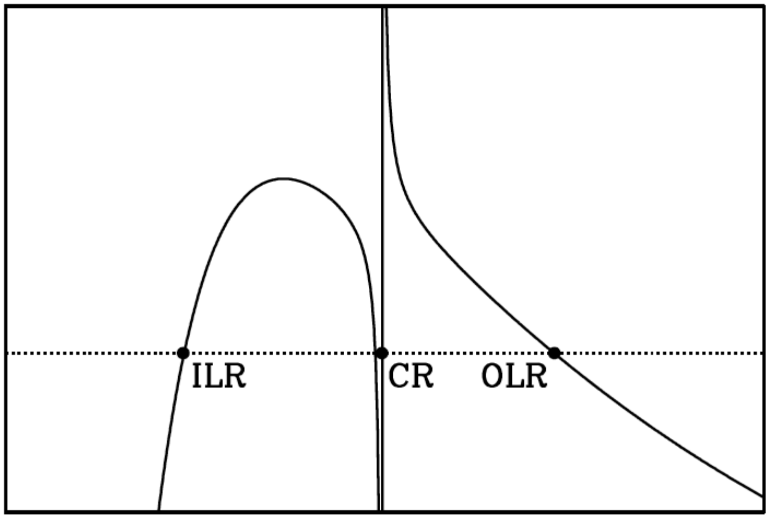


$\Rightarrow \mathcal{D}_c < 0$

WKB calculations of reflectivity/transmission:

$$\delta h = \sqrt{S/k}\left[\exp\left(-i\int_{r_{\rm IL}}^{r} kdr + \frac{\pi}{4}\right) + \mathcal{R}\exp\left(i\int_{r_{\rm IL}}^{r} kdr - \frac{\pi}{4}\right)\right]$$

$$\delta h = \sqrt{S/k}\,\mathcal{T}\exp\left(i\int_{r_{\rm OL}}^{r} kdr + \frac{\pi}{4}\right)$$

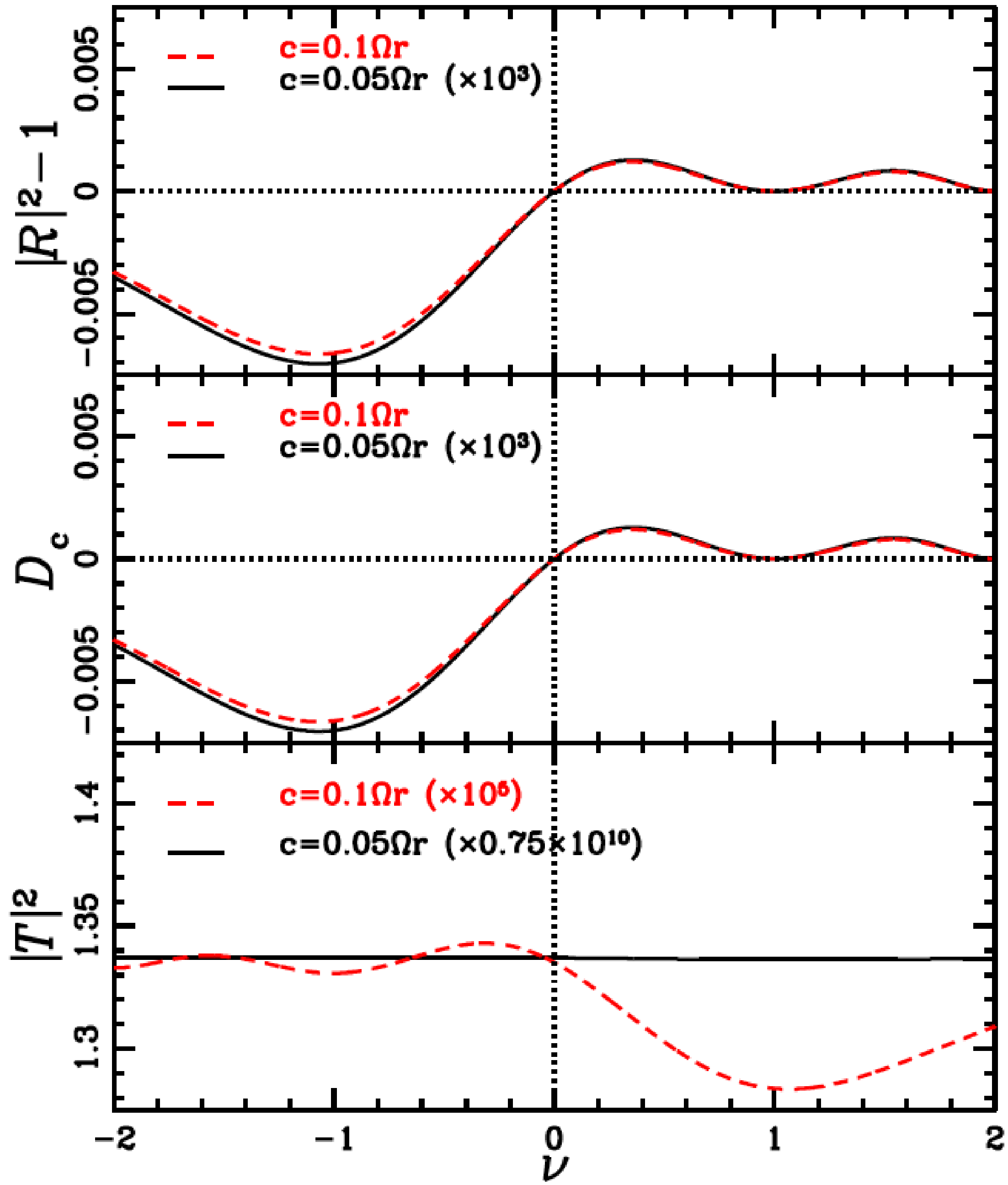


- Solve wave equation in different regions
- Match the solutions using asymptotic expansions
- Around corotation: Whittaker function; Stokes phenomenon

$$\mathcal{R} = \frac{1 + \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} + \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}{1 - \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

$$\mathcal{T} = \frac{ie^{-2\Theta_{\rm II}}e^{i\pi\nu}}{1 - \frac{1}{4}\left(e^{-i2\pi\nu} + \sin^2\pi\nu\right)e^{-2\Theta_{\rm II}} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIa}}}{(\Gamma(1-\nu))^2} - \frac{\pi\nu}{2}\frac{e^{-2\Theta_{\rm IIb}}}{(\Gamma(1+\nu))^2}}$$

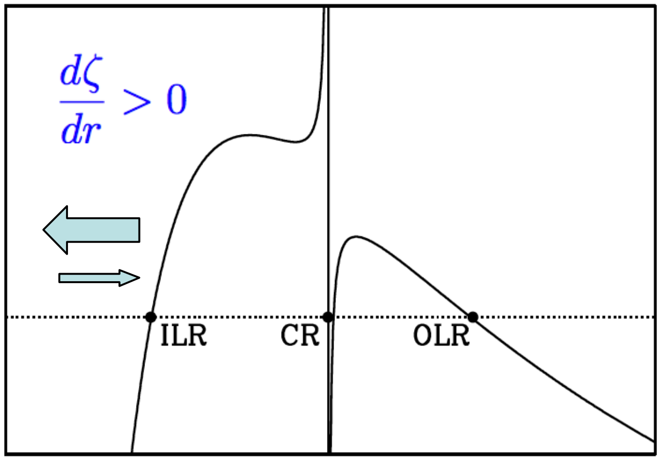
$$\Theta_{\rm IIa} = \int_{r_{\rm IL}}^{r_c} |k|\,dr \qquad \Theta_{\rm IIb} = \int_{r_c}^{r_{\rm OL}} |k|\,dr$$



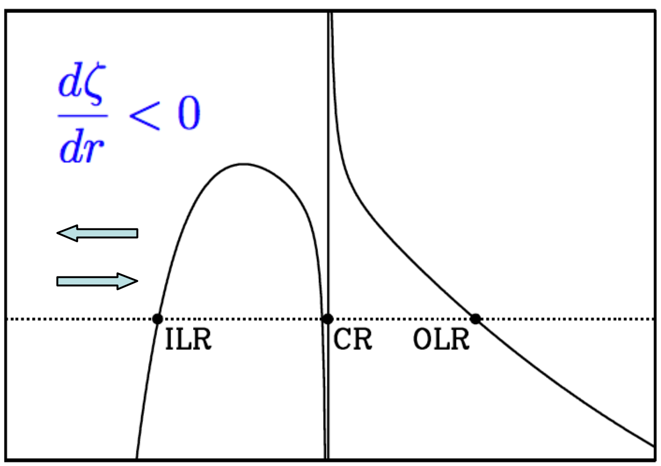
$$\nu \equiv \left(\frac{2H}{3r}\frac{d\ln\zeta}{d\ln r}\right)_{\mathrm{CR}}$$

Tsang & DL 2008

Summary: For $\dfrac{d\zeta}{dr} \neq 0: \quad |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c$

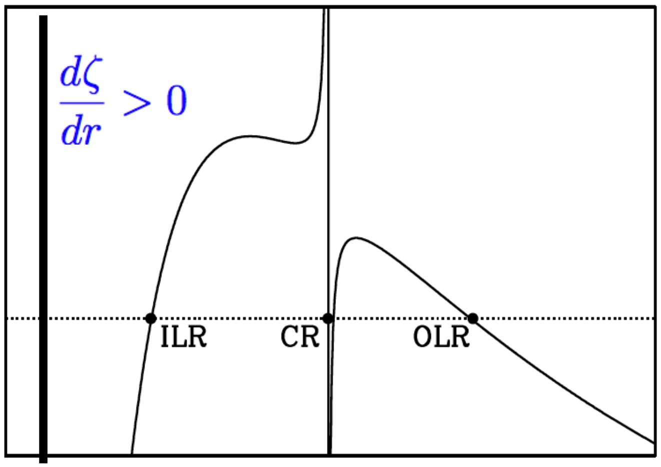


$\Rightarrow \mathcal{D}_c > 0$



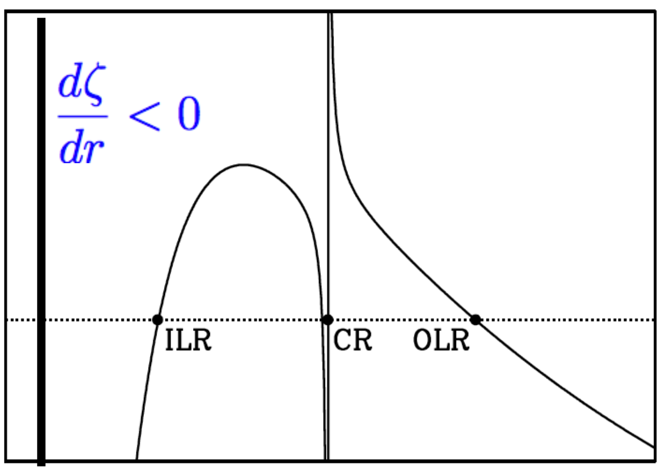
$\Rightarrow \mathcal{D}_c < 0$

Summary: For $\dfrac{d\zeta}{dr} \neq 0: \quad |\mathcal{R}|^2 = 1 + |\mathcal{T}|^2 + \mathcal{D}_c \simeq 1 + \mathcal{D}_c$



