Tidal Dissipation in Binaries

From Merging White Dwarfs to Exoplanetary Systems

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Tidal Dissipation in Binaries

I. Merging White Dwarf Binaries

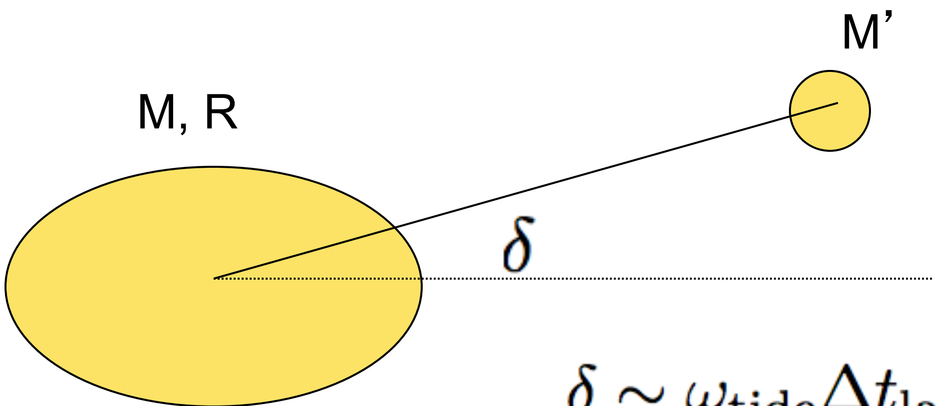
II. Kepler Heartbeat Stars (KOI-54)

III. Exoplanetary Systems

  (a) Hot Jupiters

  (b) Host Stars

Equilibrium Tide



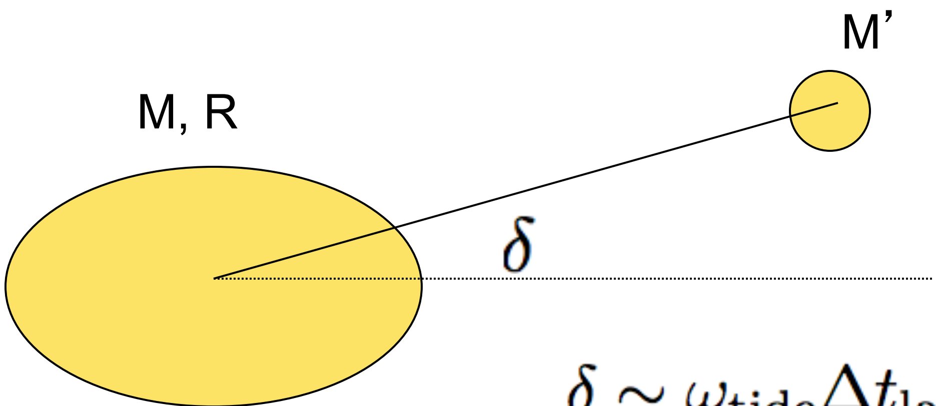
$$\delta \sim \omega_{\rm tide}\Delta t_{\rm lag} \sim 1/Q$$

$$\omega_{\rm tide} = 2(\Omega_{\rm orb} - \Omega_s)$$

$$\text{Torque} \sim G\left(\frac{M'}{a^3}\right)^2 R^5\,\frac{k_2}{Q}$$

$$k_2 = \text{Love Number}$$

Equilibrium Tide



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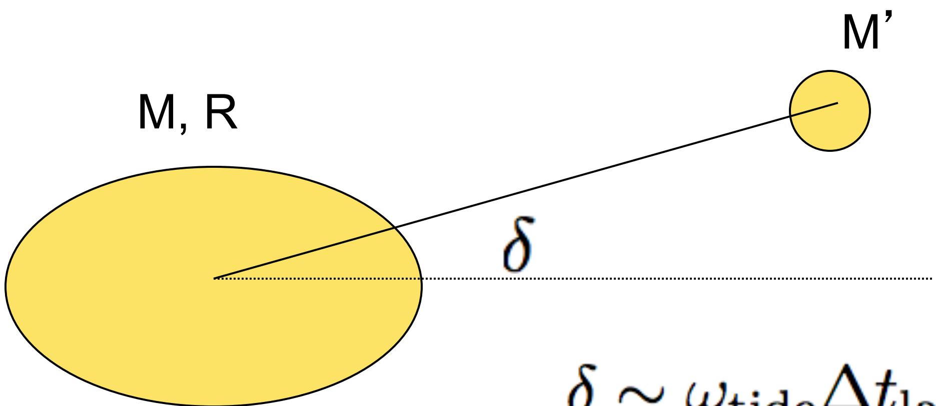
Weak friction theory: $\Delta t_{\rm lag} \sim (\omega_{\rm dyn}^2 t_{\rm visc})^{-1} = \text{const}$

Equilibrium Tide



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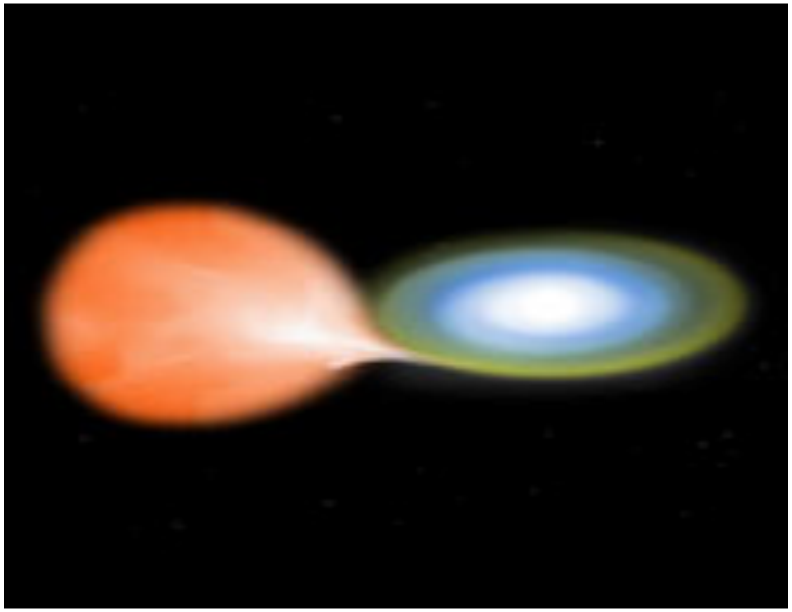
Problems:

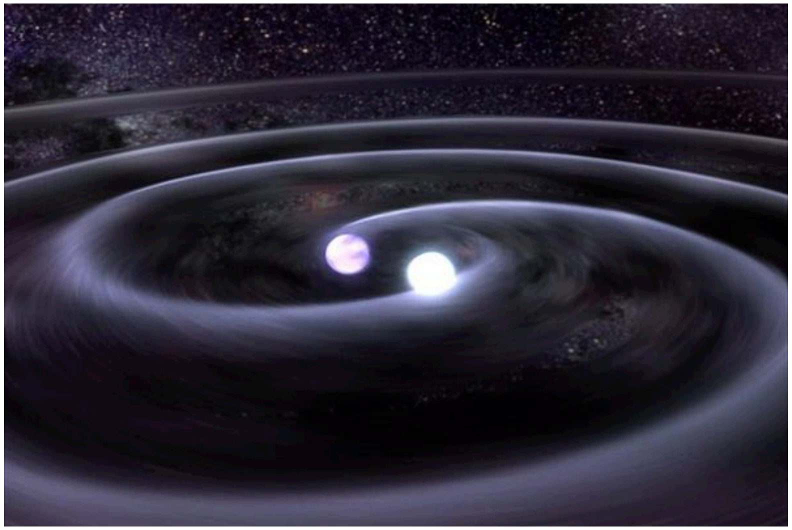
-- Parameterized theory

-- The physics of tidal dissipation is more complex:
Excitation/damping of internal waves/modes (Dynamical Tides)

-- For some applications, the parameterization is misleading

Compact White Dwarf Binaries

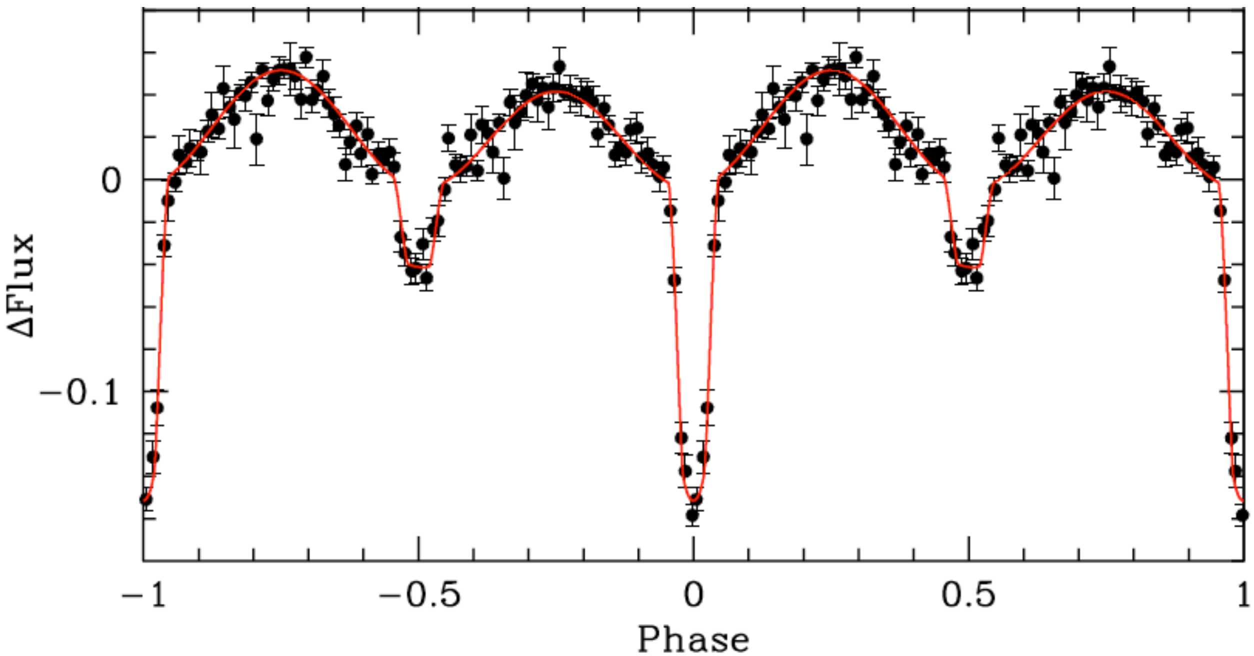




-- May lead to various outcomes: R CrB stars, AM CVn, SN Ia, AIC-->NS, transients, etc

-- Gravitational waves (eLISA-NGO)

12 min orbital period double WD eclipsing binary



SDSS J0651+2844

Primary & secondary eclipses
Ellipsoidal (tidal) distortion
Doppler boosting

Brown et al. 2011

-- Will merge in 0.9 Myr
-- Large GW strain ==> eLISA
-- Orbital decay measured from eclipse timing (Hermes et al. 2012)

Dynamical Tides in Compact WD Binaries

with Jim Fuller (Ph.D. 2013)

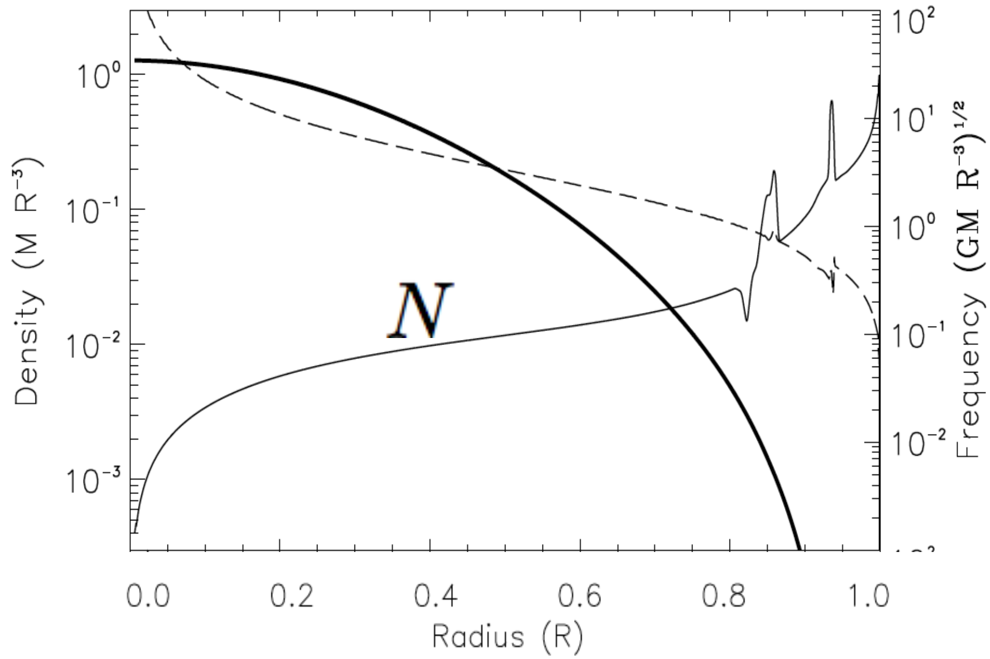
Issues:

-- Spin-orbit synchronization?

-- Tidal dissipation and heating?

