A Toy Model of the Multiple Bands and Zonal Jets on the Giant Planets

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Abstract. A simple model to explain the large-scale atmospheric dynamics of the four giant planets (Jupiter, Saturn, Uranus, and Neptune) is discussed. We propose that the large-scale rising motion, which is concentrated in some particular latitudes, drives the large-scale atmospheric structures (zonal bands and jets) on the four giant planets. The physics behind the large-scale rising motion, which is far from known, is also discussed in this note.

1. Introduction
The multiple zonal jets and the banded structures on the four giant planets have long been a puzzling feature. The studies of the banded structures are very limited, but there are various theories to explain the multiple zonal jets. The “deep-convection” hypothesis, which suggests that the multiple jets on the giant planets might be the surface expression of deep flows of cylinders parallel to the rotation axes, is proposed by Busse (1976, 1983) and is tested by a few numerical models (Sun et al., 1993; Christensen 2001; Aurnou and Olson 2001; Aurnou and Heimpel, 2004). The above theory and numerical experiments can explain the multiple jets in the low and middle latitudes of the gas giants (Jupiter and Saturn), but they do not work for the high-latitude jets in the region inside the tangent cylinder along the internal cores. The fully-developed 3-dimensional (3D) Boussinesq convection model, which is still based on the “deep convection” hypothesis, successfully extends the simulations to high latitudes by changing spherical shell geometries (Heimpel et al., 2005; Heimple and Aurnou, 2007). One recent work (Aurnou et al., 2007) shows that the 3D Boussinesq convection model is applicable to simulate the retrograde (eastward) equatorial jets on the ice giants (Uranus and Neptune) if buoyancy forces approach and exceed Coriolis forces. However, the internal heat of Uranus and Neptune, which is required to drive the deep convection, is not realistic in the 3D Boussinesq convection model (Aurnou et al., 2007). In addition, Liu et al. (2008) show that the “deep convection” hypothesis encounters difficulty when taking into account the magnetic field imposed by the deep zonal flow of an electrically conducting fluid.

The alternate hypothesis for the multiple jets on the four giant planets is a theory of zonal jets emerging from decaying turbulence, which was first suggested by Rhines (1975). This theory has already been tested in shallow-water (SW) models (Cho and Polvani, 1996; Showman, 2007) and quasi-geostrophic (QG) models (Williams, 1978; Panetta, 1993; Huang and Robinson, 1998; Marcus et al., 2000; Li et al., 2005) for the multiple jets on Jupiter and Saturn. These models all successfully generate multiple jets on Jupiter and Saturn. However, the assumption of the QG models does not hold near the equator so that the above QG models cannot simulate the equatorial jets on the four giant planets. The SW models work in the equatorial regions, but their generated equatorial jets are always westward, which is in agreement with the observed jets on Uranus and Neptune, but in disagreement with the observed eastward jets on Jupiter and Saturn.
2. Theory
The four giant planets exhibit a remarkable similarity in their large-scale dynamics. Jupiter, Uranus, and Neptune all have alternating eastward and westward jets. Anderson and Schubert (2007) suggest that multiple jets on Saturn flow both east and west in a frame of more rapid spin (Fig.1), as on the other three giant planets. The alternating eastward and westward jets on the four giant planets display good mirror symmetry across the equator even though the obliquities of the four planets are quite different (Jupiter 3°, Saturn 27°, Uranus, 98°, and Neptune 28°).

Figure 1. The observed zonal winds and their axisymmetric approximations on the four giant planets. (A) Jupiter (Porco et al., 2003). (B) Saturn (Ingersoll et al., 1984; Porco et al., 2005; Anderson and Schubert, 2007). (C) Uranus (Hammel et al., 2001). (D) Neptune (Sromovsky et al., 2001). The blue lines are the observed zonal winds. The red lines are the approximations by assuming that the wind profiles are exactly axisymmetric. The red stars represent the locations of the peaks of westward and eastward jets, which are determined by the centers of meridional cells shown in the Table 1. The observed winds on Saturn (B) are a combination of the wind measurements from Voyager (Ingersoll et al., 1984) and Cassini (Porco et al., 2005). In addition, the ~ 7-minute difference of the rotation period of Saturn between the Voyager measurement (Desch and Kaiser, 1981) and the recent measurement (Anderson and Schubert, 2007) has been taken into account, which suggests that the winds at higher latitudes of Saturn flow both east and west.