$\Rightarrow \mathcal{D}_c > 0$

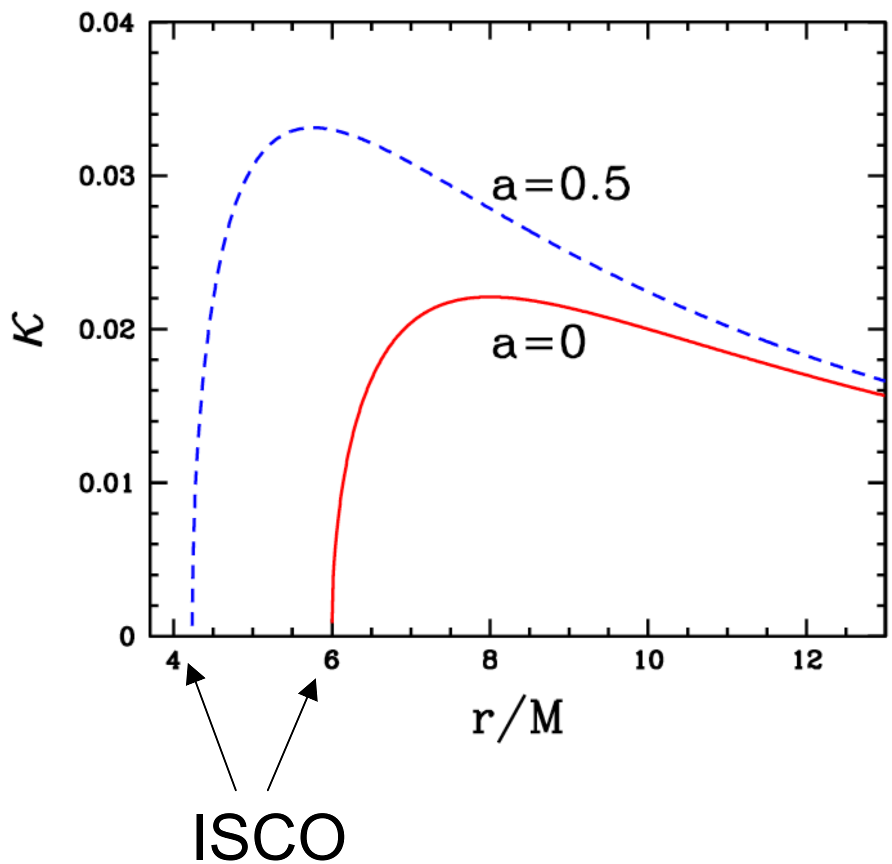
Overstable mode

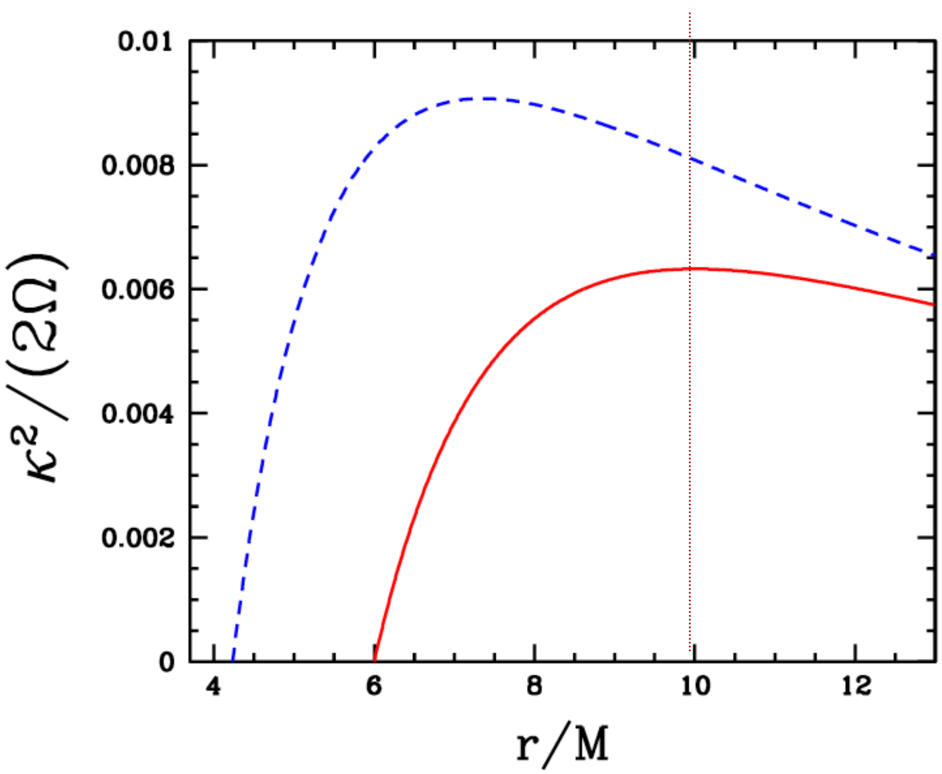


$\Rightarrow \mathcal{D}_c < 0$

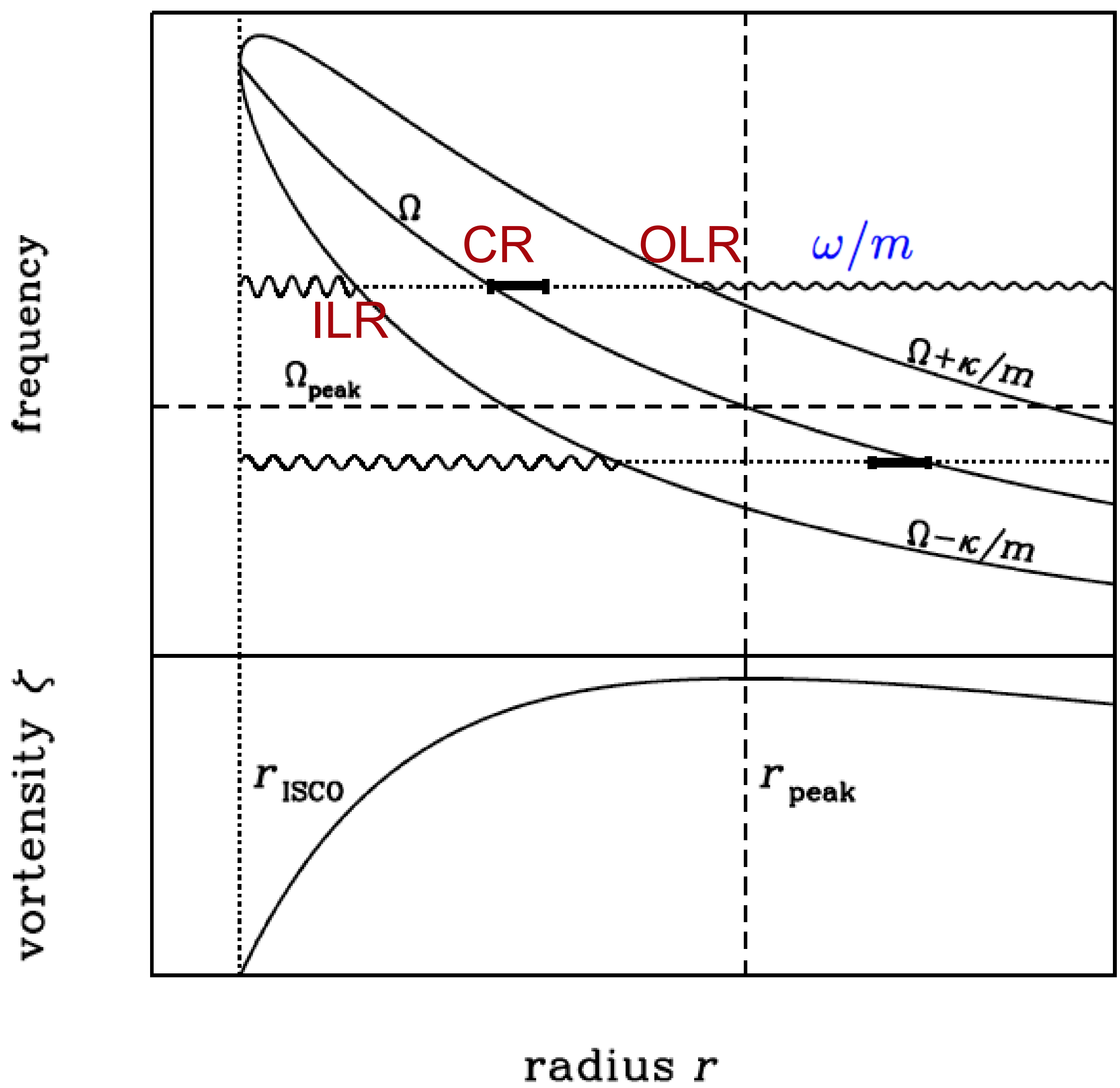
Damped mode

General Relativity Effect

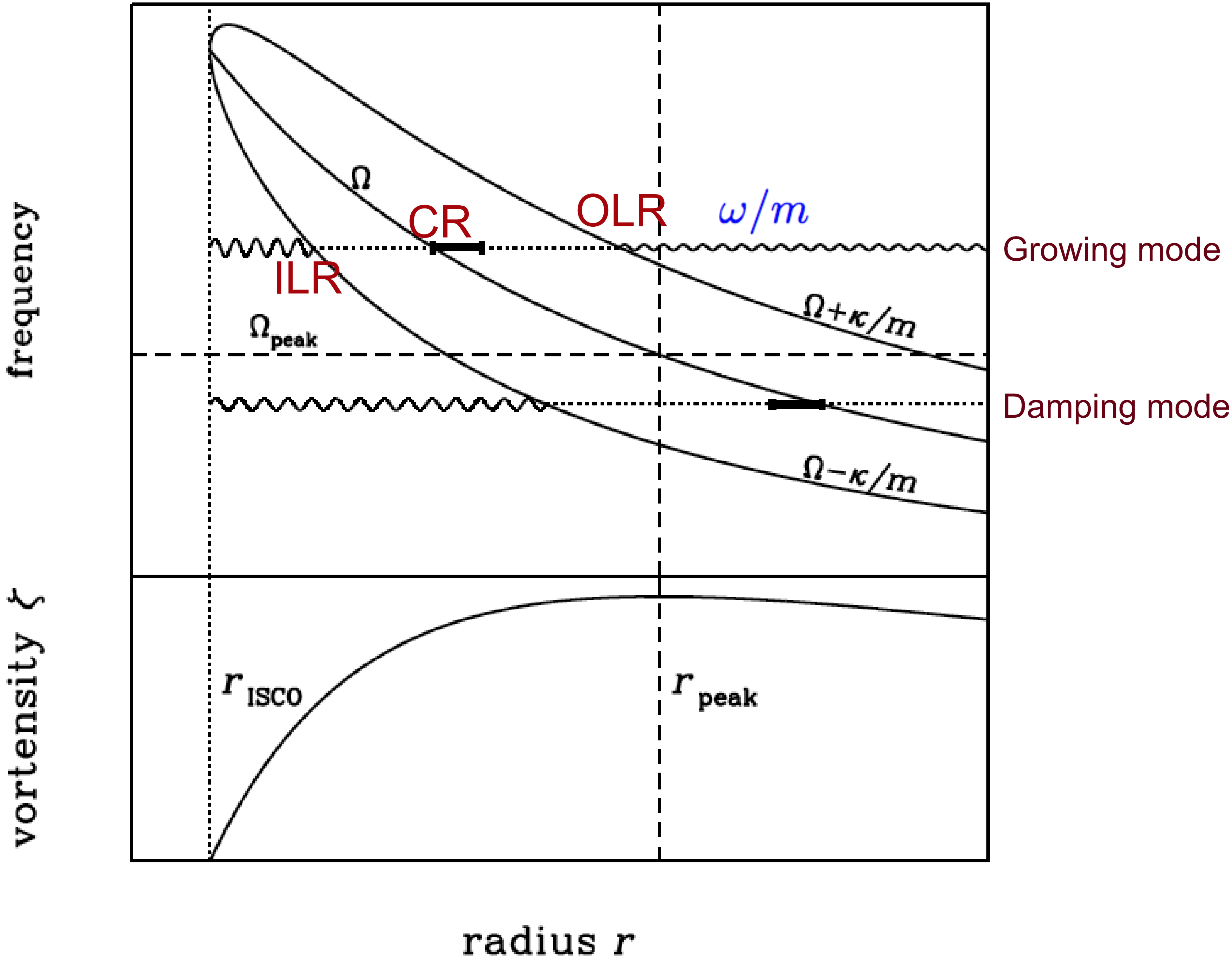




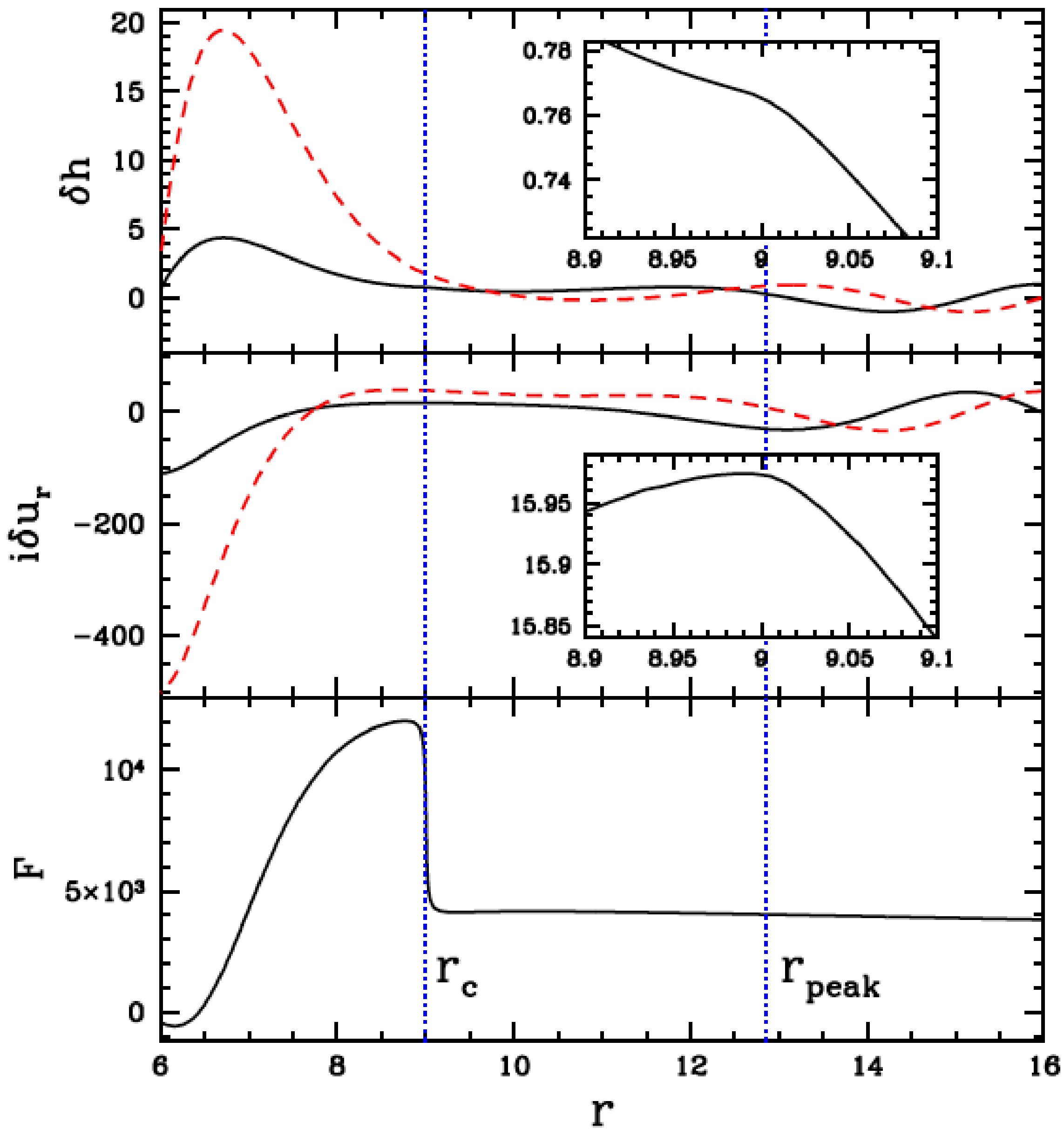
$$\zeta = \frac{\kappa^2}{2\Omega\Sigma}$$



frequency
vortensity ζ
Ω
CR
OLR
ω/m
ILR
Ω_peak
Ω+κ/m
Ω−κ/m
r_ISCO
r_peak
radius r



frequency
vortensity ζ
Ω
CR
OLR
ω/m
Growing mode
ILR
$\Omega+\kappa/m$
Ω_{peak}
Damping mode
$\Omega-\kappa/m$
r_{ISCO}
r_{peak}
radius r



$$\Sigma \propto r^{-1}, \quad c_s = 0.1 r\Omega, \quad m = 2$$
$$\omega_r = 0.93\,\Omega_{\rm ISCO}, \quad \omega_i/\omega_r = 0.0029$$

DL & Tsang 2009

Boundary Conditions at the Inner Disk Edge

- For thin disks, radial velocity goes through sonic point near $r_{ISCO.}$ (viscous slim disk solution)

We found (DL & Tsang 2009)

- Radial inflow tends to damp the mode (cf. Blaes 1987)
- The disk inner edge is partially reflective due to steep radial gradient in density and radial velocity