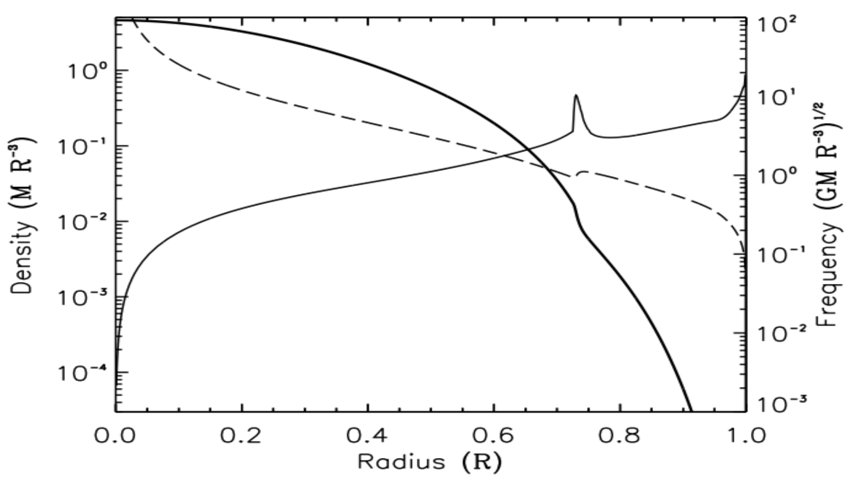
-- Effect on orbital decay rate? (e.g. eLISA-NGO)

White Dwarf Propagation Diagram



CO WD

$0.6 M_{\odot},\ 8720\,\mathrm{K}$



He-core WD

$0.3 M_{\odot},\ 12000\,\mathrm{K}$

Resonant Tidal Excitation of G-modes

As the orbit decays, resonance occurs when

$$\omega = 2(\Omega_{\rm orb} - \Omega_s) = \omega_\alpha$$

Calculation: mode amplitude evolution + orbital evolution

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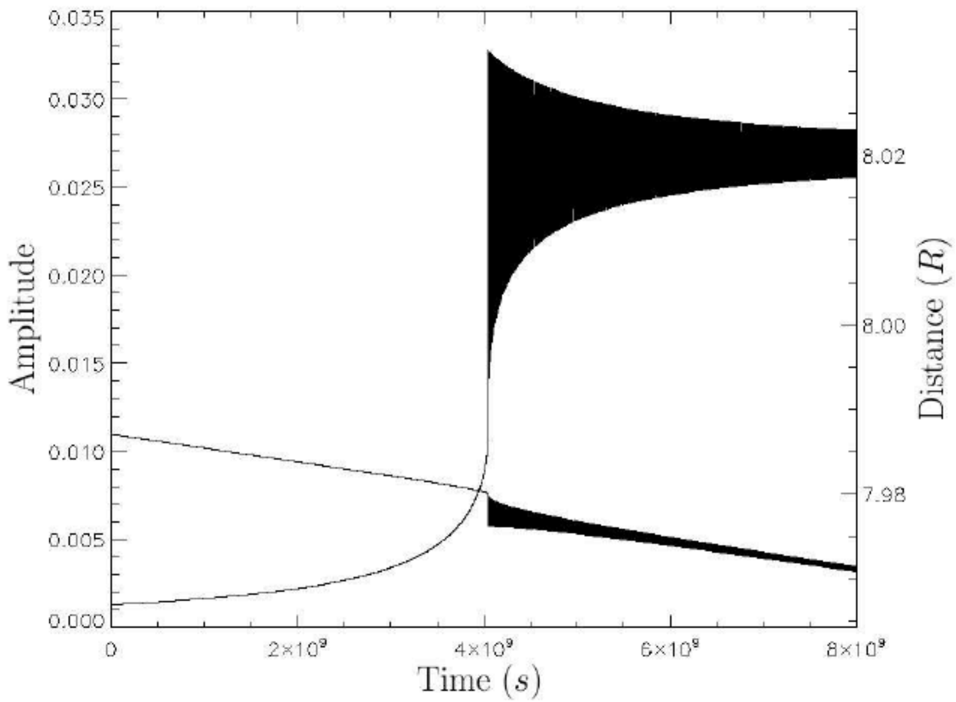

Result: Surface displacement is ~ R

==> Dissipation==> No standing wave

“Continuous” Excitation of Gravity Waves

Waves are excited in the interior/envelope, propagate outwards and dissipate near surface

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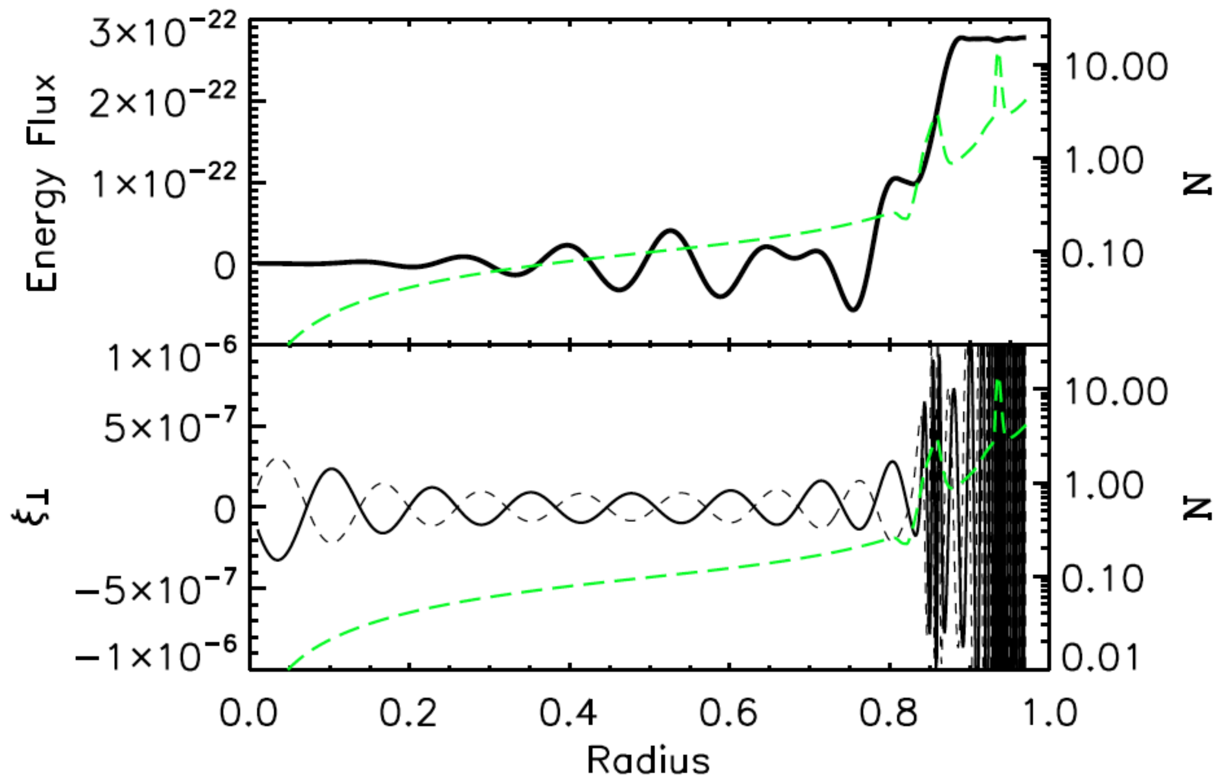
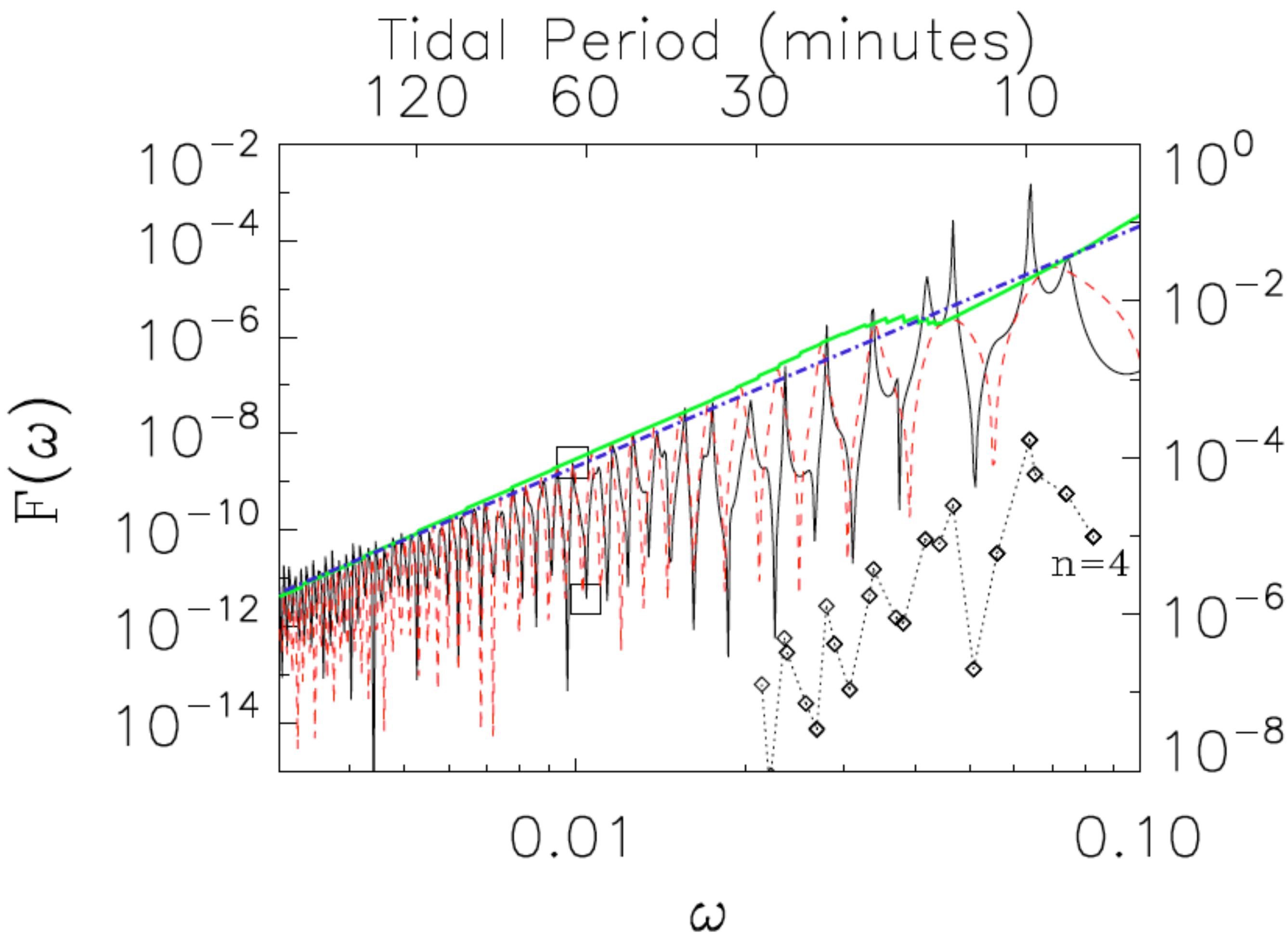


$$M = 0.6M_{\odot}, \quad \omega = 0.01$$

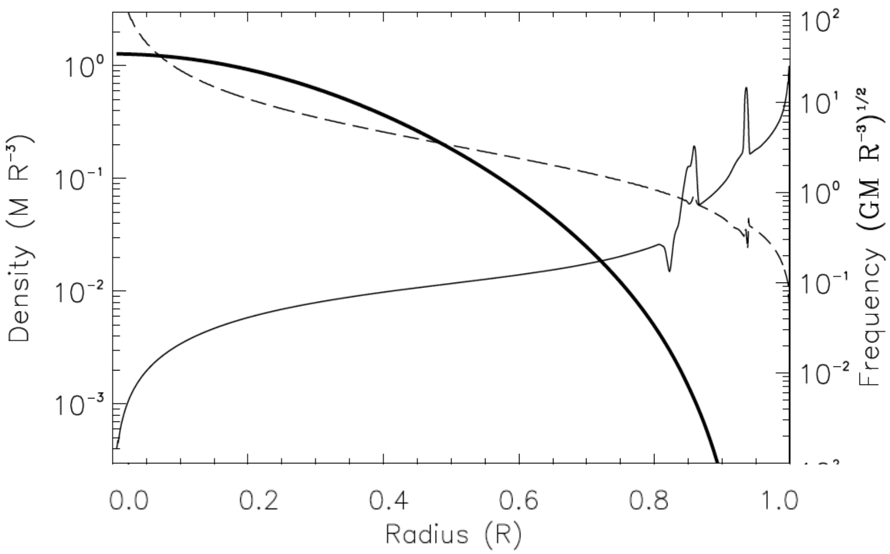
Fuller & DL 2012

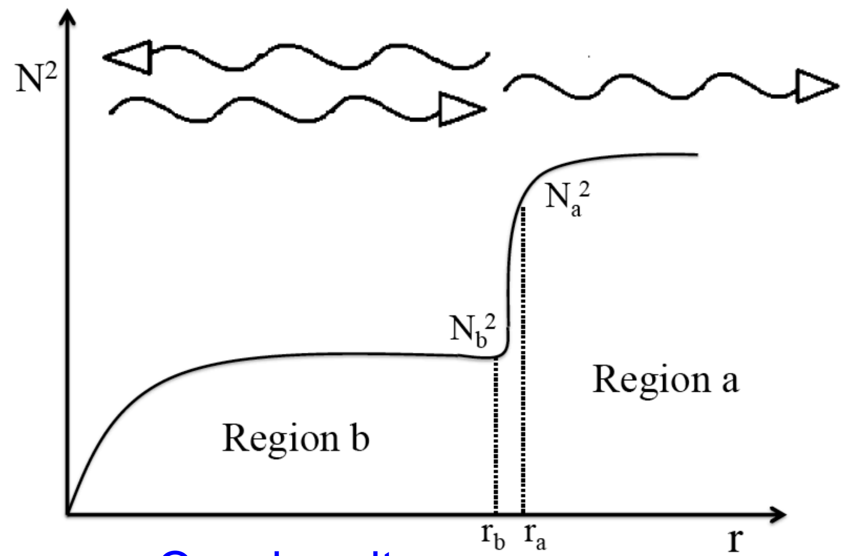
$$\text{Torque} = G\left(\frac{M'}{a^3}\right)^2 R^5 \; F(\omega)$$

$$\frac{k_2}{Q}$$

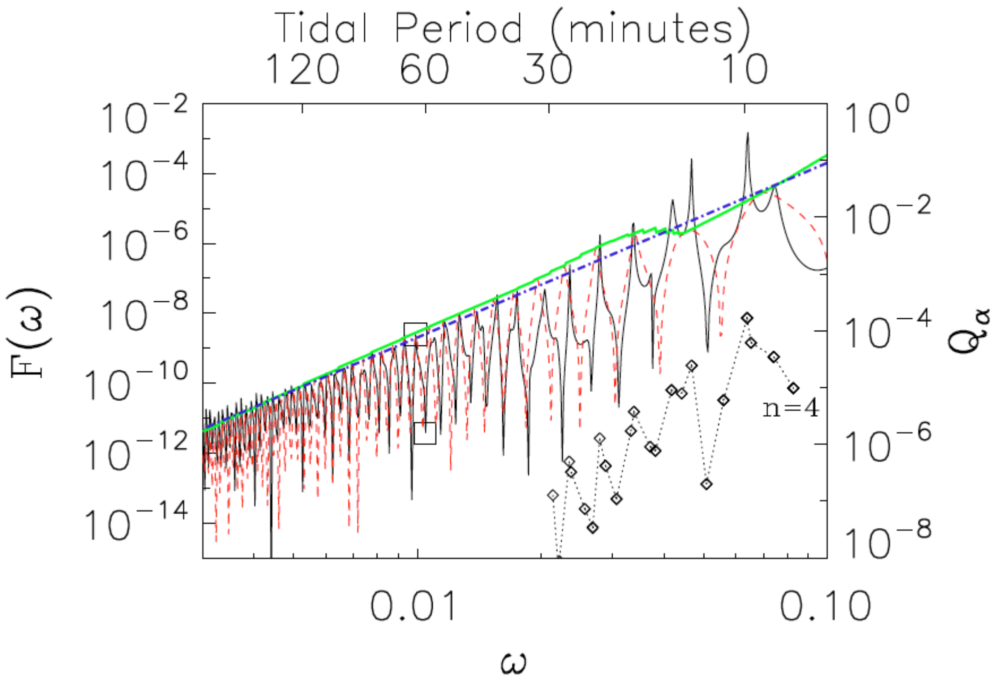


Why is $F(\omega)$ not smooth ?

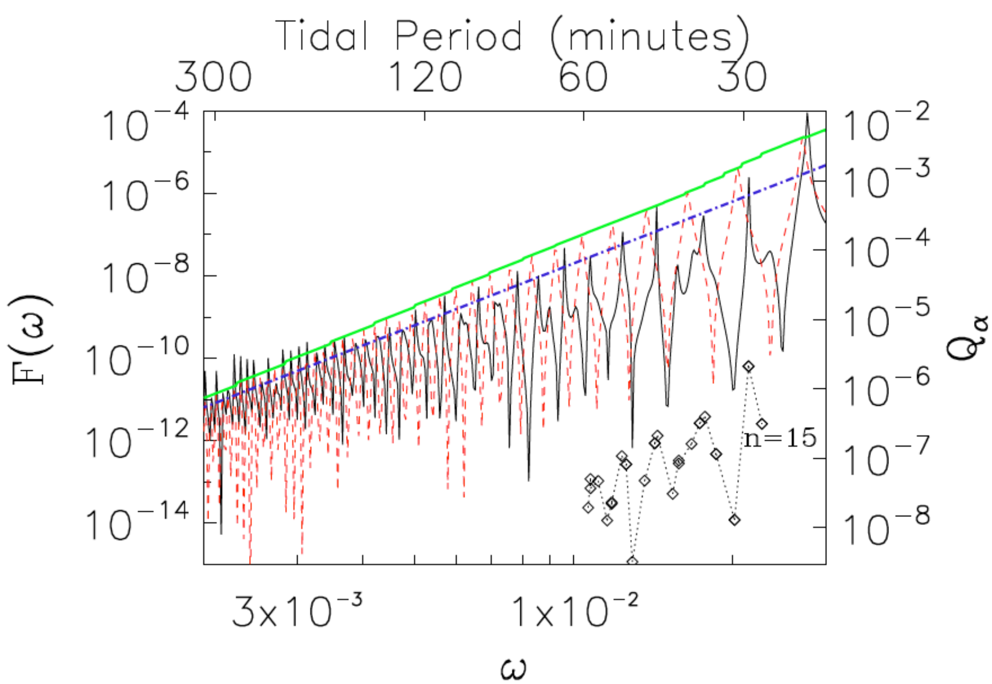




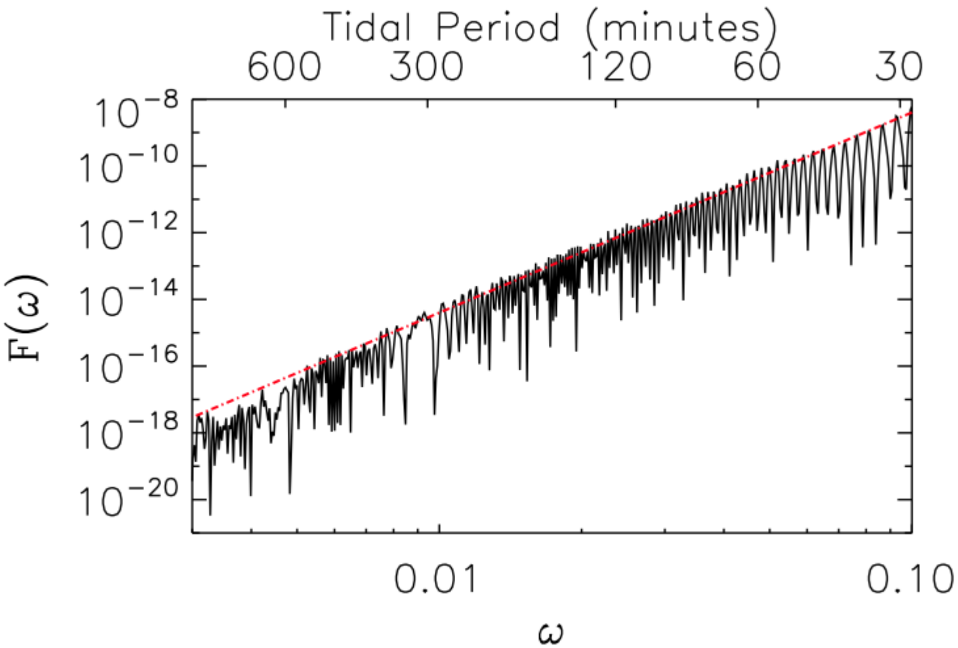
Quasi cavity



$M = 0.6M_\odot,\ T = 8720$ K

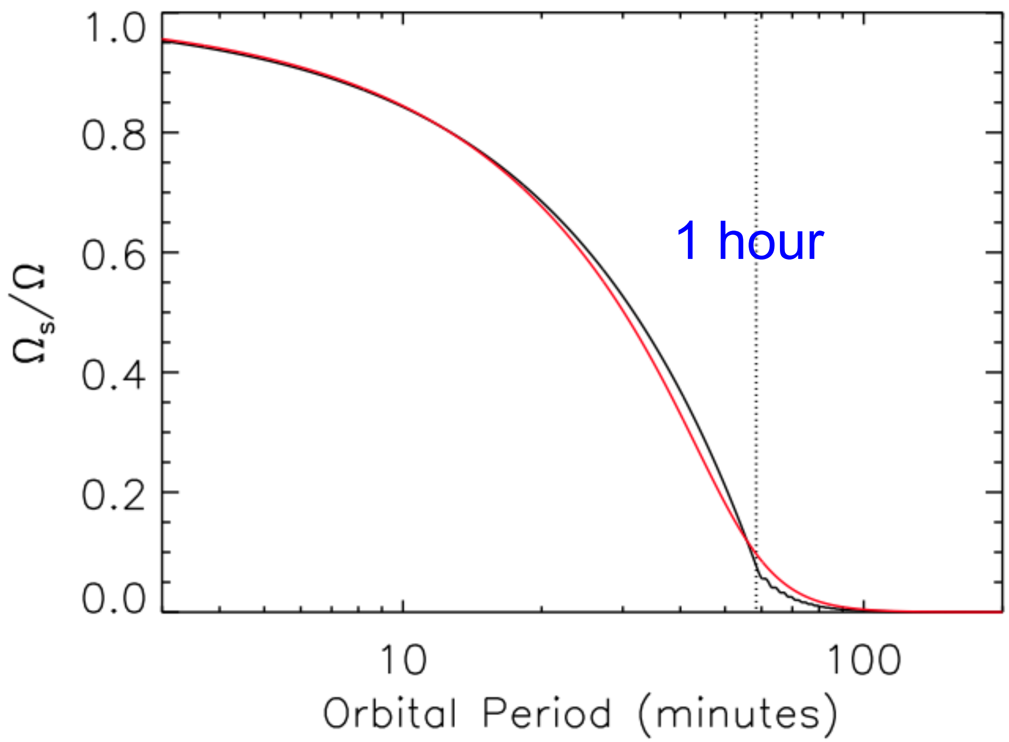


$M = 0.6M_\odot,\ T = 5080$ K



$M = 0.3M_\odot,\ T = 12000$ K

Spin-Orbit Synchronization

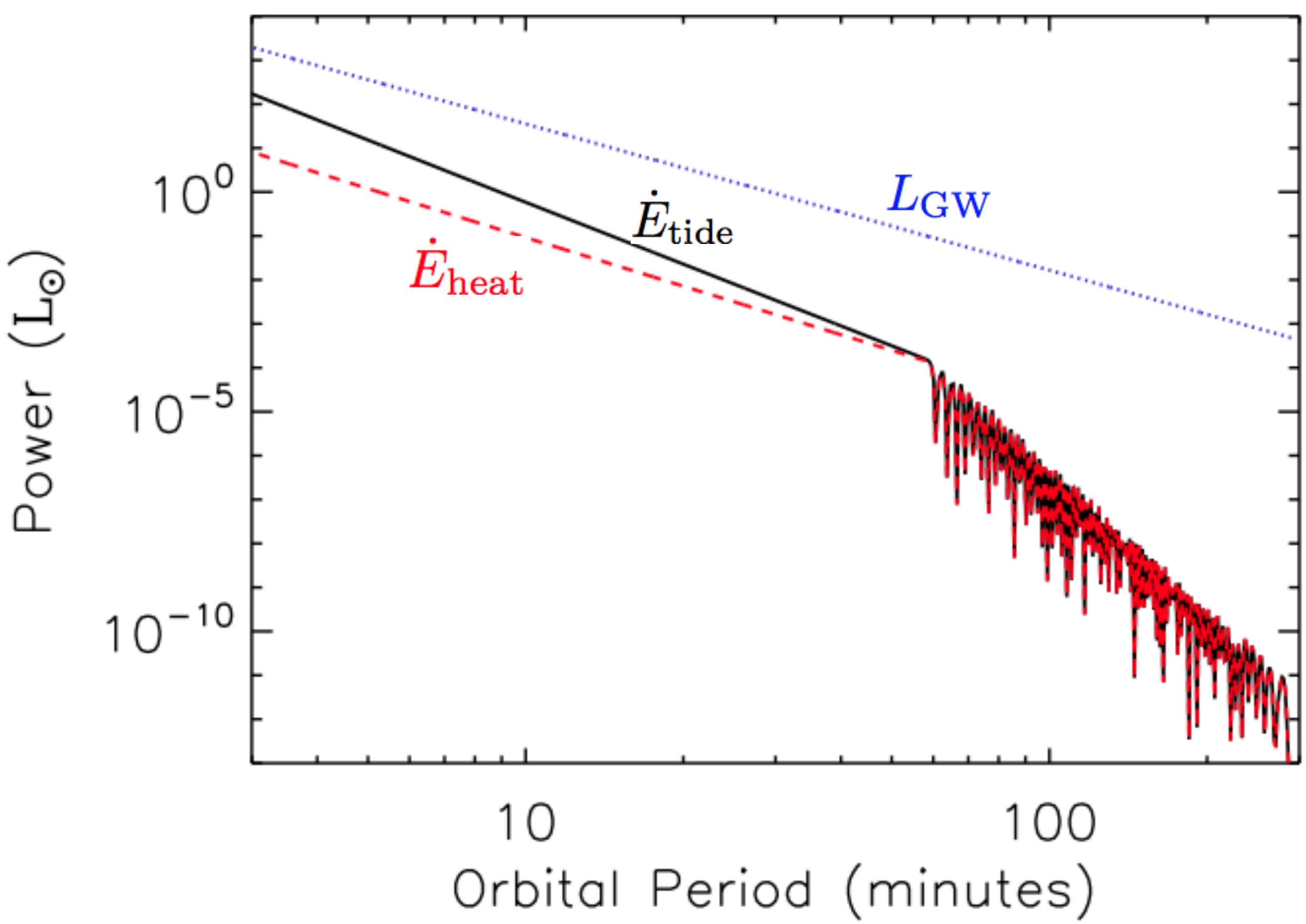


$$\text{Critical orbital } \; \Omega_c : \quad \dot{\Omega}_s = \frac{\text{Torque}}{I} \simeq \dot{\Omega}_{\rm orb} = \frac{3\Omega_{\rm orb}}{2t_{\rm GW}}$$

$$\text{For } \Omega_{\rm orb} > \Omega_c : \quad \dot{\Omega}_s > \dot{\Omega}_{\rm orb}$$

$$\dot{\Omega}_s - \dot{\Omega}_{\rm orb} \ll \dot{\Omega}_{\rm orb} \Longrightarrow \dot{E}_{\rm tide} = \Omega_{\rm orb} T \simeq \frac{3I\Omega_{\rm orb}^2}{2t_{\rm GW}}$$

Tidal Heating Rate



Consequences of Tidal Heating

Depend on where the heat is deposited …

If deposited in shallow layer:

thermal time short

==> change T_{eff}

Explain SDSS J0651+2844

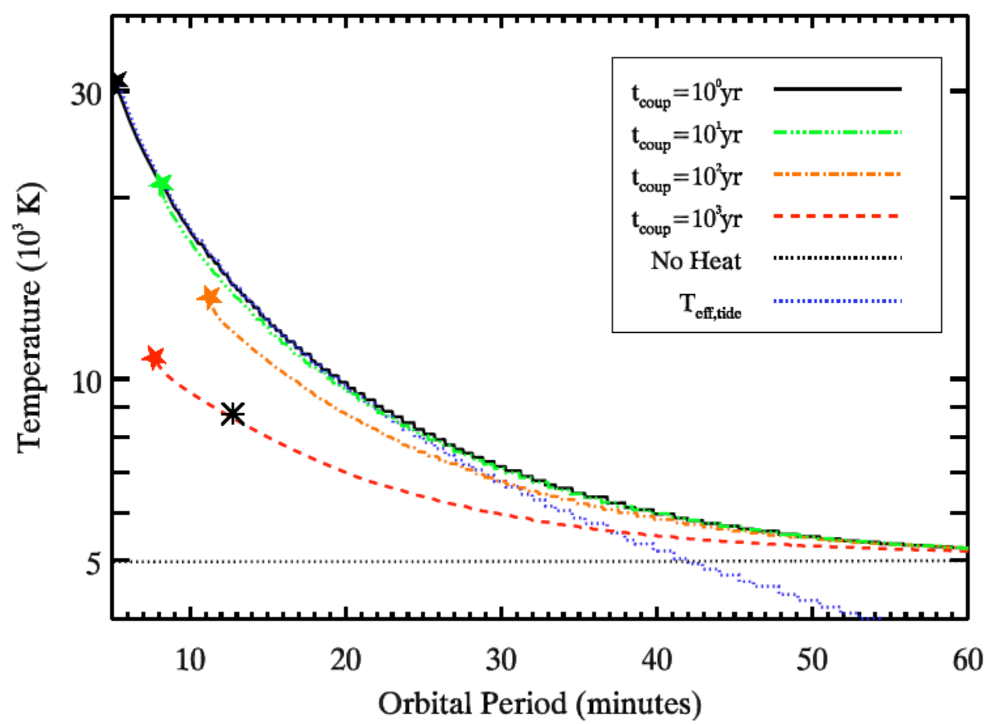
If deposited in deeper layer:

(common: critical layer…)

thermal time longer than orbital

==> Nuclear flash

* "Tidal Nova"



Summary: Tides in White Dwarf Binaries

-- Dynamical tides: Continuous excitation of gravity waves, outgoing, nonlinear breaking/critical layer…

-- Spin synchronized prior to merger (but not completely)

-- Tidal heating important… Tidal novae

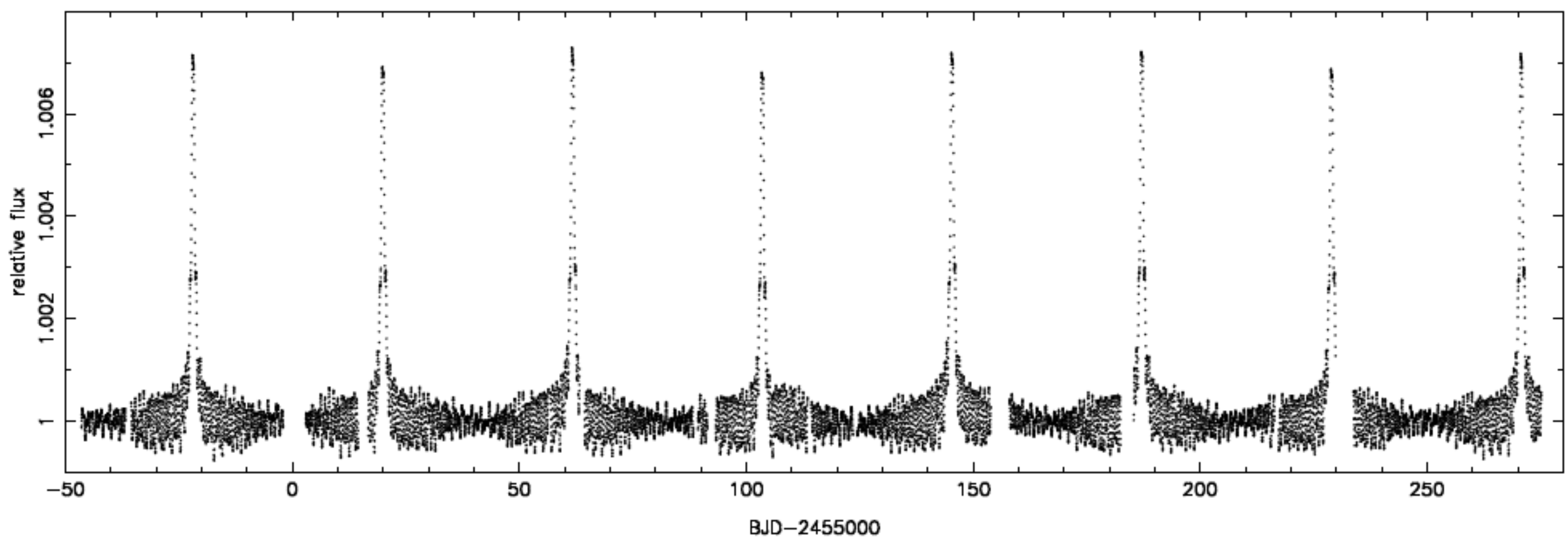
“Heartbeat Stars”