===> Even with radial inflow, global unstable mode is still possible under some (realistic) conditions (when the density/velocity gradient is large).

Real accretion disk is more complicated…

- Global MHD simulations of inner disks
  (still in the early of development. e.g. Gammie, Hawley, Krolik, Blaes, Fragile, etc)
- Magnetic fields may accumulate in the inner disk and inside
  (e.g. Lovelace et al 2009)
  ==> The inner disk edge may be more reflective than transonic flow
  (Toy-model example: Tsang & DL 2009b)

Implications: Growing modes (QPOs) may appear only in certain state of accreting BHs (e.g. damped in the thermal state, but appears in "steep power-law state")

A bit more theory…

So far I have assumed the disk is barotropic fluid (P is a function of density)

$$\zeta \equiv \frac{\kappa^2}{2\Omega\Sigma} \quad \text{(vortensity)}$$

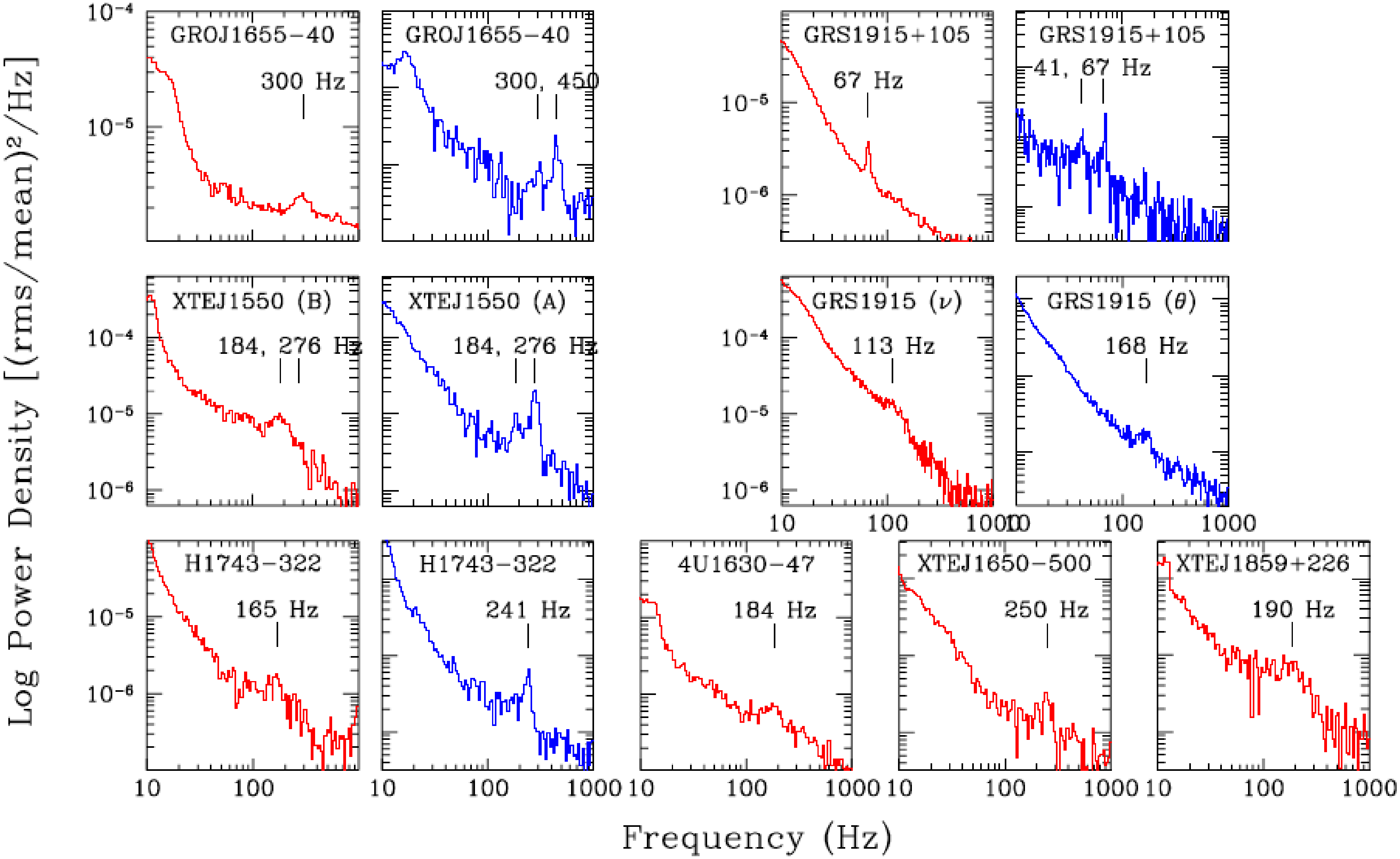
For non-barotropic disk (Tsang & DL 2009; cf. Lovelace et al.1999; Baruteau & Masset 2008)

$$\zeta_{\rm eff} \equiv \frac{\kappa^2}{2\Omega\Sigma S^{2/\Gamma}}$$

where $S \equiv P/\Sigma^{\Gamma}$ ("entropy")

For alpha-disks, this typically makes mode grow faster

High-Frequency QPOs in BH X-Ray Binaries



Remillard & McClintock 2006

Ideas/Models of High-Freq QPOs

- Orbiting blobs (hot spots) in disks (e.g., Stella et al 1999; Schnittman & Bertschinger 2004)
- Acoustic modes in torus (Rezzolla el al 2003; Lee, Abramowicz & Kluzniak 2004; Blaes et al. 2007; Sramkova et al 2007; Horak 2008)
- Nonlinear resonances of some kind (Abramowicz, Kluzniak, Horak, Rebusco)
- Disk/Magnetosphere Boundary Layer Oscillations (Li & Narayan 2004; Tsang & DL 2009c)

- Diskoseismology in relativistic disks (Kato; Wagoner & collaborators)

  -- Oscillation modes (m=0 inertial modes) excited by global disk deformation (e.g. warps) (Kato 2003,2008; Ferreira & Ogilvie 2008; Henisey et al.2009?)

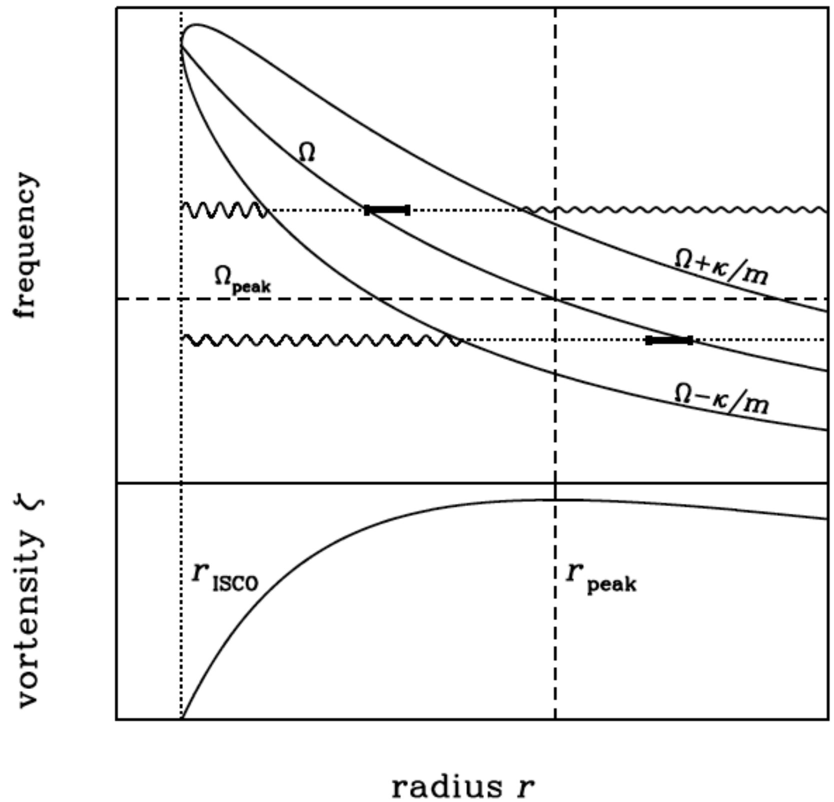
  -- Oscillation (p-) modes driven by Accretion-Ejection/Rossby wave Instabilities in magnetized disks (Varinere & Tagger 2002; Tagger & Varniere 2006 etc)

  -- Oscillation modes: Corotational effects, magnetic fields (Tsang & DL 2008,2009a,b; DL & Tsang 2009; Fu & DL 2009,2010)

Diskoseismic Modes in Black-Hole Accretion Disks

- Inertial-Acoustic Modes (P-Modes) = Global spiral density waves

= What I have been discussing so far…



- Other modes: G-modes and C-modes

High-Freq. QPOs from Overstable P-Modes
(My story so far…)

Low-order p-modes trapped between inner boundary and ILR provides a possible explanation for HFQPOs

$$\omega \simeq \beta m \Omega(r_{\rm in})$$

$\beta = 0.55\text{–}0.75$ depending on disk models and inner BC

- Mode frequency varies (~10%) as Mdot changes (~3)
- Frequency ratio close to 1:2:3:4…, but not exactly

Overstable due to corotation resonance (vortensity gradient and GR plays important role)

- Mode growth rate: m=1 much slower than m=2,3,4…
- Damping at ISCO: growth rate depends on inner BC

==> net mode growth maybe possible only in certain spectral state ("steep power-law state"?)

Effects of Magnetic Fields on P-Modes

Effects of Magnetic Fields on P-Modes

$$\frac{d\xi_r}{dr} = A_{11}\xi_r + A_{12}\delta h,$$

$$\frac{d\delta h}{dr} = A_{21}\xi_r + A_{22}\delta h,$$

where

$$A_{11} = \frac{r\tilde{\omega}^2\left[(\omega_{A\phi}^2 - \Omega^2)\tilde{\omega}^2 + \omega_{A\phi}^2\omega^2\right]}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)(\tilde{\omega}^2 - \omega_s^2)} + \frac{g\tilde{\omega}^2}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - \omega_s^2)} - \frac{\tilde{\omega}^2 + 2m\tilde{\omega}\Omega + m^2\omega_{A\phi}^2}{r(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)},$$

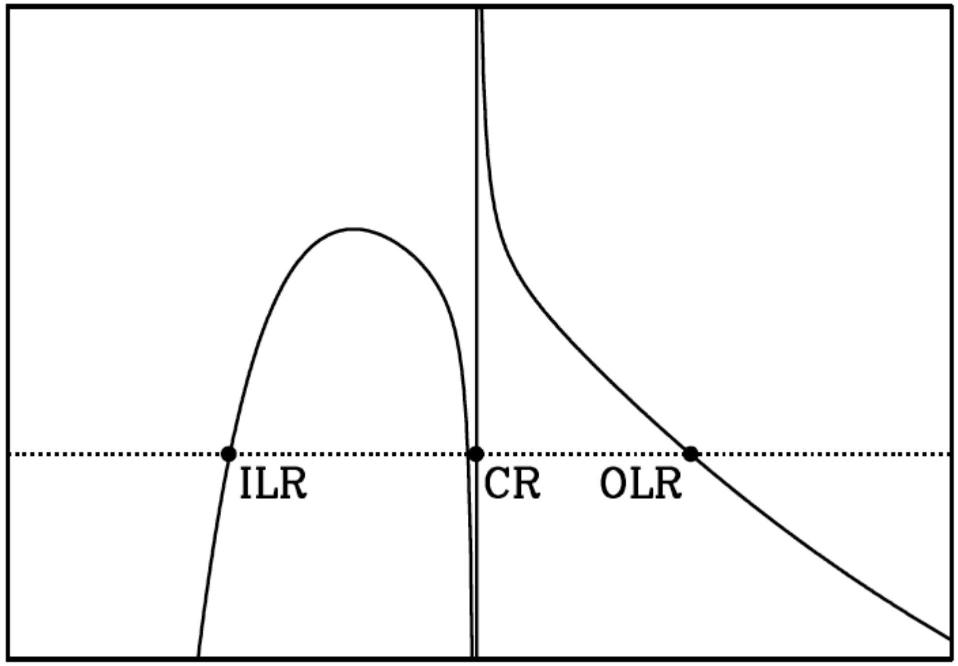
$$A_{12} = -\frac{\tilde{\omega}^4}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)(\tilde{\omega}^2 - \omega_s^2)} + \frac{m^2}{r^2(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)},$$

$$A_{21} = \tilde{\omega}^2 - m^2\omega_{A\phi}^2 - \frac{4(m\omega_{A\phi}^2 + \tilde{\omega}\Omega)^2}{\tilde{\omega}^2 - m^2\omega_{A\phi}^2} + r\frac{d}{dr}(\omega_{A\phi}^2 - \Omega^2) + (\omega_{A\phi}^2 - \Omega^2)\frac{r}{\rho}\frac{d\rho}{dr} + \frac{g}{\rho}\frac{d\rho}{dr}$$

$$+ \frac{1}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)(\tilde{\omega}^2 - \omega_s^2)}\left\{r\left[(\omega_{A\phi}^2 - \Omega^2)\tilde{\omega}^2 + \omega_{A\phi}^2\omega^2\right] + g(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)\right\}^2,$$

$$A_{22} = -\frac{r\tilde{\omega}^2\left[(\omega_{A\phi}^2 - \Omega^2)\tilde{\omega}^2 + \omega_{A\phi}^2\omega^2\right]}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)(\tilde{\omega}^2 - \omega_s^2)} - \frac{g\tilde{\omega}^2}{(c_s^2 + v_A^2)(\tilde{\omega}^2 - \omega_s^2)} + \frac{2m(m\omega_{A\phi}^2 + \tilde{\omega}\Omega)}{r(\tilde{\omega}^2 - m^2\omega_{A\phi}^2)} - \frac{1}{\rho}\frac{d\rho}{dr}.$$