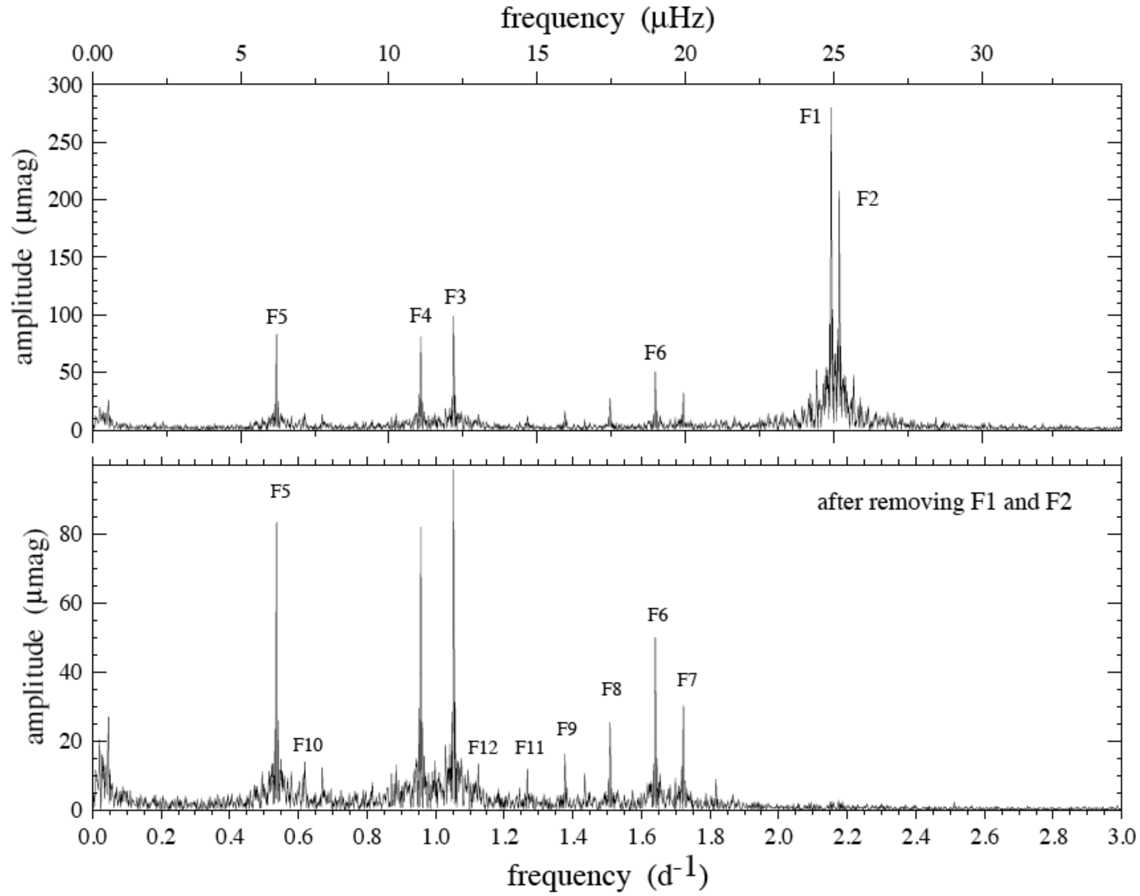
Tidally Excited Oscillations in Eccentric Binaries

KOI-54a,b Binary

Welsh et al 2012

A-type stars: 2.32, 2.38 M_{sun}
P=42 days, e=0.834, face-on (5.5 deg)
--> At periastron: $a_p = 6.5R, \quad f_p = 20 f_{\rm orb}$





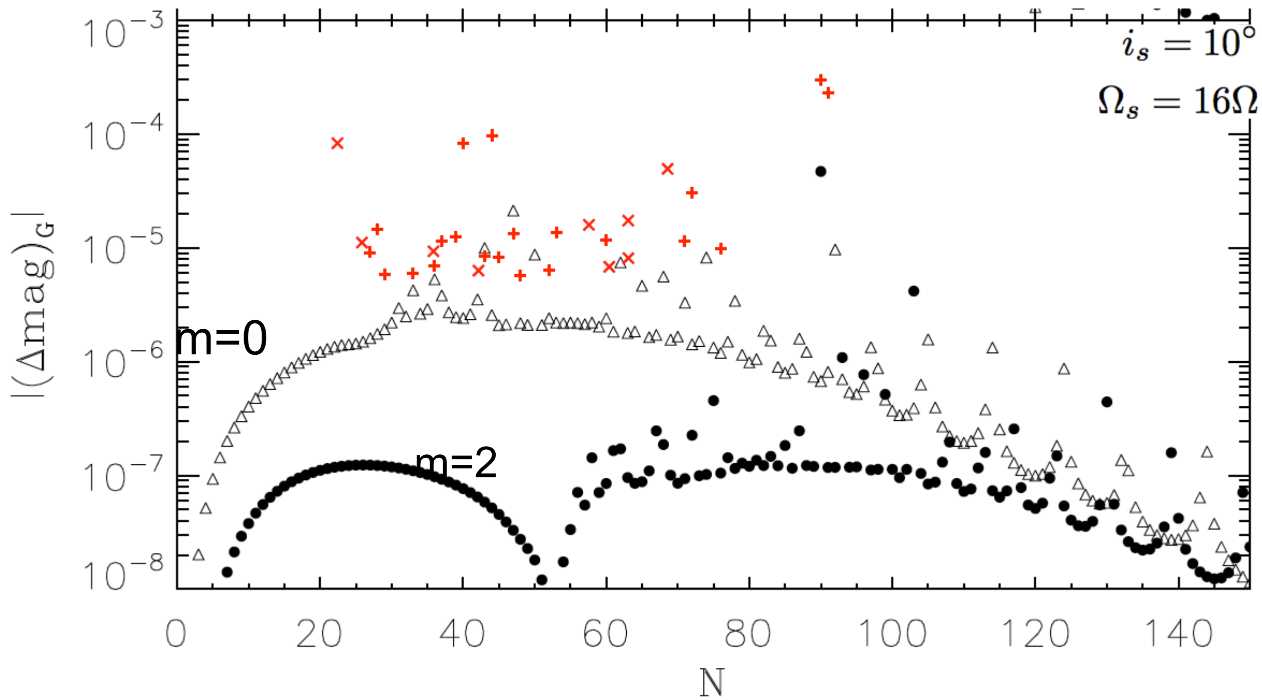
30 pulsations (21 are integer $\times f_{\rm orb}$)

$22.42 f_{\rm orb} \rightarrow 91 f_{\rm orb}$

Welsh et al 2012

amplitude (μmag)	f/f_{orbit}
297.7	90.00
229.4	91.00
97.2	44.00
82.9	40.00
82.9	22.42
49.3	68.58
30.2	72.00
17.3	63.07
15.9	57.58
14.6	28.00
13.6	53.00
13.4	46.99
12.5	39.00
11.6	59.99
11.5	37.00
11.4	71.00
11.1	25.85
9.8	75.99
9.3	35.84
9.1	27.00
8.4	42.99
8.3	45.01
8.1	63.09
6.9	35.99
6.8	60.42
6.4	52.00
6.3	42.13
5.9	33.00
5.8	29.00
5.7	48.00

Tidally Forced Oscillations: Flux Variations



Fuller & DL 2012

Most of the observed flux variations are explained by m=0 modes (more visible for near face-on orientation)

Variations at 90,91 harmonics require very close resonances $(N\Omega = \omega_\alpha)$

Why N=90,91 ?

The probability of seeing high-amplitude modes

Consider mode near resonance $\omega_\alpha = (N + \epsilon)\Omega$

- By chance

$$P_{|\epsilon|<\epsilon_0} \simeq 2\epsilon_0$$

likely for N=20-80 ($\epsilon_0 \sim 0.1$)

- If mode dominates tidal energy transfer

$$P_{|\epsilon|<\epsilon_0} = \frac{\Delta t_{\rm res}}{\Delta t_{\rm nonres}} \sim \frac{8\pi^2}{3}\epsilon_0^3$$

unlikely for N=90,91 $(\text{require } \epsilon_0 < 0.01)$

Resonance Locking

- Tidal excitation of modes ==> Orbitdal decay, spinup of star,
  change mode frequency

$$\omega_\alpha = \omega_\alpha^{(0)} + m B_\alpha \Omega_s$$

- At resonance, $\dfrac{\omega_\alpha}{\Omega} = N$

- Mode can stay in resonance if $\dfrac{d}{dt}\left(\dfrac{\omega_\alpha}{\Omega}\right) = 0$ or $\left(\dfrac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide} = \left(\dfrac{\dot{\Omega}}{\Omega}\right)_{\rm tide}$

==> $N_c = m\left(\dfrac{B_\alpha \mu a^2}{3I}\right)^{1/2} \simeq 130 - 145$

$$\left(\frac{\dot{\Omega}}{\Omega}\right)_{\rm tide} = \left(\frac{N}{N_c}\right)^2 \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide}$$

Resonance Locking (continued)

Including intrinsic stellar spin-down torque:

$$\dot{\Omega}_s = (\dot{\Omega}_s)_{\rm tide} + (\dot{\Omega}_s)_{\rm sd}$$

===>

$$\frac{\dot{\omega}_\alpha}{\omega_\alpha} = \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide} + \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm sd}$$

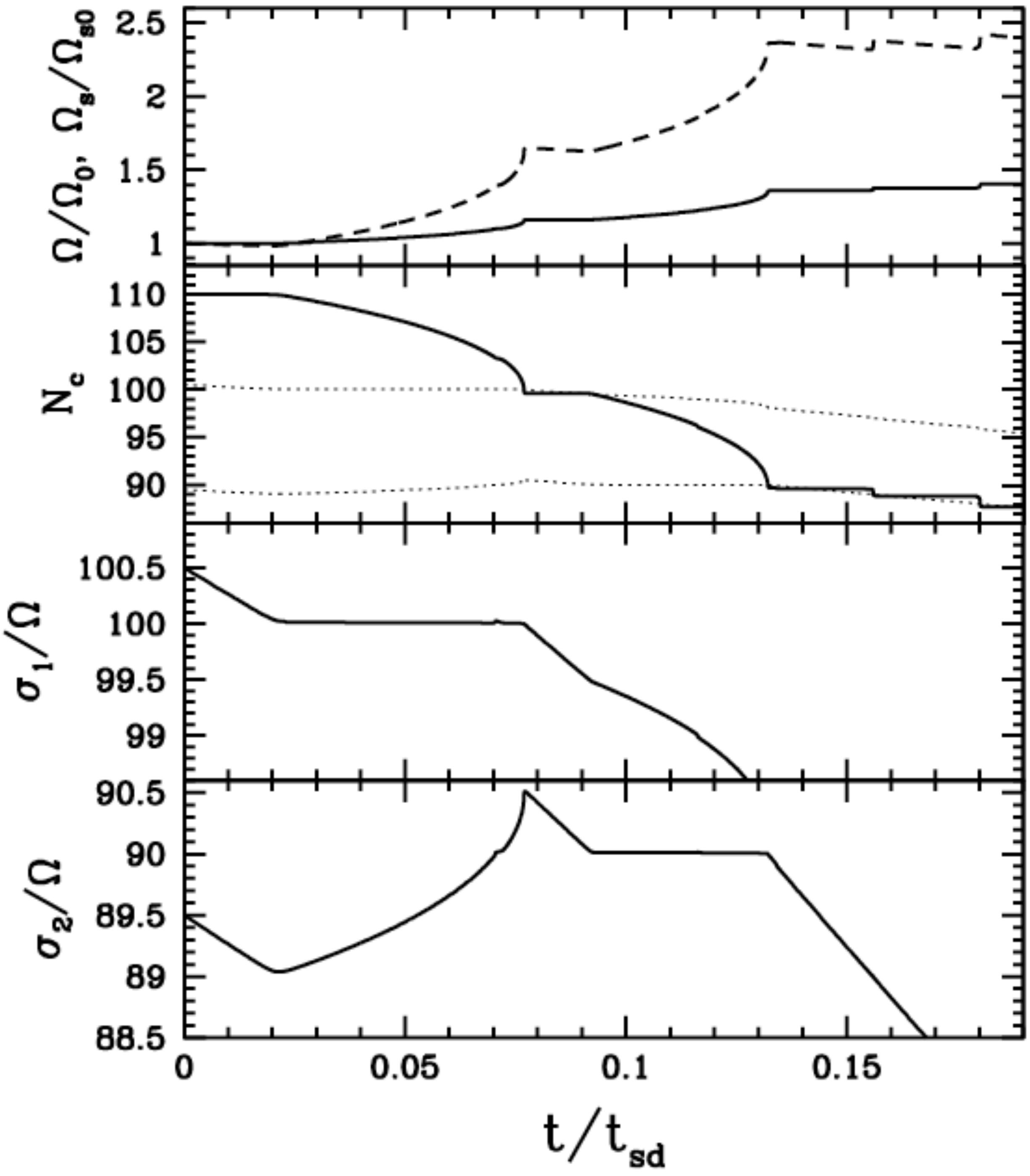
$$\frac{\dot{\Omega}}{\Omega} = \left(\frac{\dot{\Omega}}{\Omega}\right)_{\rm tide} = \left(\frac{N}{N_c}\right)^2 \left(\frac{\dot{\omega}_\alpha}{\omega_\alpha}\right)_{\rm tide}$$