Effective Potential for p-modes: B=0



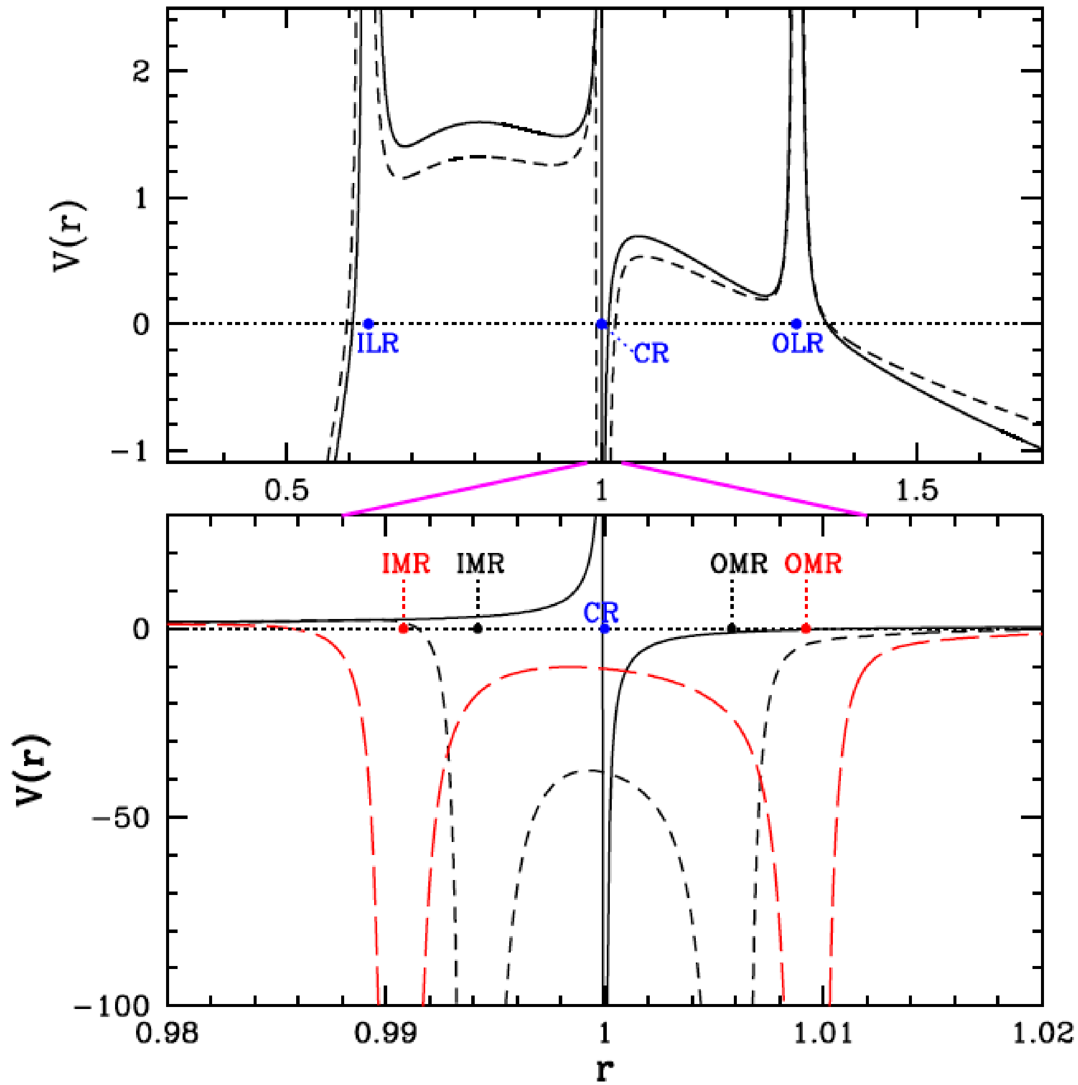
Effective potential for p-modes: with toroidal fields

- Wave zone is largely unaffected:

  acoustic wave ---> fast wave

$$\tilde{\omega}^2 = \kappa^2 + k_r^2(c_s^2 + v_A^2)$$

==> mode frequency slightly affected by B-field



Fu & DL 2010

Effective potential for p-modes: with toroidal fields

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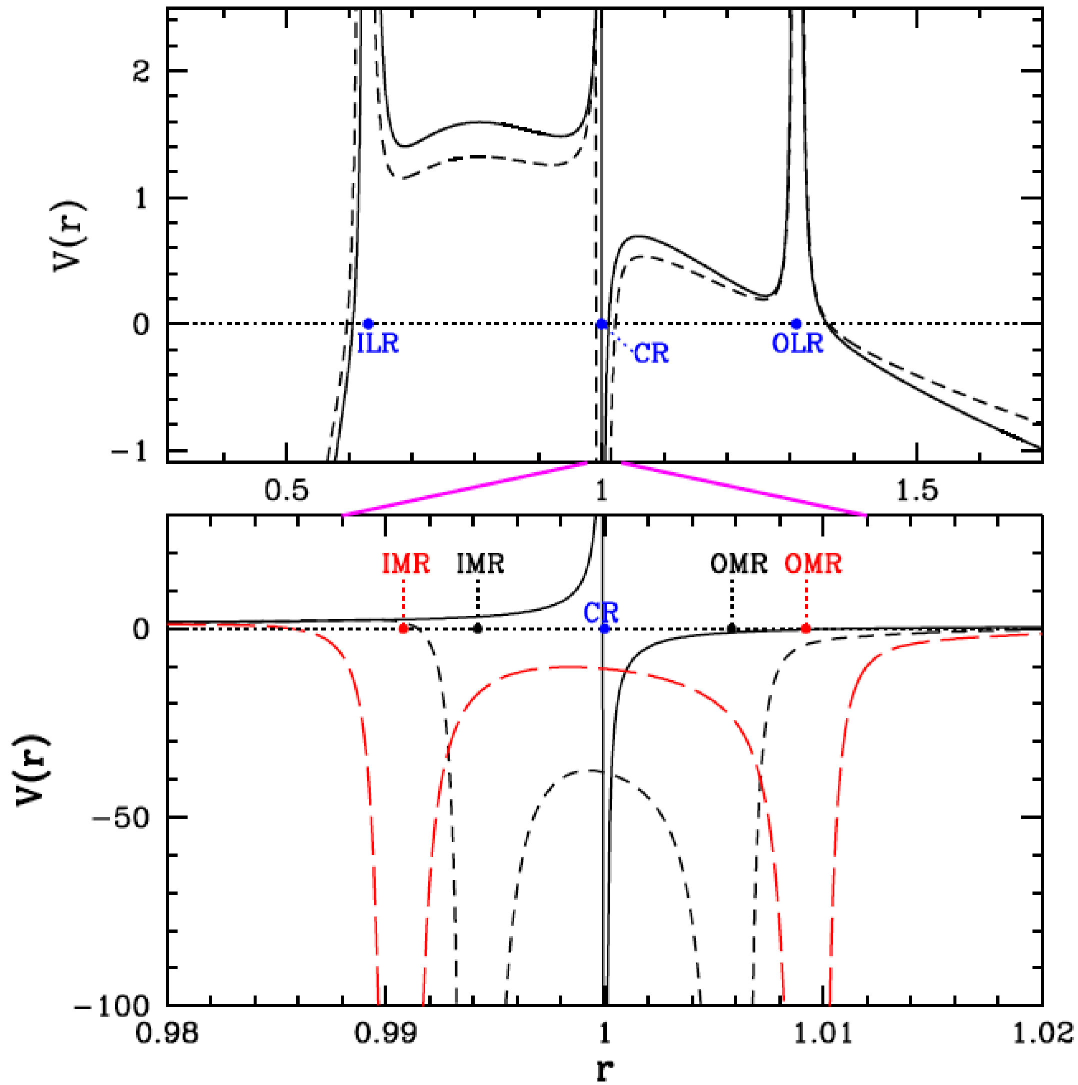
$$\tilde{\omega}^2 = \kappa^2 + k_r^2(c_s^2 + v_A^2)$$

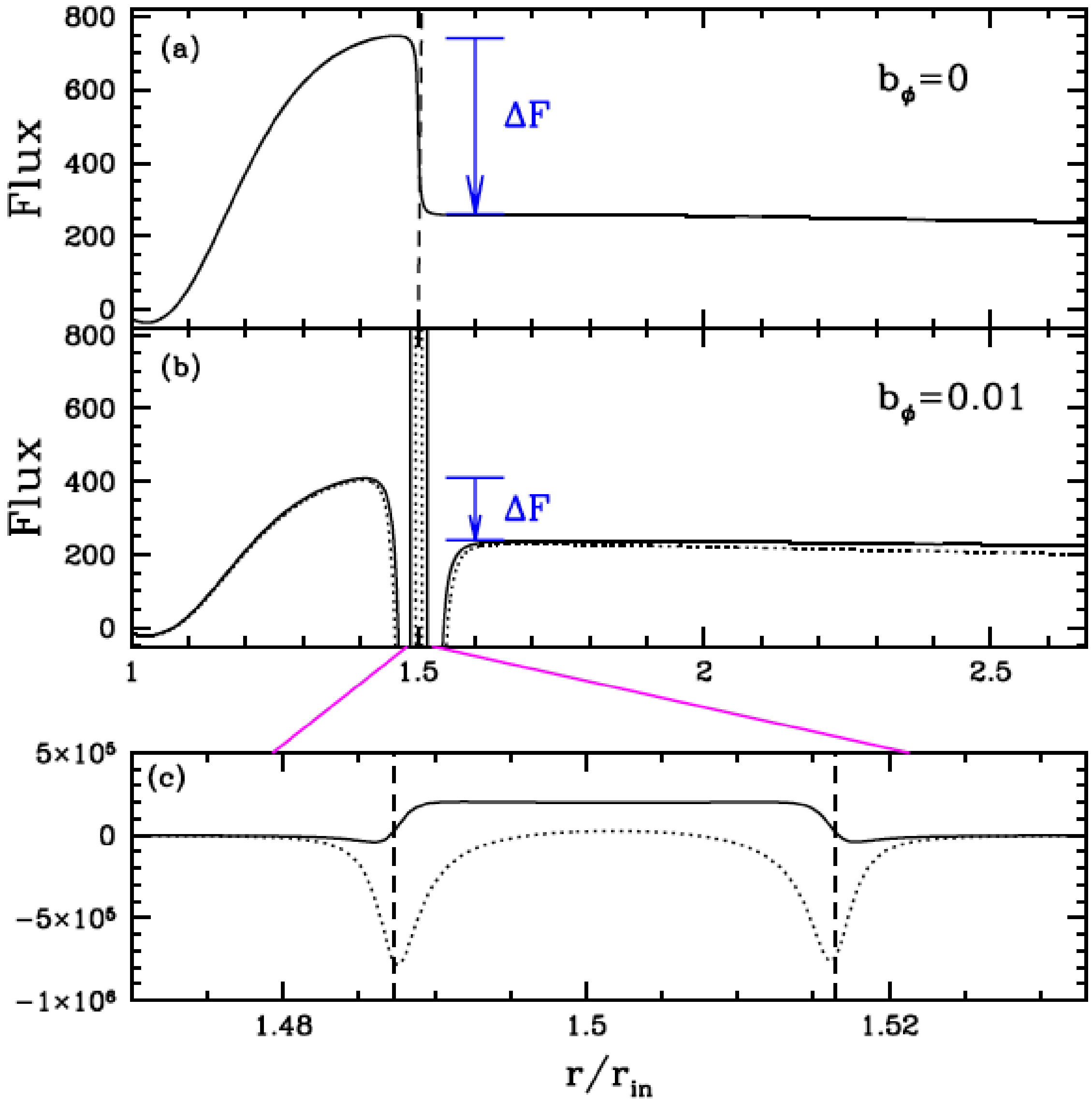
==> mode frequency slightly affected by B-field