===> **Mode can lock into resonance if** $N < N_c$

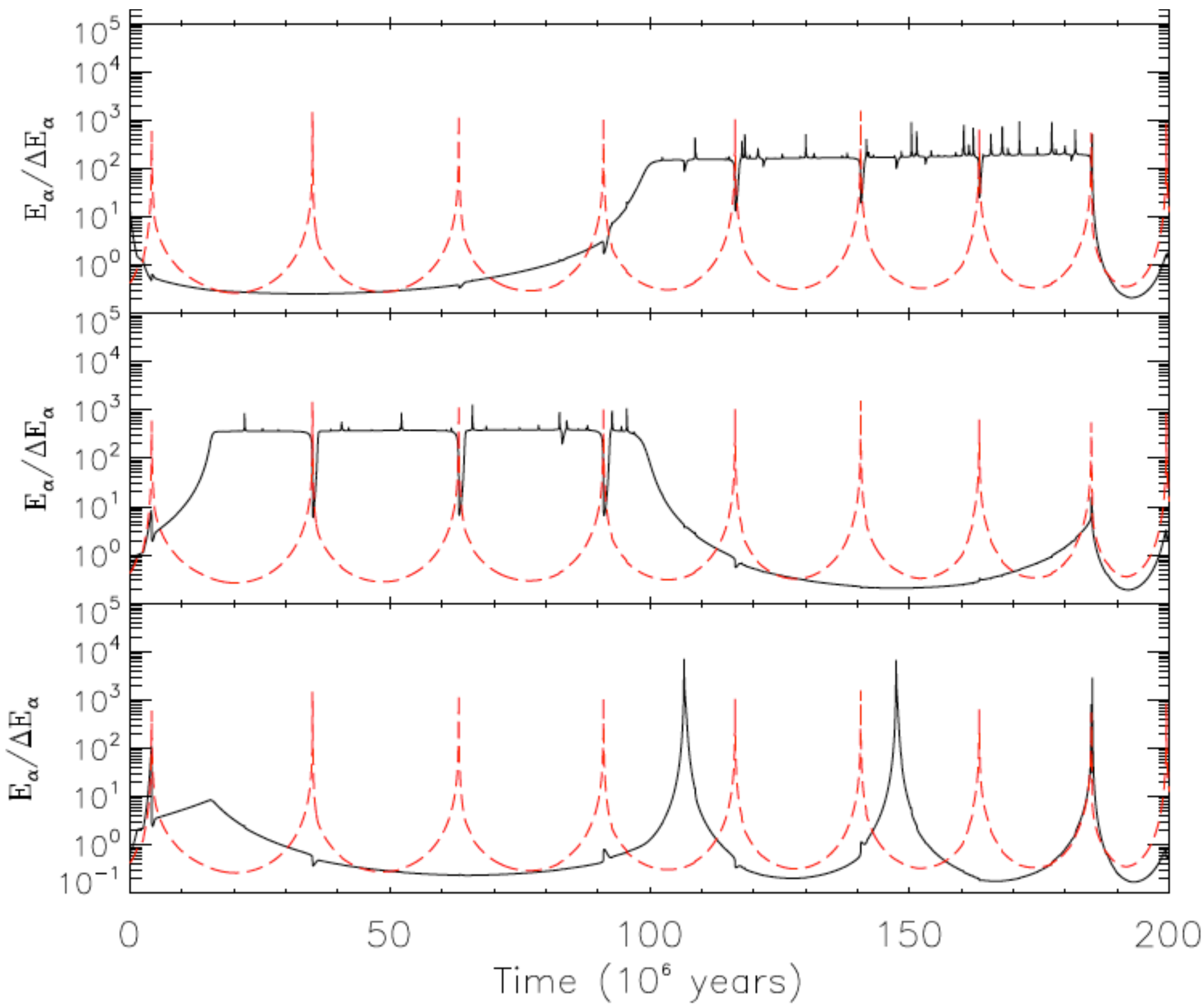
$$\frac{\omega_\alpha}{\Omega} < N_c$$

Resonance Locking: Numerical Examples

Coupled evolution of orbit, spin and mode amplitudes…



Ω/Ω_0, Ω_s/Ω_{s0}
2.5
2
1.5
1
N_c
110
105
100
95
90
σ_1/Ω
100.5
100
99.5
99
σ_2/Ω
90.5
90
89.5
89
88.5
0
0.05
0.1
0.15
t/t_{sd}



10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
$E_\alpha/\Delta E_\alpha$
0
50
100
150
200
Time (10^6 years)

Resonance Locking in Both Stars

- Locking in one star:

$$N_c = m \left(\frac{B_\alpha \mu a^2}{3I} \right)^{1/2} \simeq 130 - 145$$

- Similar modes are locked simultaneously in both stars

$$N_c = 92 - 102$$

- Explain the observed N=90,91 harmonics

Non-Linear Mode Coupling

- 9 oscillations detected at non-integer multiples of orbital frequencies
- Could be produced by nonlinear coupling to daughter modes

$$\omega_p = \omega_{d1} + \omega_{d2}$$

- In KOI-54,

$$\frac{\omega_2}{\Omega} = 91.00 \qquad \frac{\omega_5}{\Omega} = 22.42 \qquad \frac{\omega_6}{\Omega} = 68.58$$

- Other non-integer modes likely due to nonlinear coupling in which one of the daughter modes is invisible

amplitude (μmag)	f/f_{orbit}
297.7	90.00
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Summary: Lessons from KOI-54 (Heartbeat Stars)

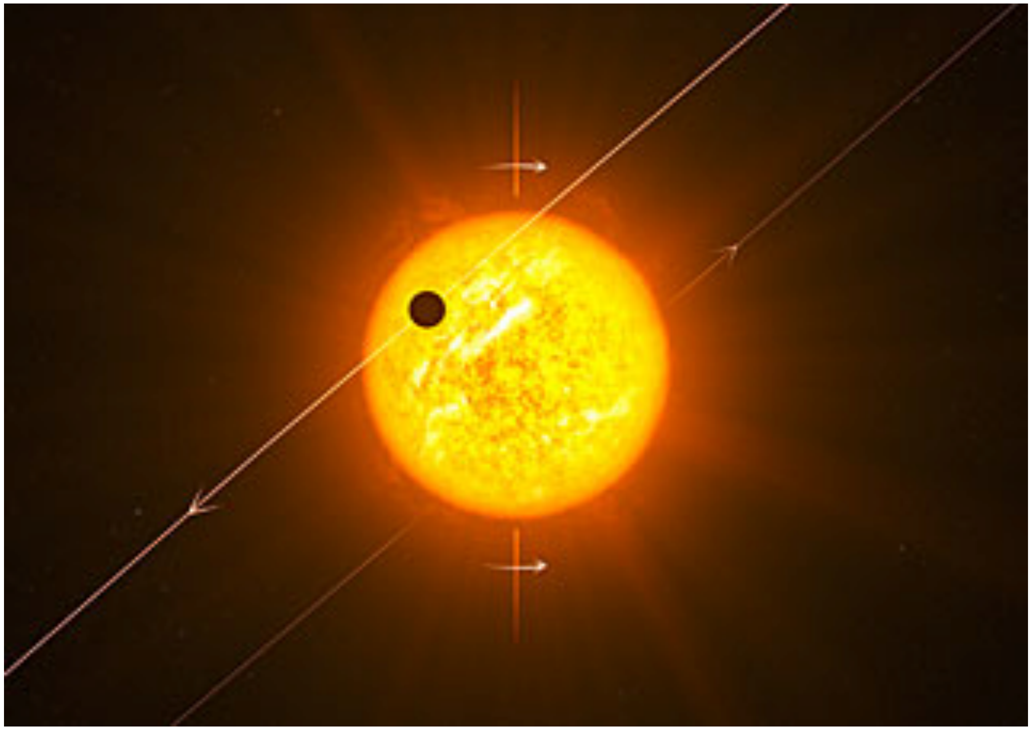
- Direct detection of tidally excited oscillations in eccentric binary
  ==> Dynamical tides at work

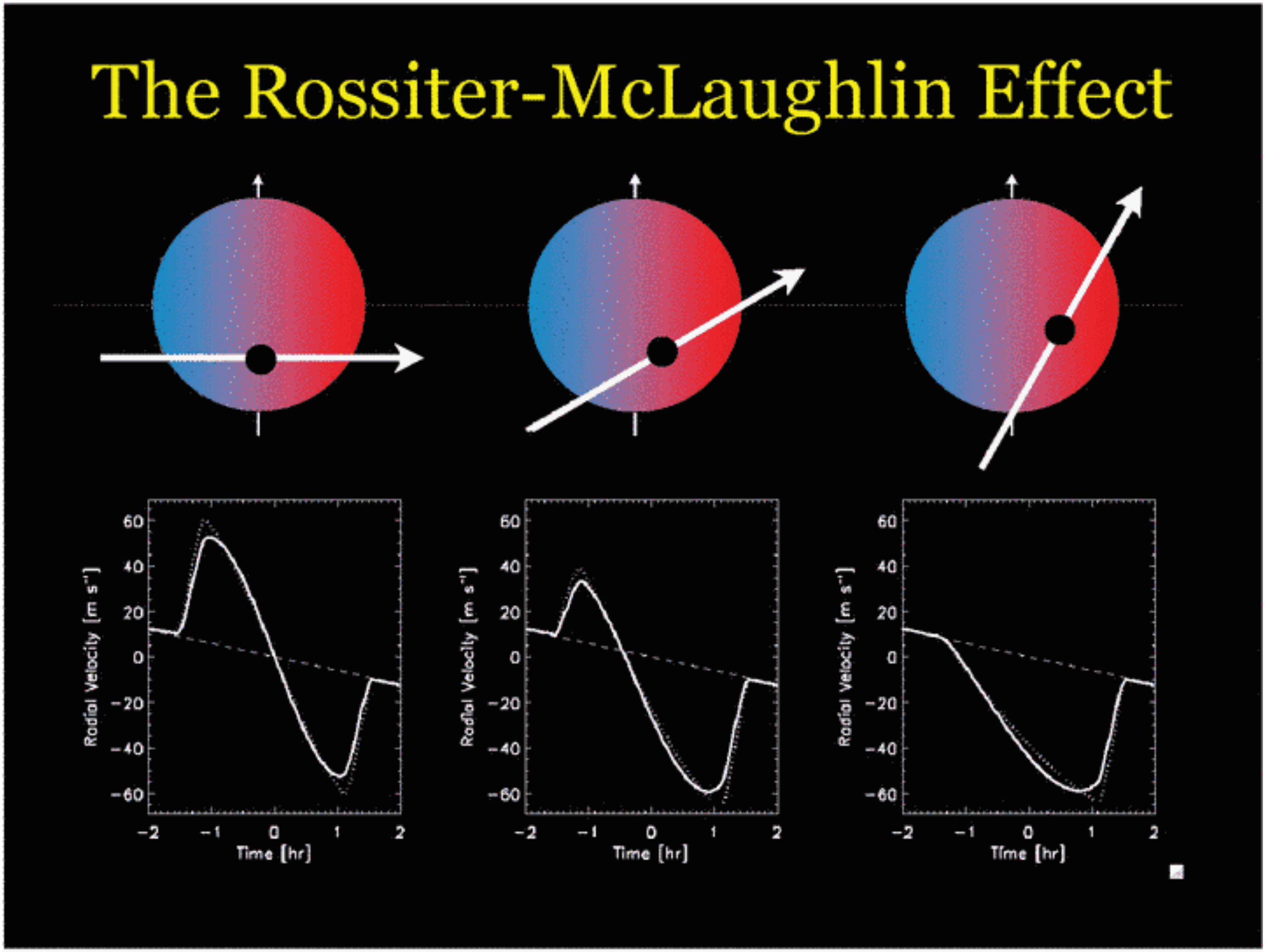
- Resonance locking

- First direct evidence of nonlinear mode coupling

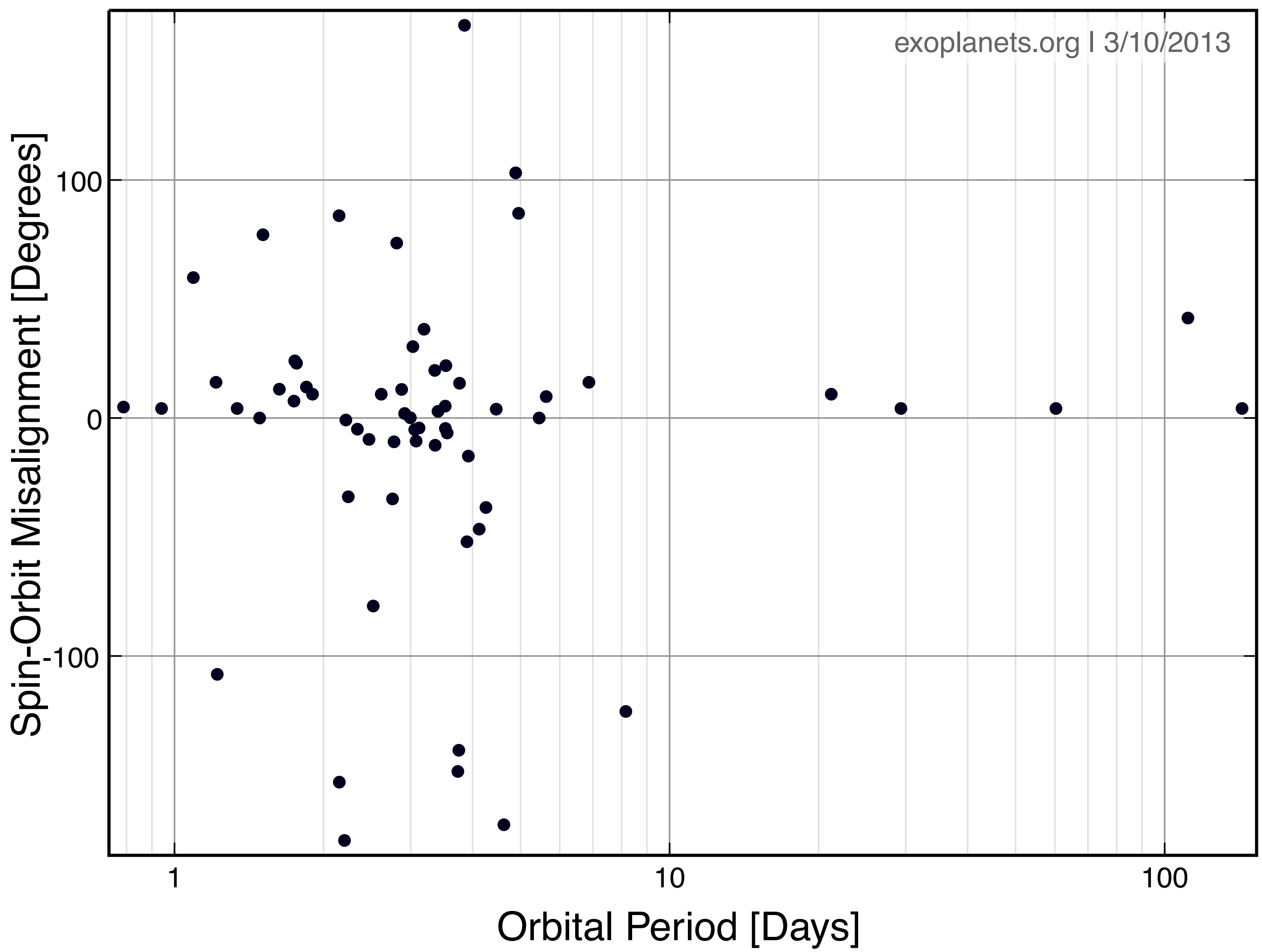
- More such systems …

Tides in Exoplanetary Systems





Slide from Josh Winn


exoplanets.org | 3/10/2013
Spin-Orbit Misalignment [Degrees]
100
0
-100
1
10
100
Orbital Period [Days]

S^*-L_p misalignment in Exoplanetary Systems
➔ The Importance of few-body interactions

1. Kozai + Tide migration by a distant companion star/planet

   (e.g., Wu & Murray 03; Fabrycky & Tremaine 07; Naoz et al.12)

2. Planet-planet Interactions

   -- Strong scatterings

   (e.g., Rasio & Ford 96; Chatterjee et al. 08; Juric & Tremaine 08)

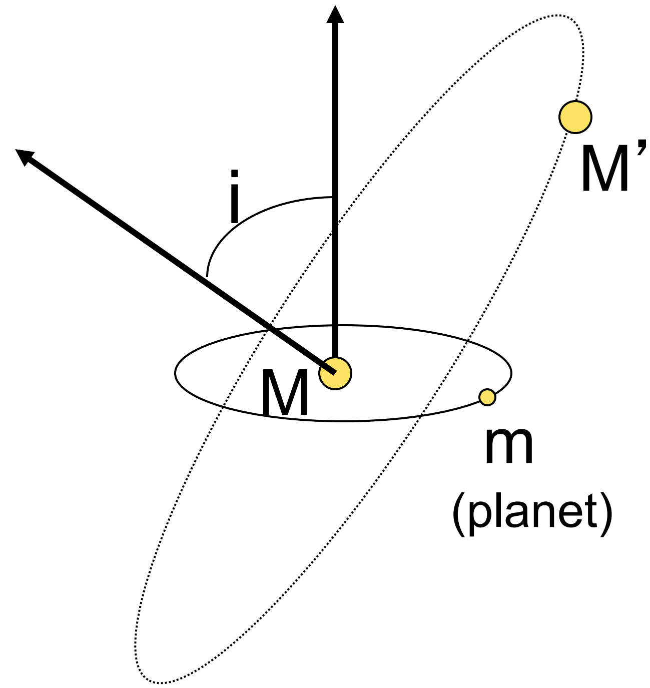
   -- Secular interactions (“Internal Kozai”, chaos) + Tide

   (e.g Nagasawa et al. 08; Wu & Lithwick 11; Naoz et al.11)

Misaligned protostar - protoplanetary disk ? (e.g. Solar system)

(Bate et al.2010; DL, Foucart & Lin 2011; Batygin 2012)

Kozai Migration with Tide



Kozai (1962), Lidov (1962):

When $i > \cos^{-1}\sqrt{\frac{3}{5}} \simeq 40^\circ$ (and $i < 140^\circ$),

the orbit of planet oscillates in *e* and *I*

$$\sqrt{GMa(1-e^2)}\,\cos i = \text{const}, \quad a = \text{const}$$

$$\Longrightarrow \quad e_{\max}^2 = 1 - \frac{5}{3}\cos^2 i_{\text{initial}}$$

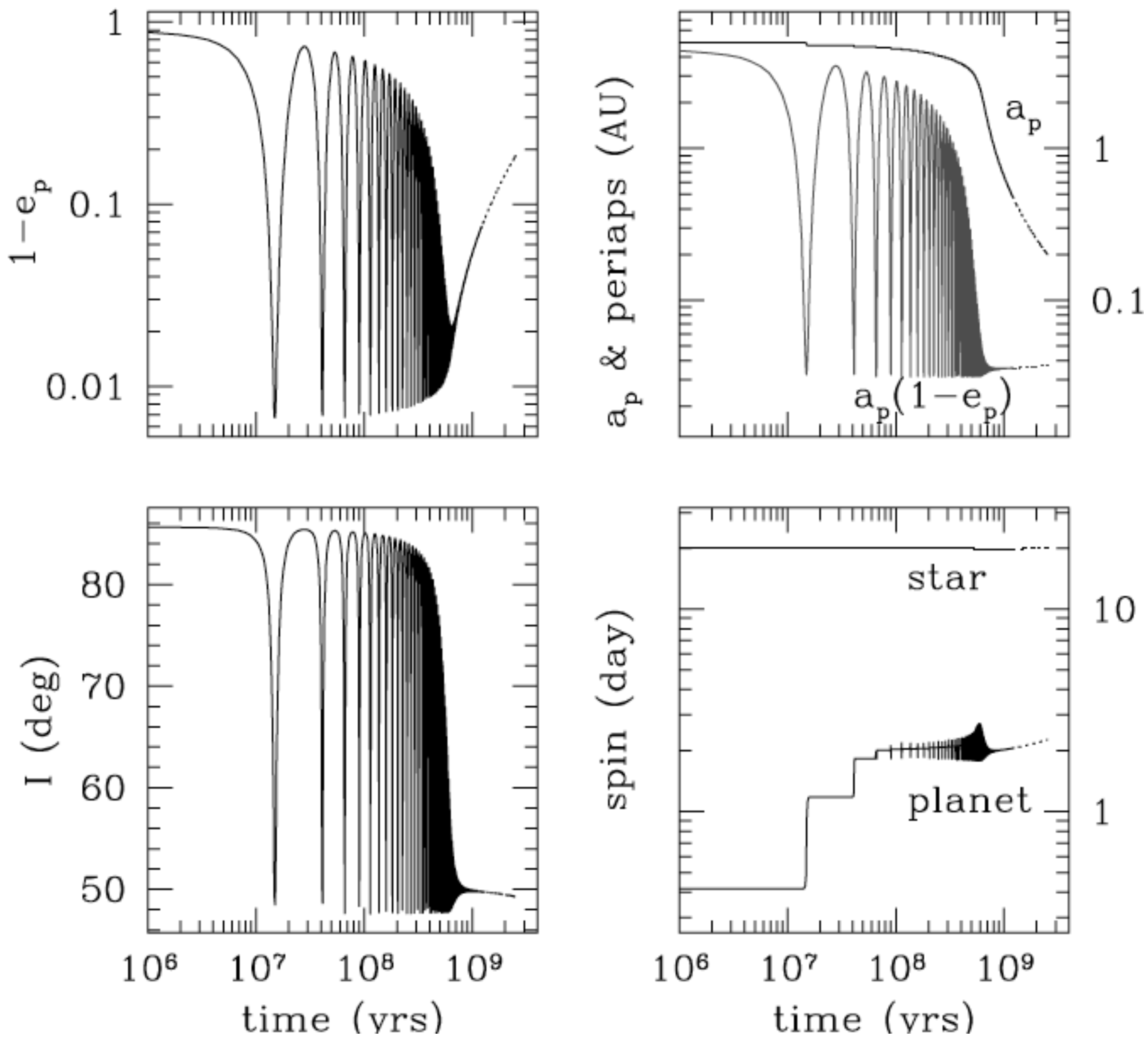
$$P_{\text{Kozai}} \sim \frac{M}{M'}\frac{(P')^2}{P_p}$$

Importance of higher-order effect (Naoz et al.2011; Katz et al.2011)

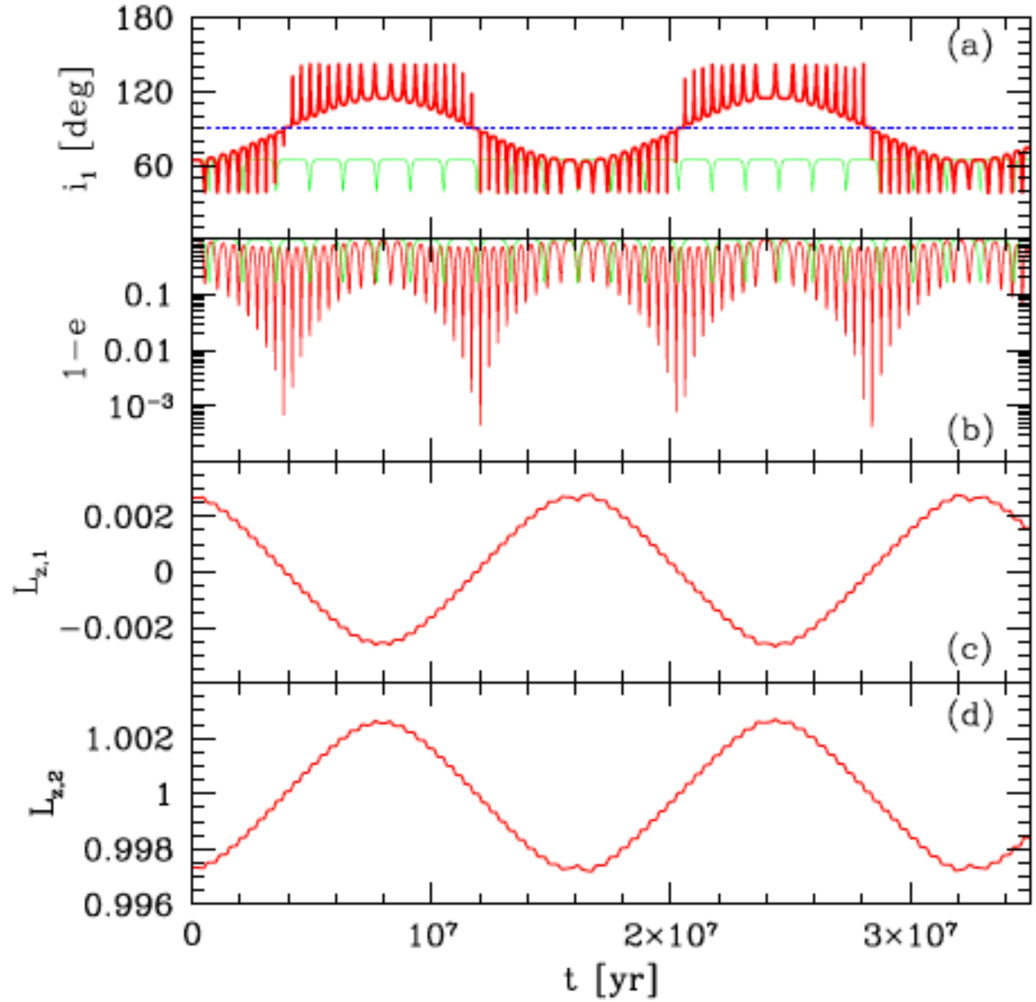
Tidal dissipation in planet:

Circluarize the orbit at small radius

Kozai Migration with Tide



Wu & Murray 2003



Naoz et al. 2011

High-e Migration requires tidal dissipation in giant planets

Tidal Q of Solar System Planets

Measured/constrained by orbital evolution of their satellites
(Goldreich & Soter 1966,……)

Jupiter:

$$6 \times 10^4 \lesssim Q \lesssim 2 \times 10^6 \qquad P_{\rm tide} = 6.5\ {\rm hr}$$

$Q \simeq 4 \times 10^4$ (Lainey et al. 2009)

Saturn:

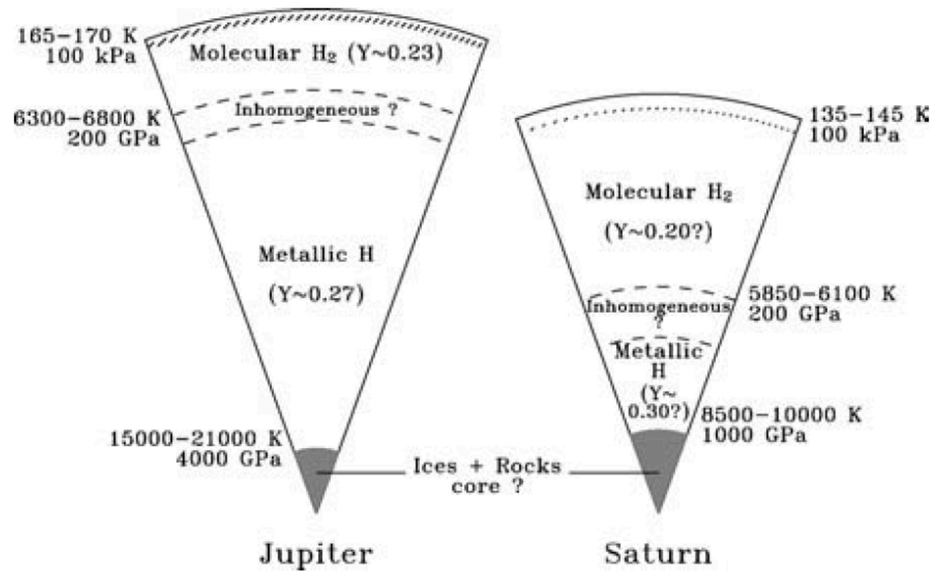
$$2 \times 10^4 \lesssim Q \lesssim 10^5$$

$Q = (1 - 2) \times 10^3$ (Lainey et al. 2012)

Theory of Tidal Q of Giant Planets

-- Viscous (turbulent) dissipation of equilibrium tide in convective envelope ➔ $Q>10^{13}$

-- Gravity waves in outer radiative layer ➔ $Q>10^{10}$



-- Inertial waves

(Ogilvie & Lin 2004,07; Ogilvie 2009,13; Goodman & Lackner 2009)

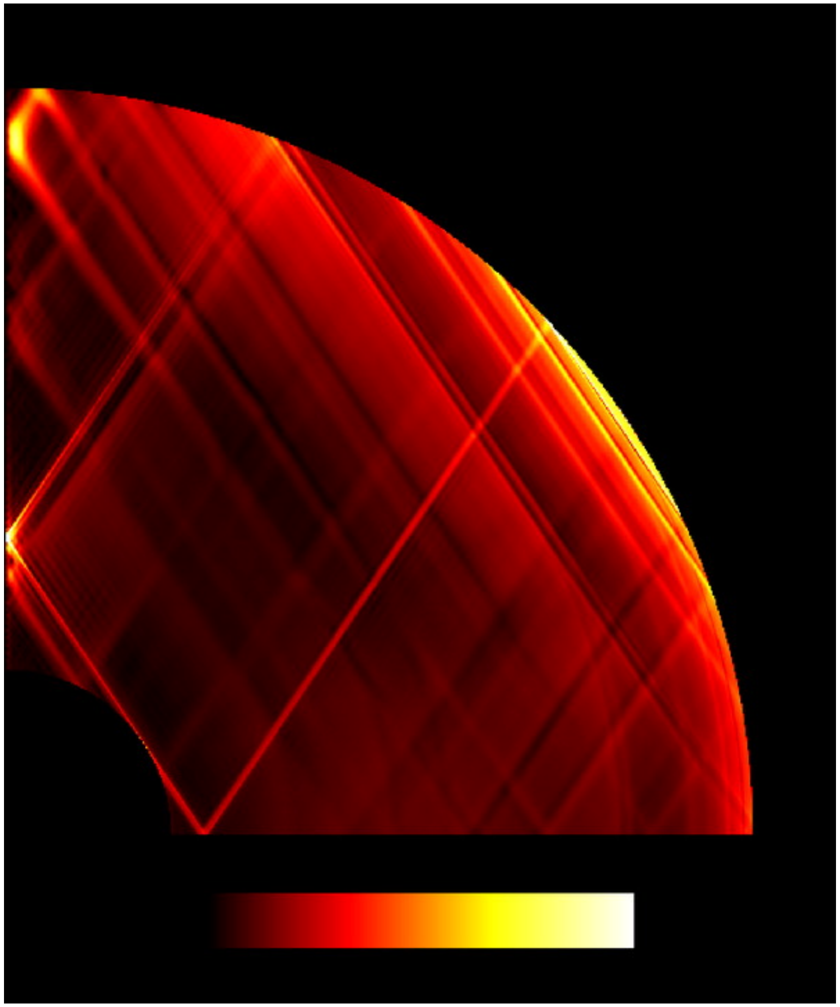
Inertial Waves in Rotating Fluid

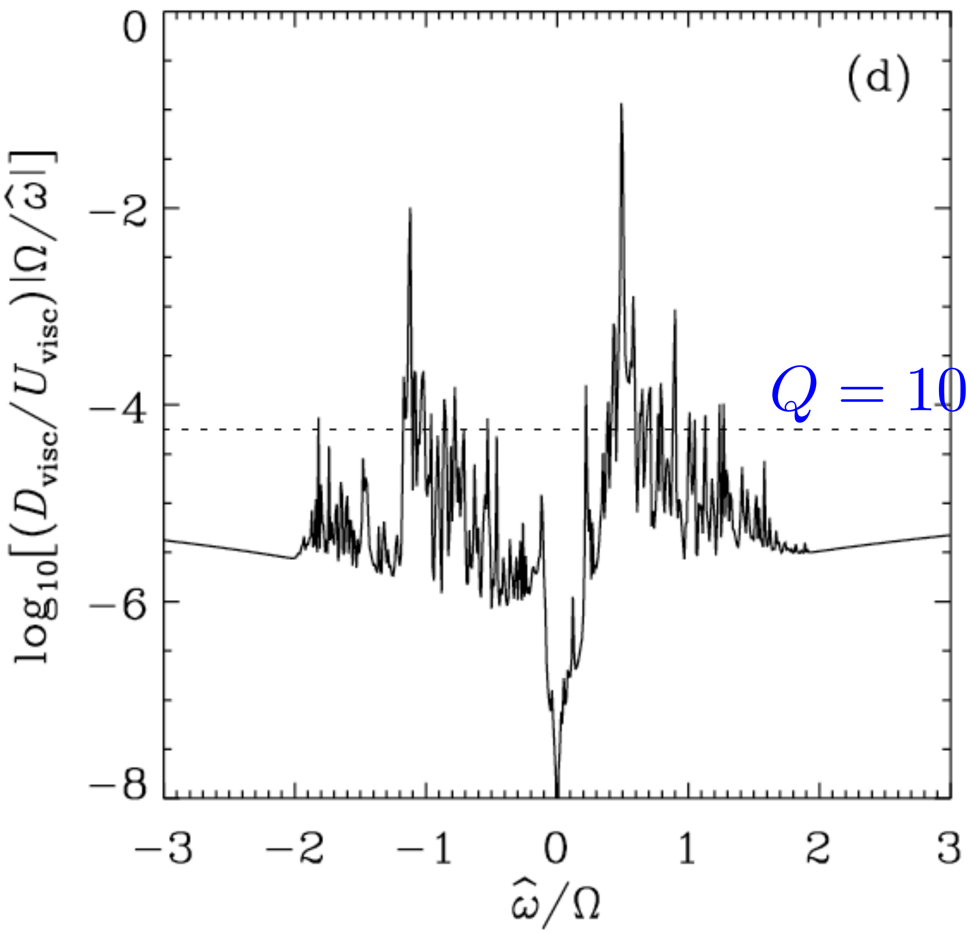
Dispersion relation (in rotating frame)