- Corotation resonance splits into two **magneic (slow-wave) resonances**:

$$\tilde{\omega} = \pm \frac{c_s v_A}{\sqrt{c_s^2 + v_A^2}} \frac{m}{r}$$



Fu & DL 2010

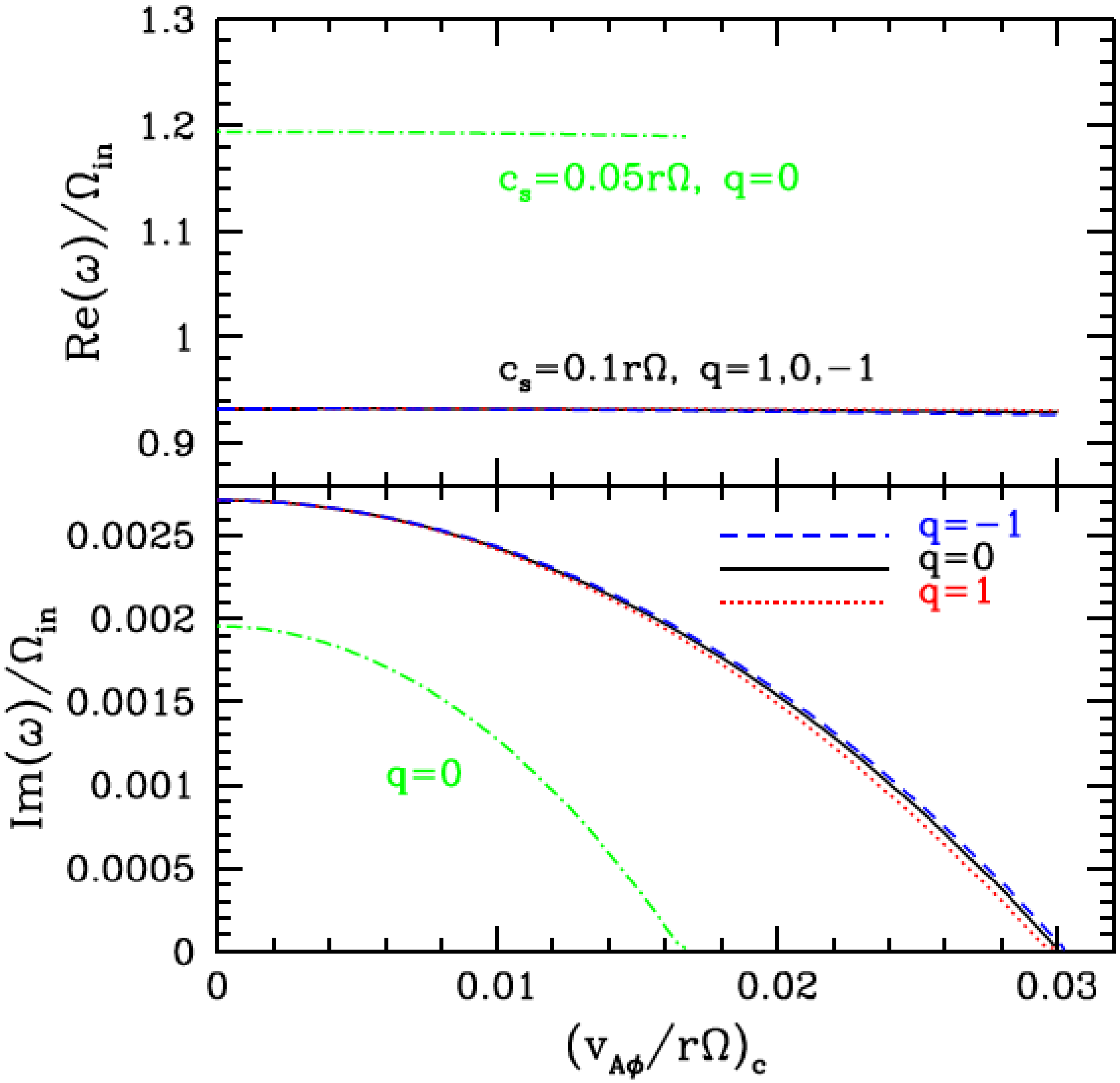


$$\omega = (0.9324 + i0.0027)\Omega_{\rm in}$$

$$\omega = (0.9312 + i0.0018)\Omega_{\rm in}$$

Fu & DL 2010

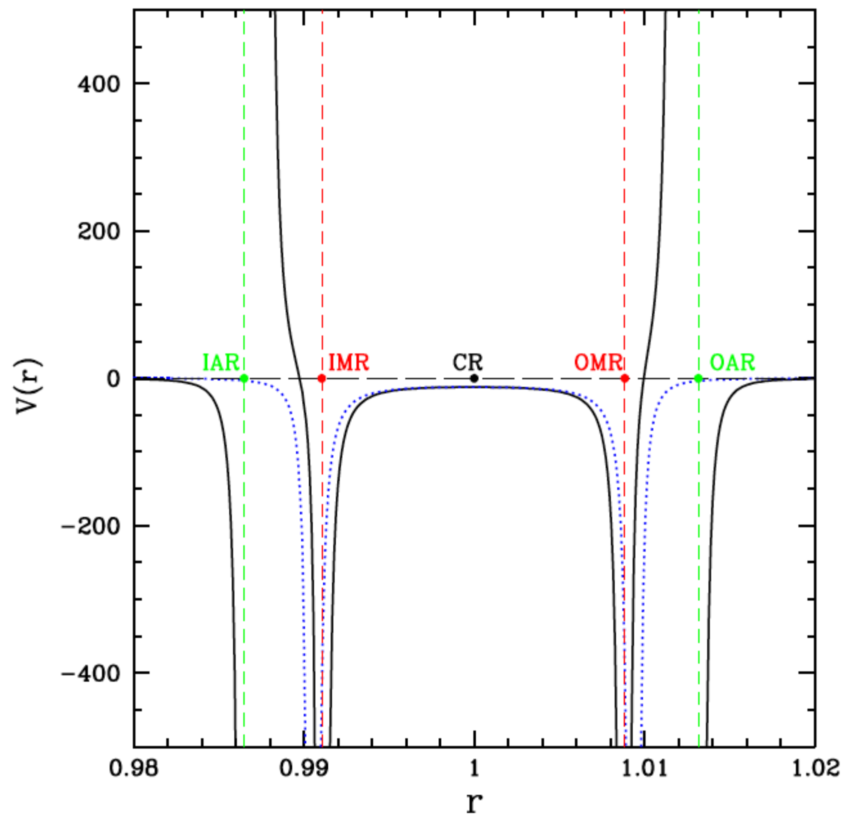
Toroidal fields tend to suppress corotational instability (bad news!?)



Fu & DL 2010

Toroidal fields tend to suppress corotational instability (bad news!?)

With both toroidal and (uniform) poloidal fields…



Fu & DL 2010

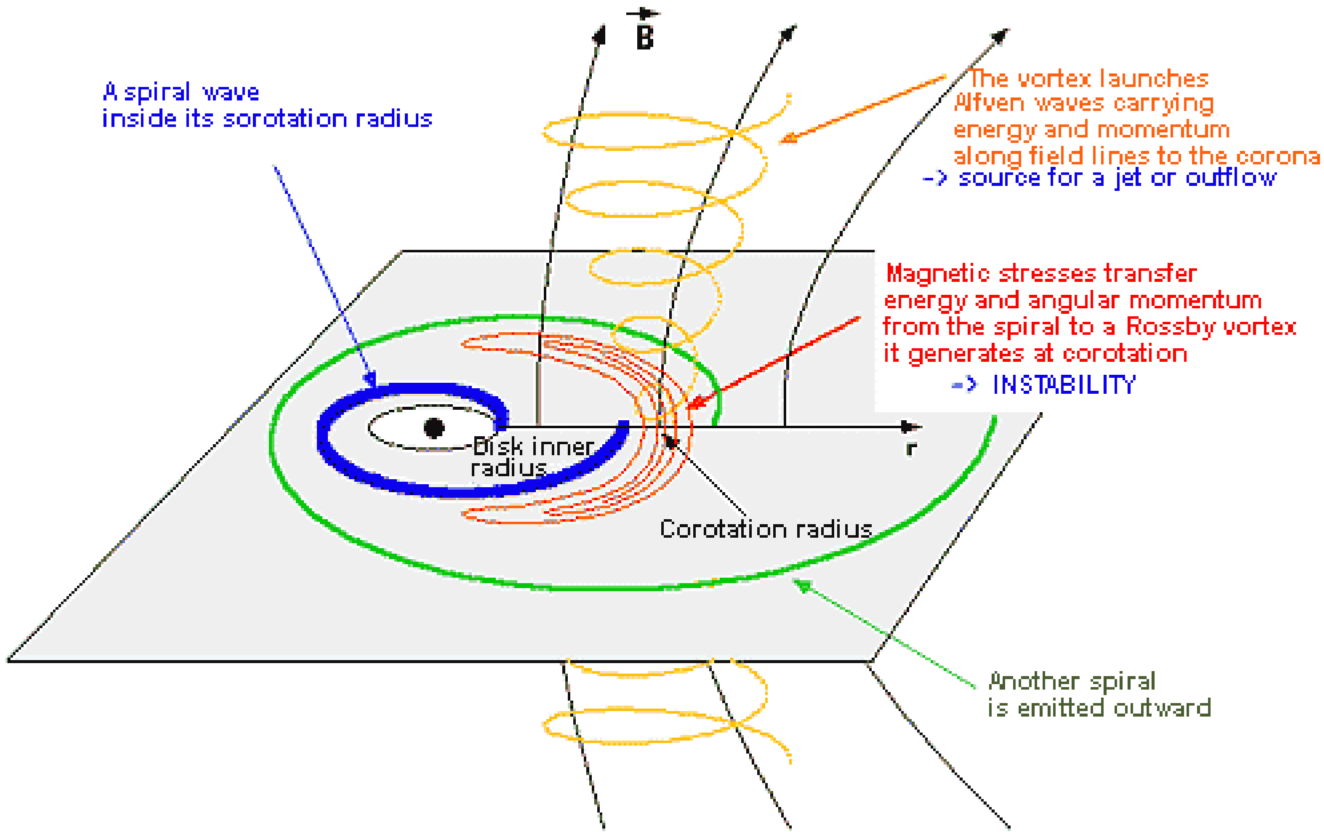
Corotation resonance splits into four:

- Inner/outer **Magnetic (Slow) Resonances**
- Inner/outer **Alfven Resonances**: $\tilde{\omega} = \pm \frac{m v_{A\phi}}{r}$

Accretion-Ejection Instability

Tagger, Varniere, etc

Disks threaded by large-scale vertical field embedded in vacuum or tenuous corona ==> Instability



P. Varniere

Unsolved Issue: Effects of disk turbulence

- P-modes are robust in the presence of MHD turbulence?

  (Arras, Blaes & Turner 2006; Reynolds & Miller 2009)

- Turbulent viscosity excites p-modes?

- Vorticity perturbation (of turbulence) excites p-mdes?