$$\omega = \pm 2\,\boldsymbol{\Omega}_s \cdot \hat{\mathbf{k}}$$

Can be excited if tidal forcing frequency satisfies

$$|\omega| < 2\Omega_s$$





Ogilvie & Lin 2004

Tidal Dissipation in High-e Migration: Phenomenological Approach

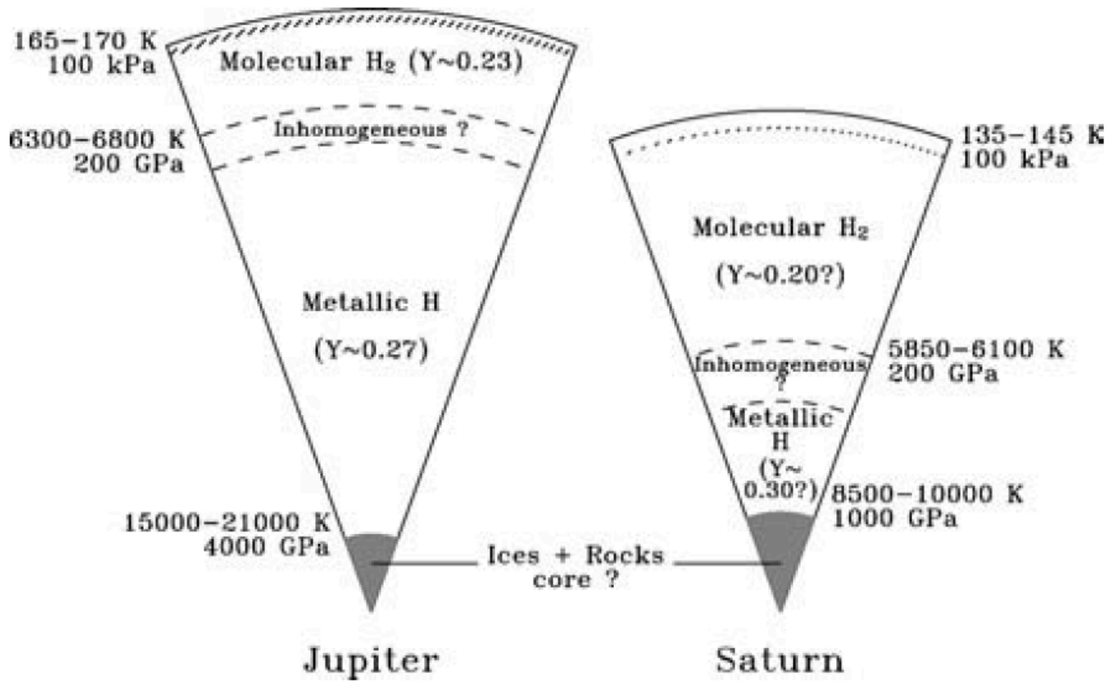
Using measured Q for Jupiter (single freq), extrapolate to high-e (many frequencies) using weak friction theory: (many papers…)

$$\delta \sim \omega_{\rm tide} \Delta t_{\rm lag} \sim 1/Q \qquad \text{with} \quad \Delta t_{\rm lag} = \text{const}$$

➔ Hot Jupiters need to be >10 times more dissipative than our Jupiter (e.g., Socrates et al 2013)

Tidal Dissipation in Solid Core of Giant Planets

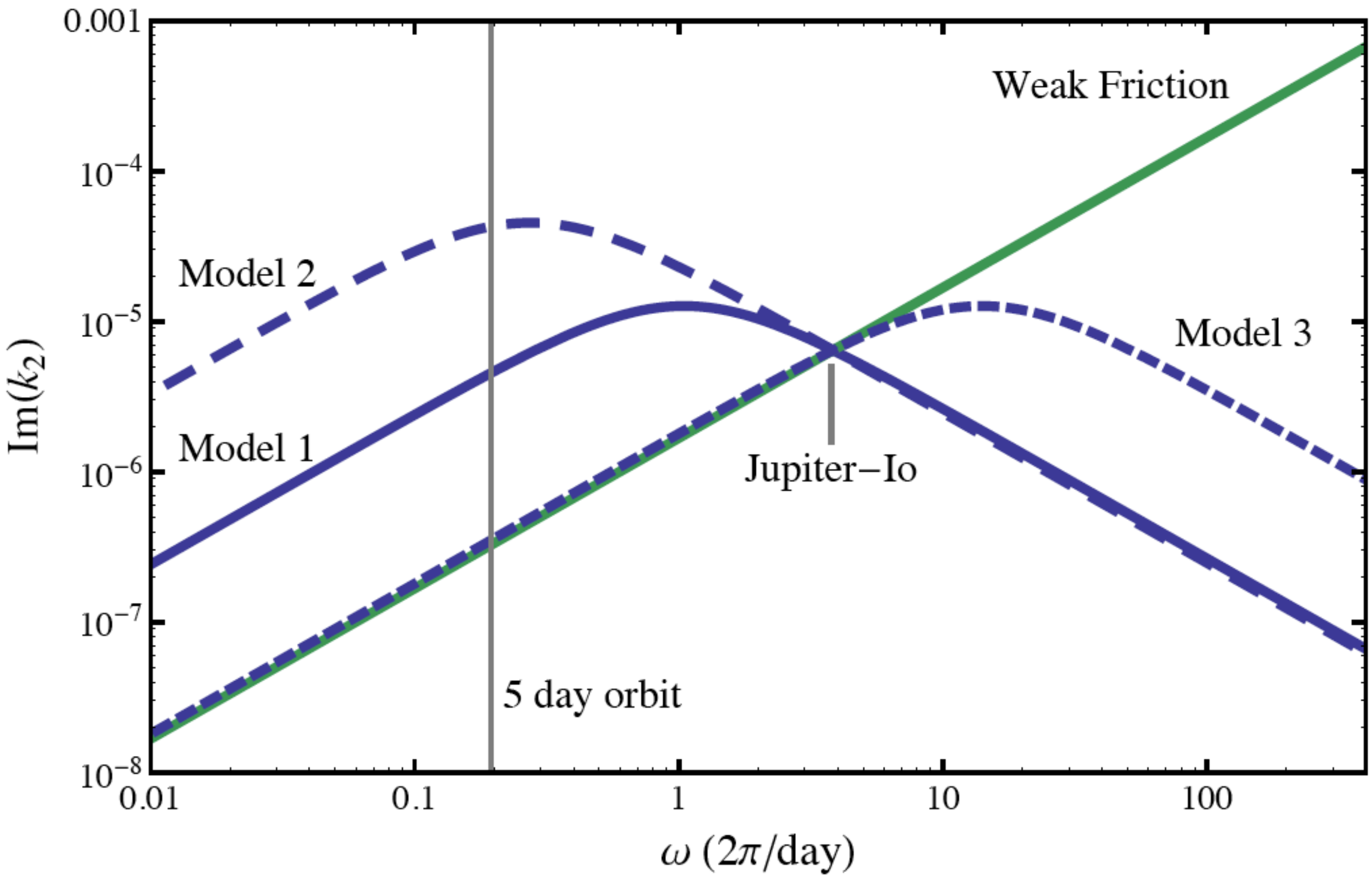
with Natalia Shabaltas



Rocky/icy core: highly uncertain …
Can be important if $R_{\rm core} \gtrsim 0.1 R_p$

Tidal response of Jupiter with visco-elastic core

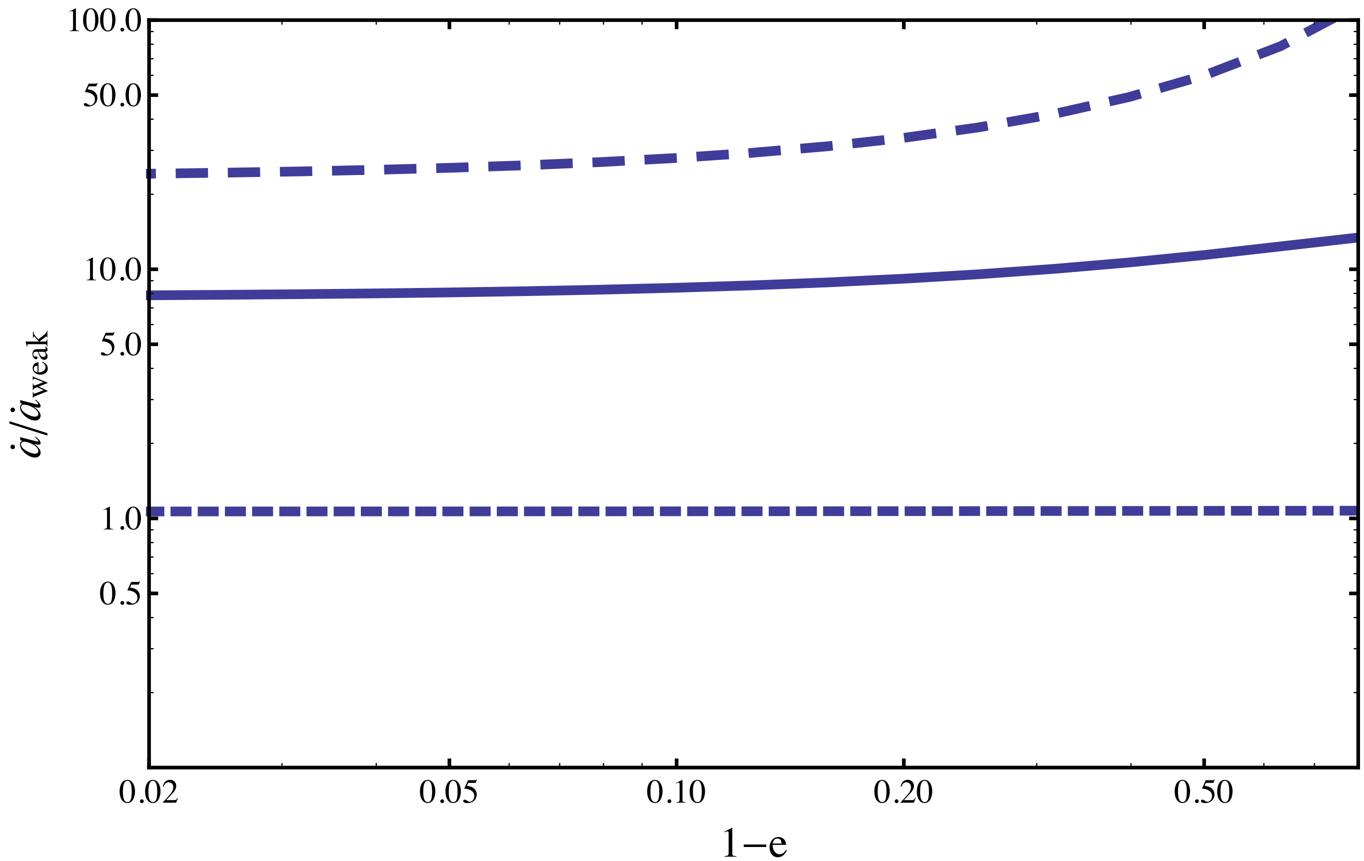
$$\frac{k_2}{Q}$$



Shabaltas & DL 2013

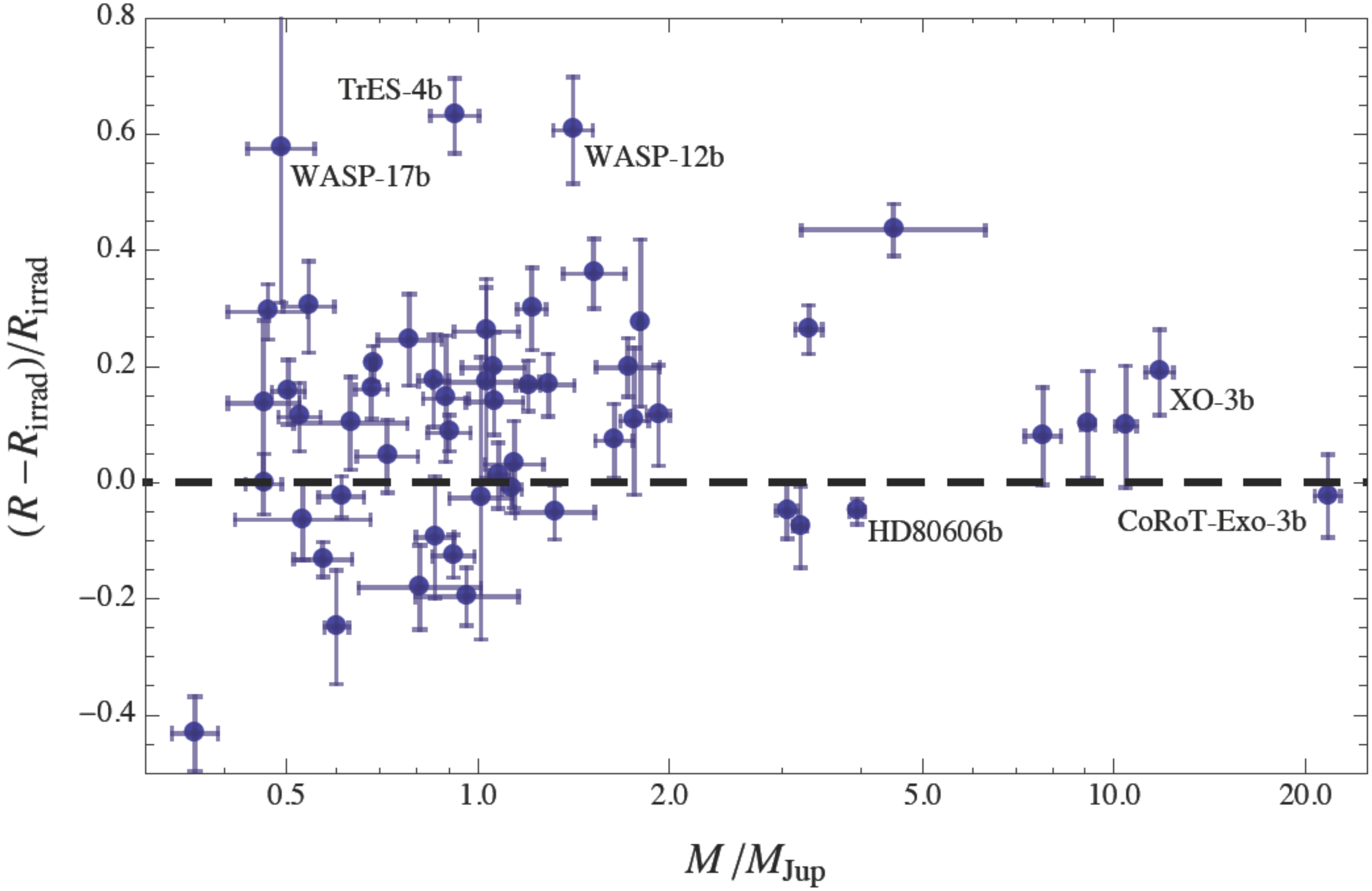
Orbital decay of proto hot Jupiters

(visco-elastic core vs weak friction)



Shabaltas & DL 2013

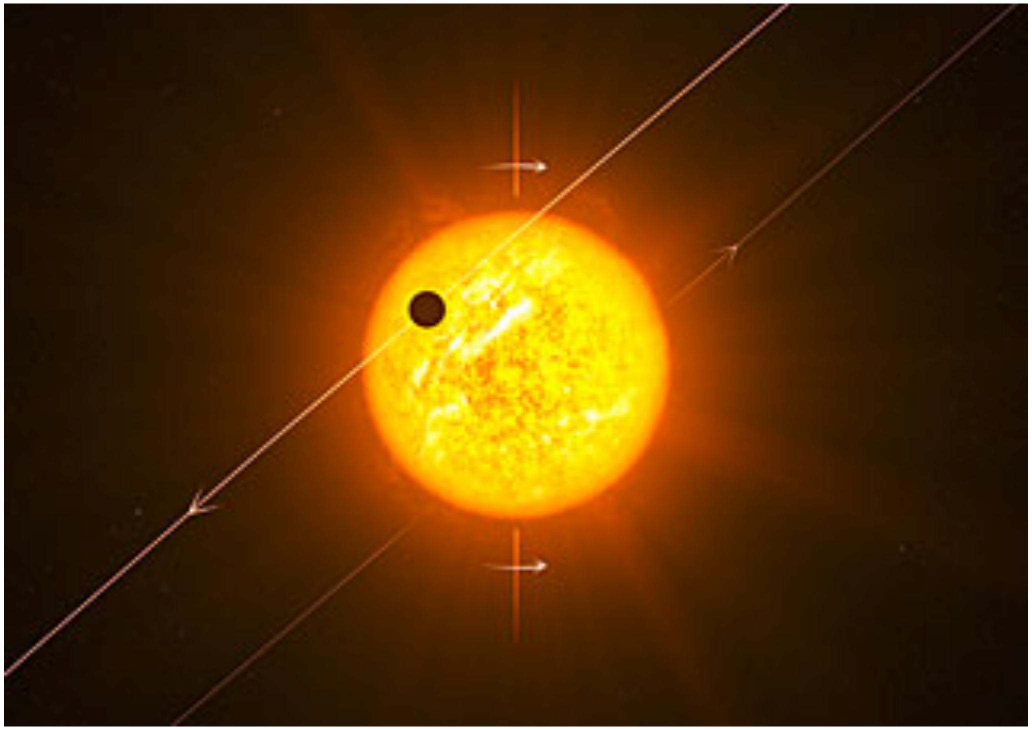
Hot Jupiter Radius Anomaly



Leconte et al. 2010

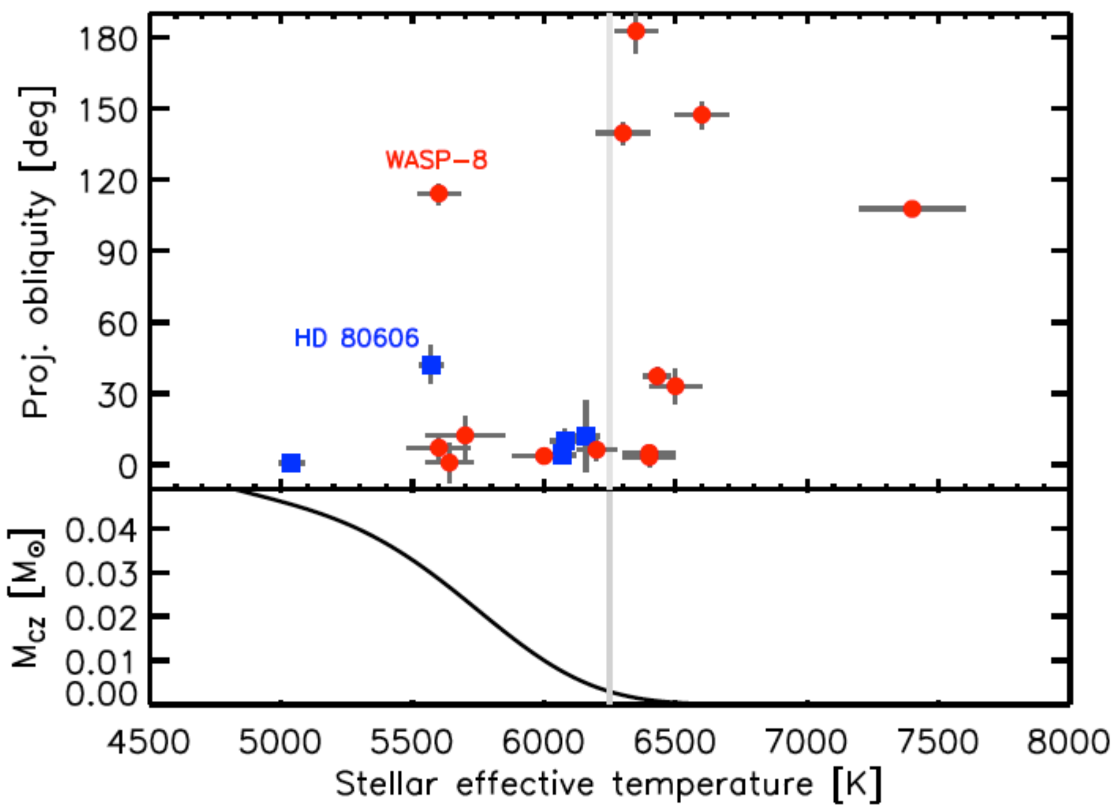
Tidal Dissipation in Planet Host Stars:

Misalignment Damping and Survival of Hot Jupiters



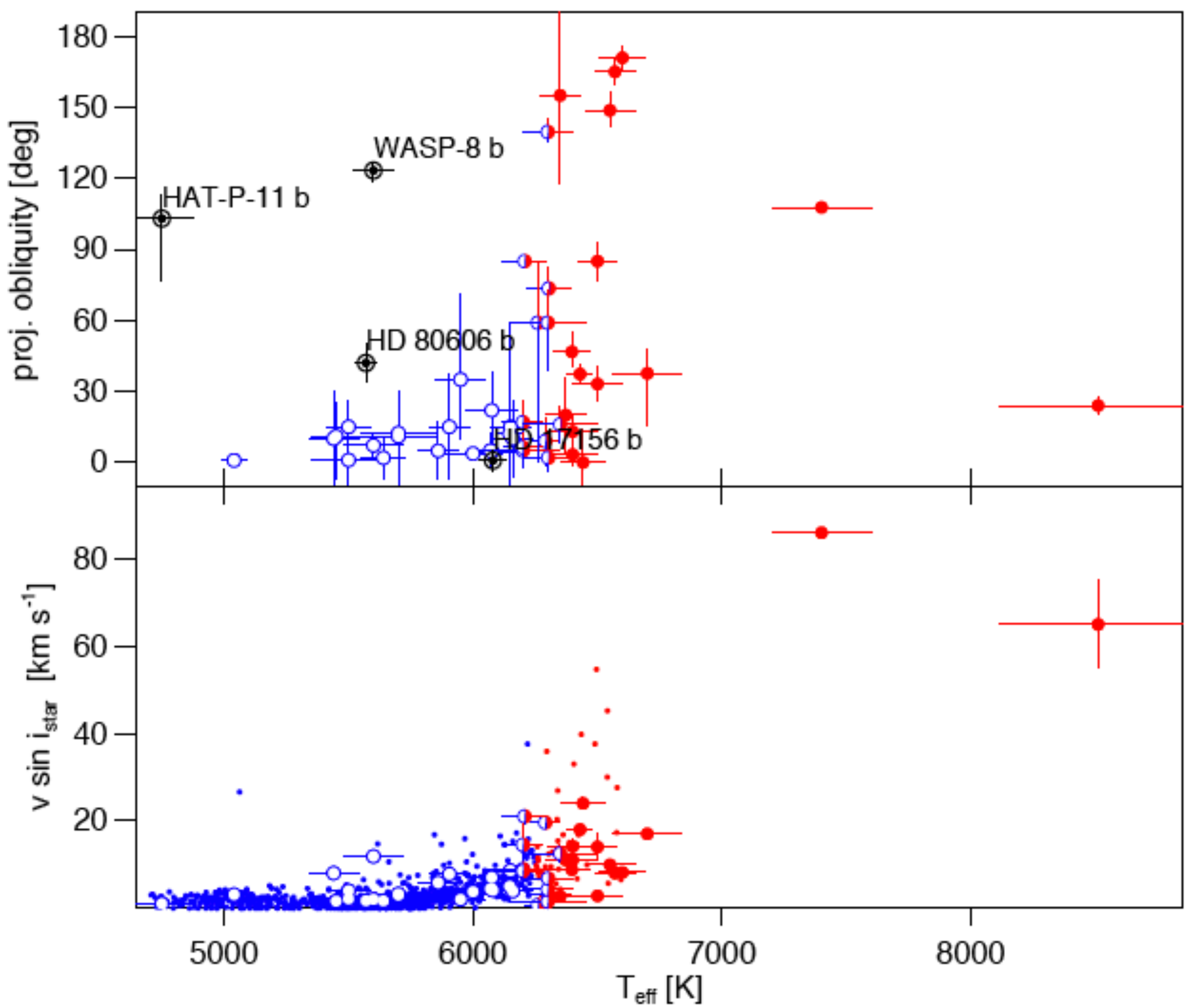
Correlation: Misalignment -- Stellar Temperature/Mass

Winn et al. 2010; Schlaufman 2010



Winn et al.2010

Correlation: Misalignment -- Stellar Temperature/Mass



Albrecht, Winn, et al 2012

Correlation: Misalignment -- Stellar Age

Triaud 2011

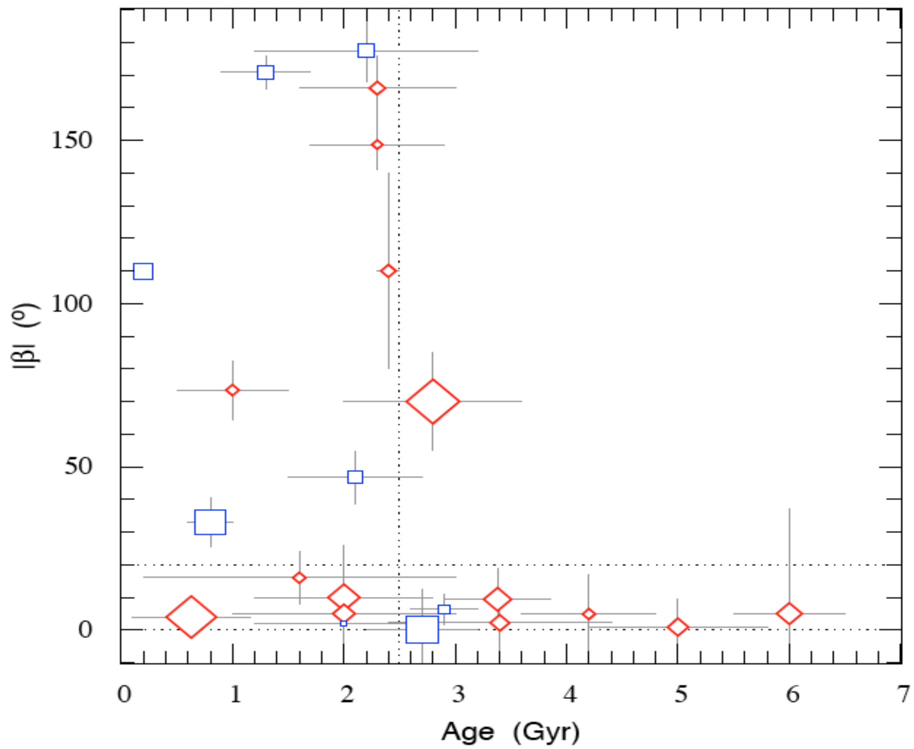


Fig. 2. Secure, absolute values of β against stellar age (in Gyr), for stars with $M_\star \geq 1.2\,M_\odot$. Size of the symbols scales with planet mass. In blue squares, stars with $M_\star \geq 1.3\,M_\odot$; in red diamonds $1.3 > M_\star \geq 1.2\,M_\odot$. Horizontal dotted line show where aligned systems are. Vertical dotted line shows the age at which where misaligned planets start to disappear.

Reasonable Hypothesis:

Many hot Jupiters are formed in misaligned orbits

Tidal damping of misalignment (especially for cooler stars)

Problem with Equilibrium Tide (with the parameterization…)

$$t_{\rm decay} \simeq 1.3 \left(\frac{Q'_\star}{10^7}\right) \left(\frac{M_\star}{10^3 M_p}\right) \left(\frac{P_{\rm orb}}{1\,{\rm d}}\right)^{13/3} {\rm Gyr}$$

$$\frac{t_{\rm align}}{t_{\rm decay}} \simeq \frac{2S_\star}{L} \simeq 2 \left(\frac{M_\star}{10^3 M_p}\right) \left(\frac{10\,{\rm d}}{P_s}\right) \left(\frac{1\,{\rm d}}{P_{\rm orb}}\right)^{1/3}$$

Possible Solution: (see DL 2012)

Different Tidal Q's for Orbital Decay and Alignment ?

Tidal Forcing Frequency=?

For aligned system

$$\omega = 2(\Omega_{\rm orb} - \Omega_s)$$

For misaligned system

$$\omega = m'\Omega_{\rm orb} - m\Omega_s \qquad m, m' = 0, \pm 1, \pm 2$$

7 physically distinct components

==> Effective tidal evolution equations with 7 different Q’s

Inertial Waves in Rotating Fluid

Dispersion relation (in rotating frame)

$$\omega = \pm 2\,\mathbf{\Omega}_s \cdot \hat{\mathbf{k}}$$

Can only be excited if tidal forcing frequency satisfies

$$|\omega| < 2\Omega_s$$

Stellar Tides in Hot Jupiter Systems

For aligned system:

$$\omega = 2(\Omega_{\rm orb} - \Omega_s) \gg \Omega_s$$

==> Cannot excite inertial waves

For misaligned system:

$$\omega = m'\Omega_{\rm orb} - m\Omega_s$$

The m’=0, m=1 component has $\omega = -\Omega_s$

This component leads to alignment, but not orbital decay

Summary: Tides in Hot Jupiter Systems

Tidal dissipation in giant planets:

-- Required for high-e migration
-- Inertial wave excitation?
-- Dissipation in solid core?

Tidal dissipation in host stars:

-- Spin-orbit misalignment may be damped faster than orbital decay
-- Different Q's for different processes (equilibrium tide parameterization misleading)

Thanks !