  (e.g., Heinemann & Papaloizou 2009)

Other Diskoseismic Oscillation Modes Around BH

G-Modes (Inertial-Gravity Modes)

(studied extensively by Kato, Wagoner, etc)

- Dispersion relation:

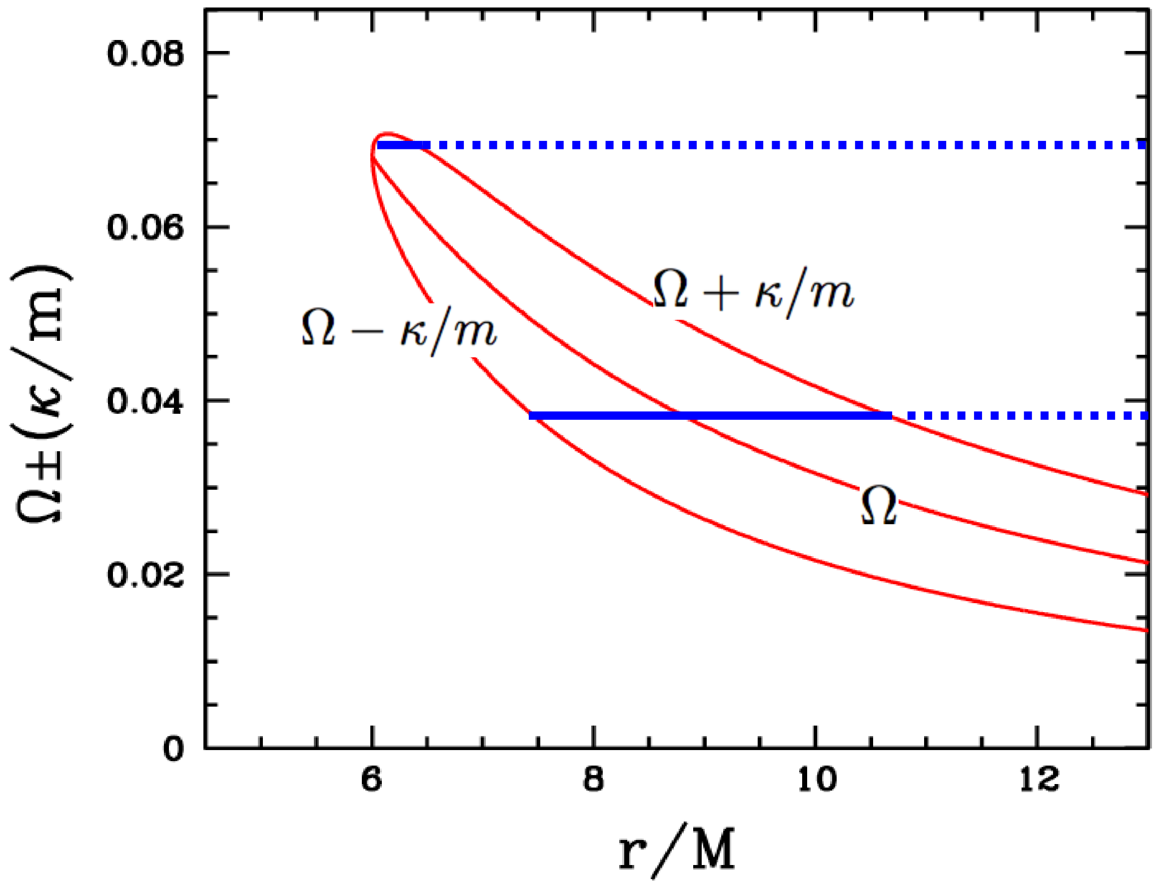
$$(\tilde{\omega}^2 - \kappa^2)(1 - n_z \Omega_\perp^2/\tilde{\omega}^2) = k_r^2 c_s^2 \qquad n_z > 0$$

- Can propagate only between ILR and OLR

$$\Omega - \frac{\kappa}{m} < \frac{\omega}{m} < \Omega + \frac{\kappa}{m}$$

(self-trapped!!)

Non-axisymmetric G-Modes (Inertial-Gravity Modes)

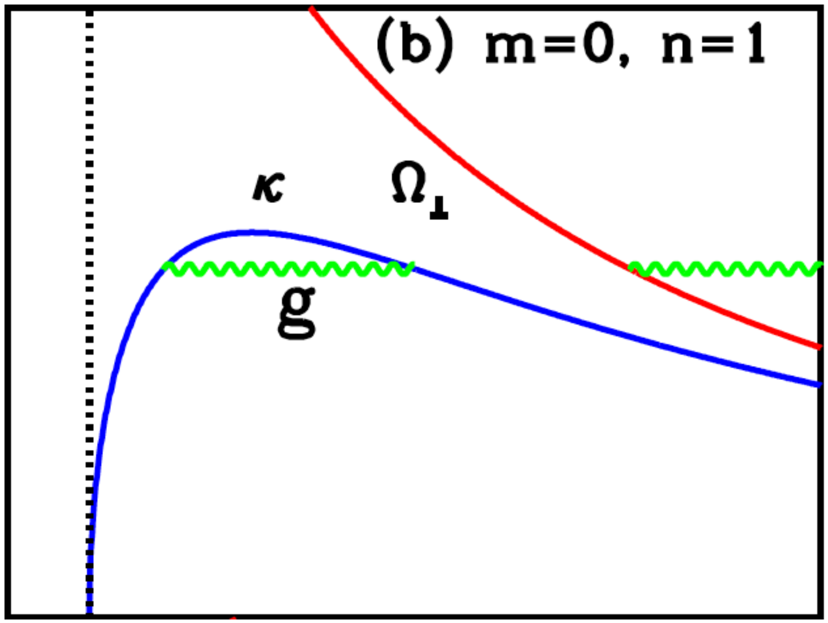


$\omega/m \simeq \Omega(r_{\rm in})$
This is too high (for the measured BH Mass)

Damping at Corotation
(Kato 2003; Li, Goodman & Narayan 2003)

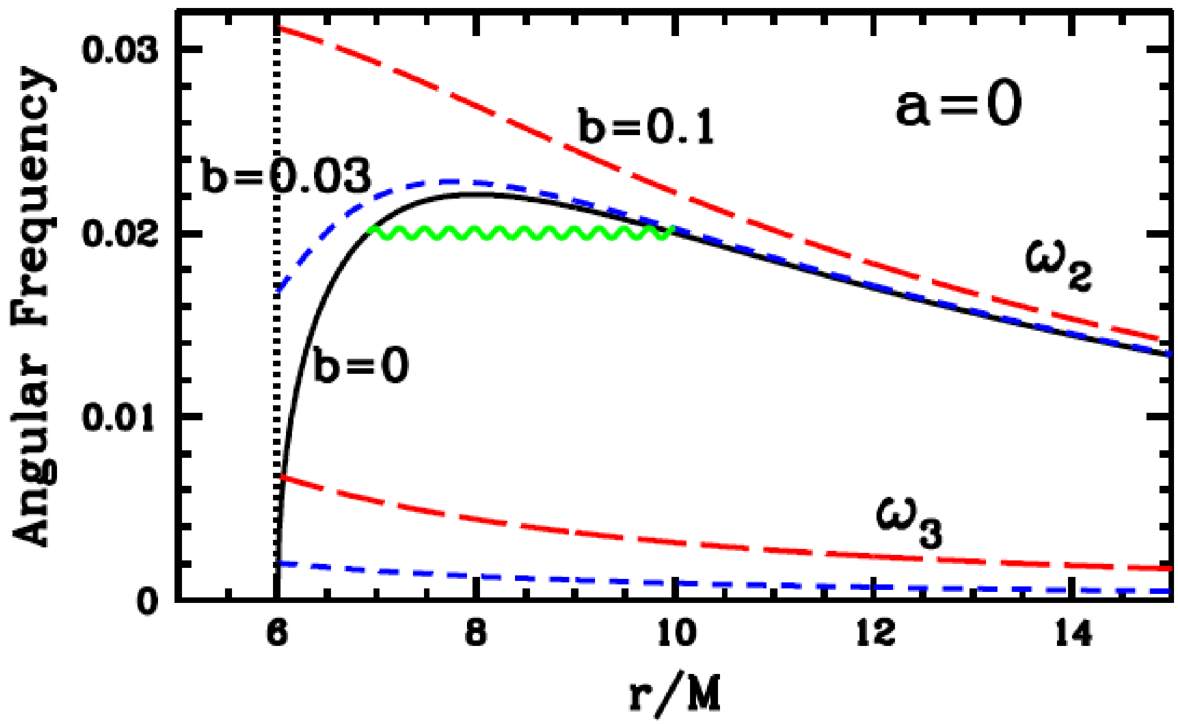
- Maybe strongly affected ("destroyed") by MRI turbulence??? (Arras et al; Reynolds & Miller)
- Cavity is destroyed by B-field (Fu & DL 2009)

Axisymmetric (m=0) G-modes (Inertial-Gravity Modes)



- Self-trapped!
- Can be excited by global disk distortion (Kato, Ferreira & Ogilvie)

Axisymmetric (m=0) G-modes (Inertial-Gravity Modes)

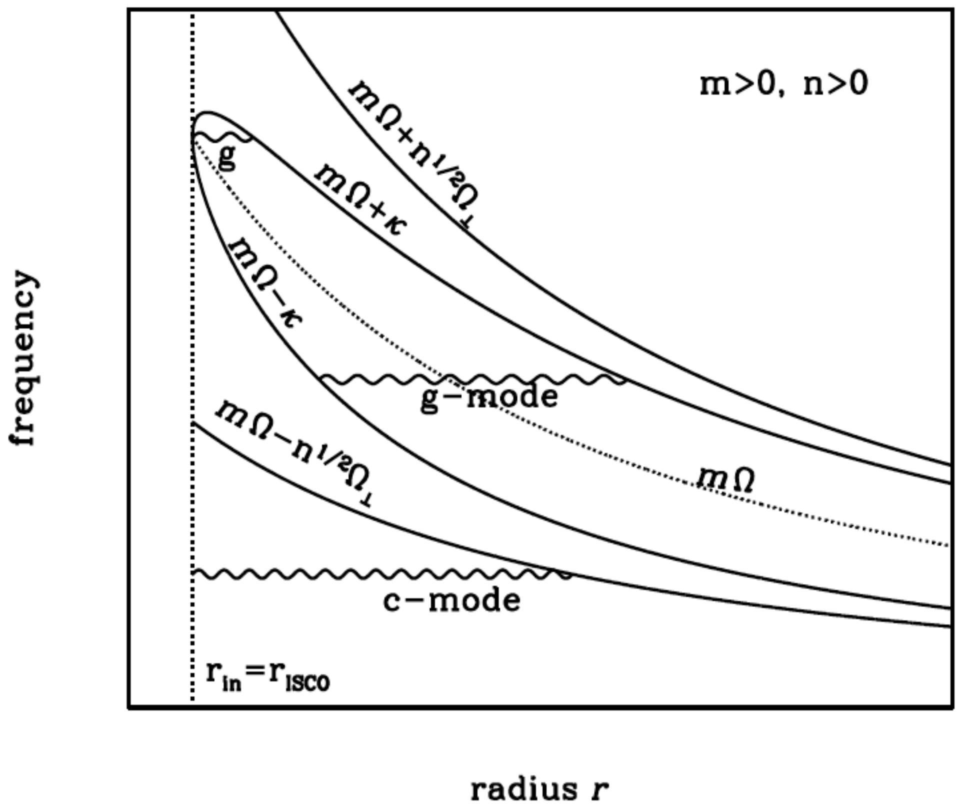


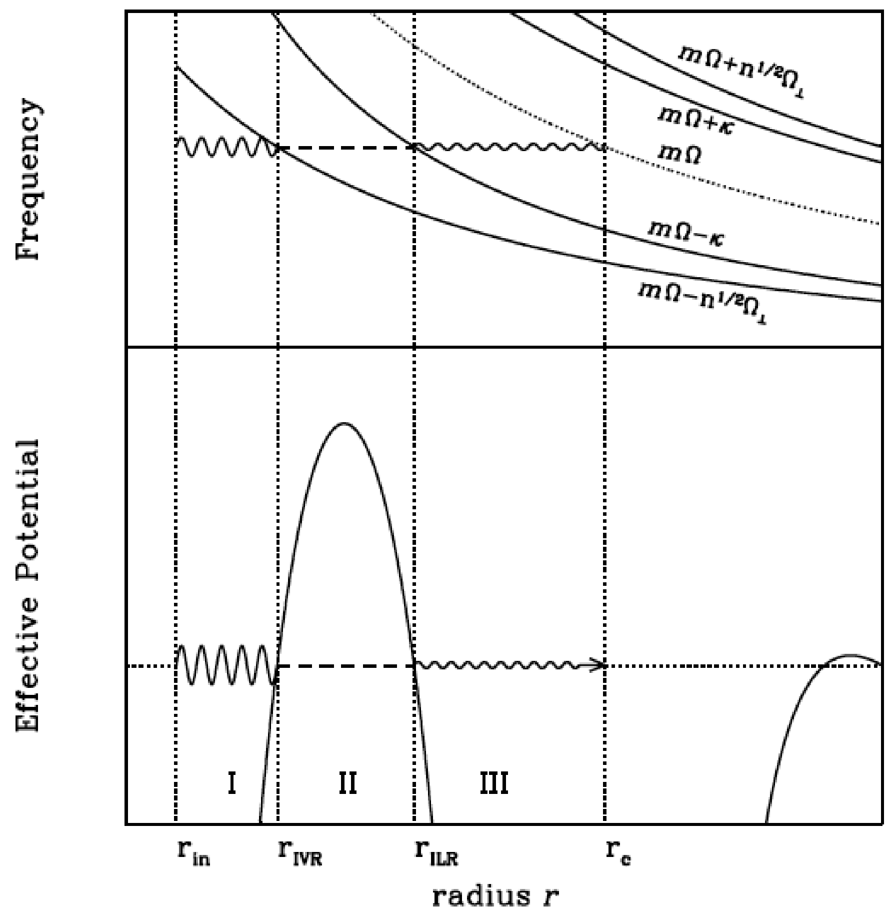
- Self-trapped!
- Can be excited by global disk distortion (Kato, Ferreira & Ogilvie)
- Problem: Cavity (trapping zone) disappears with B field (sub-thermal)

(Fu & DL 2009)

C-Modes

- Dispersion relation: $(\tilde{\omega}^2 - \kappa^2)(1 - n_z \Omega_\perp^2 / \tilde{\omega}^2) = k_r^2 c_s^2$
- Trapped between inner disk edge and IVR (inner-vertical resonance)
- Damped due to wave tunneling and absoption at corotation (Tsang & Lai 2009)





Summary

Global instabilities/modes in accretion disks

- Super-reflection
- Role of corotation resonance (vortensity gradient)
- Trapped p-modes: overstable due to wave absorption at corotation
- GR effects
- Magnetic field effects

QPOs in black-hole X-ray binaries

- Overstable p-modes **maybe** promising
- Other modes (g- and c-modes) **may** have problems

Many unsolved issues

- Understand magnetic field effects
- Role of MHD turbulence (or no turbulence? MRI surpressed by strong B)
- How disk oscillation manifests observationally?

Final Remarks:

The same processes/dynamics are also relevant in other astrophysical systems/contexts

Examples….

Planet-disk interaction (planet migration)

advective transport. Substituting equation (53) into equation (27), we obtain

$$F_A = -\frac{\pi^2}{4} m \operatorname{sgn}(x) \left[\frac{\varphi_1{}^2}{d\Omega/dr} \frac{d}{dr}\left(\frac{\sigma}{B}\right) \right]_{r_c} \exp(-|qx|) , \tag{55}$$

to lowest order in $(c/\Omega r)$. Note that $F_A \to 0$ as $|x| \to \infty$, so angular momentum is not transported to infinity. Curiously, there is a net flux of angular momentum into the resonance:

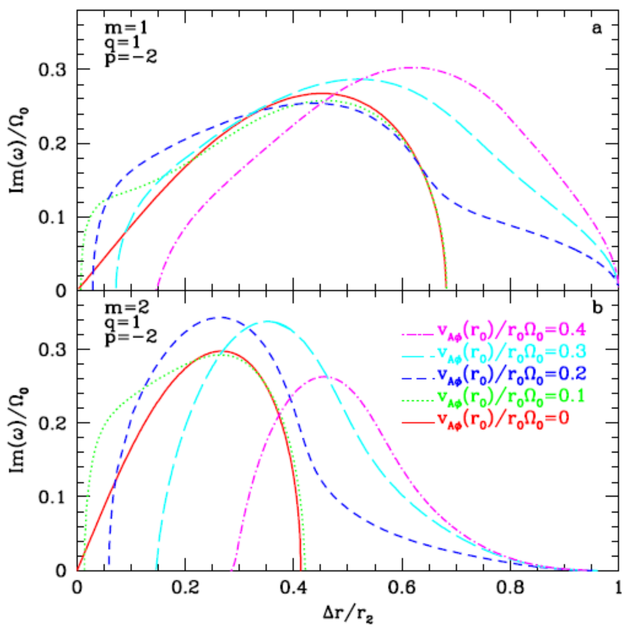
$$F_A(-0) - F_A(+0) = \frac{\pi^2 m}{2} \left[\frac{\varphi_1{}^2}{d\Omega/dr} \frac{d}{dr}\left(\frac{\sigma}{B}\right) \right]_{r_c} . \tag{56}$$

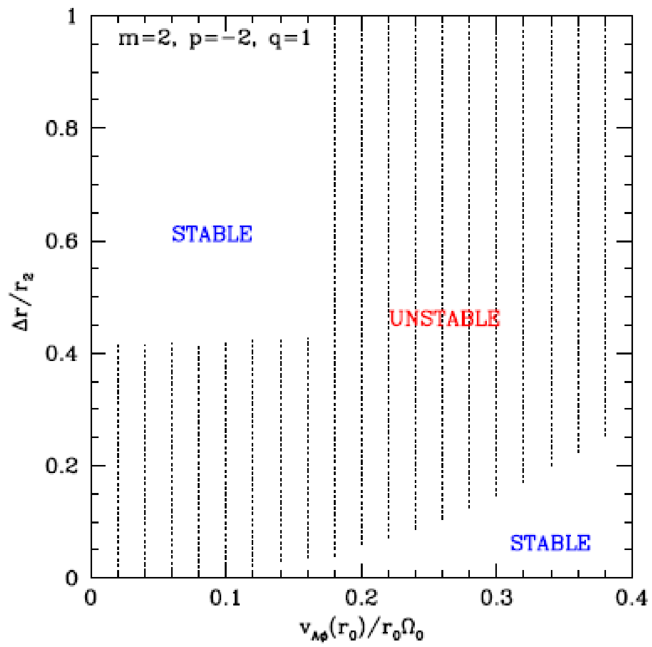
Goldreich & Tremaine 1979

Many recent papers ….

Global instability of accretion tori

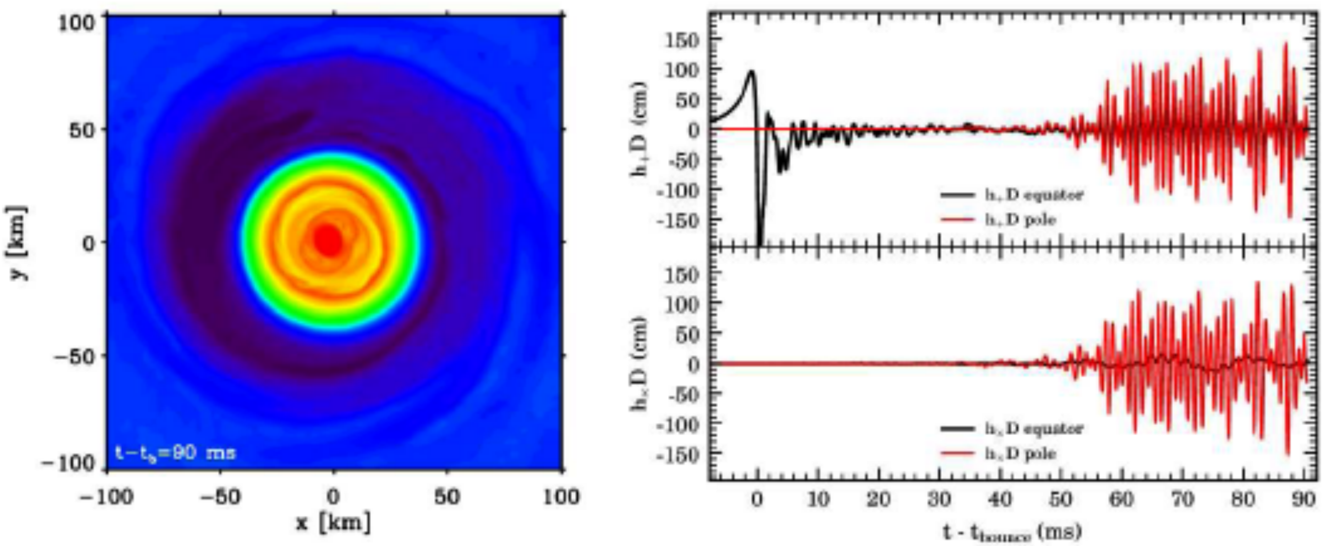
- May be formed in the collapse of rotating massive star (colapsar) or NS binary merger (e.g. Duez et al. 2009; Rezzolla et al. 2010)
- May be unstable due to corotation (Papaloizou-Pringle instability) ==> Variability of GRB central engines?
- Magnetic field effect on the instability (Fu & DL 2010)





Stabilize thin torii, while destabilize thick torii

Global rotational instability of differentially rotating proto-neutron stars



Ott et al. 2007

- Could affect the GW signature of core-collapse SNe.
- Can operate with T/|W|~0.03, a regime where bar-mode instability doesn't work
- Might be a class of corotation shear instability
- Effects of B field need to be explored

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Thanks