The emitted power, which is related to the energy balance, also shows some mirror symmetry across the equator (Fig. 2). Figure 2 further shows that a center of minimal emitted power is located around the equator on Jupiter and Saturn, which corresponds to the bright equatorial region and the corresponding large-scale upwelling (Conrath and Gierasch, 1984; Conrath et al., 1990) on the two gas giants. Such a bright zonal band with relatively uniform high clouds is called a zone. On the other hand, the relatively dark band, which emits thermal radiation from a deeper level of the troposphere so that the emitted power is maximal there, is called a belt. In general, the belt lacks the uniform high clouds, but small-scale features including very bright small-scale clouds related to moist convection can disperse there (Little et al., 1999; Porco et al., 2003; Li et al., 2004). There are multiple zones and belts on Jupiter and Saturn. The equatorial region of Uranus and Neptune shows the maximal emitted power (Fig. 2), which means that the thermal emission escapes from a relatively deep atmosphere. Such a region with deficiency of large-scale high clouds in the troposphere suggests that it is a belt, which has the corresponding large-scale sinking motion (Flasar et al., 1987; Read, 1987; Conrath et al., 1990). Likewise, the minimal emitted power in the middle latitudes (Fig. 2) implies a zone on the two ice giants.
Compared to the clearly banded structures on Jupiter and Saturn, the contrast of zonal bands on Uranus and Neptune is not very noticeable. Therefore, in this note, the division of zone and belt on the two ice giants is based mainly on the meridional distribution of emitted power (Fig. 2 and Table 1). In summary, zones with the minimal emitted power and the relatively uniform high cloud deck imply a large-scale rising motion, and belts with the maximal emitted power and the deficiency of large-scale cloud suggest a large-scale sinking motion (Gierasch et al., 1986; West et al., 1986; Ingersoll et al., 1995). A relationship between the banded structures (zones and belts) and multiple jets, in which zonal jets locate at the interfaces of zones and belts, was suggested for Jupiter in previous studies (Gierasch, 1996). Such a relationship is basically applicable to Saturn by comparing the nomenclature of bands (Price, 2000) and the profile of zonal winds (Fig. 1). The division of zones and belts based on the meridional distribution of emitted power (Fig. 2 and Table 1) suggests that the relationship between the pattern of bands and the profile of zonal wind also works for Uranus and Neptune.

Based on the relationship between the pattern of bands and the profile of zonal wind, we propose a simple hypothesis to explain the banded structures and the multiple zonal jets on the four giant planets. The hypothesis is illustrated by a simple two-layer (the upper weather layer and the deep atmosphere layer) model. The weather layer, which is composed by multiple cloud layers, extends from the deep water cloud (~5 bar for Jupiter and Saturn, and ~50 bar for Uranus and Neptune) to the top cloud layer (~0.5 bar for the four giant planets). Most of our measurements of zonal winds on the four giant planets are made by tracking the features in the top cloud layer. The weather layer is assumed to sit on a uniformly deep atmosphere, which has same rotating period as the period of the magnetic field. The deep atmosphere has a much greater thickness and mass than the weather layer, so that the interaction between them has an important influence on the weather layer but little influence on the deep atmosphere. We describe our hypothesis based on the above two-layer model as following: The large-scale rising motion, which is concentrated in some particular latitudes, triggers the development of uniform bright high-cloud in these bands, which results in the formation of zones. The rising motion in the zones will induce a corresponding divergence at the top of the weather layer by the principle of mass continuity. The divergence at the top of each zone will result in an outflow away from the

![Figure 2](image-url). Meridional distribution of the emitted power on the four giant planets. (A) Jupiter. (B) Saturn. (C) Uranus. (D) Neptune. Panel (A) comes from Pirraglia (1984). Panels (B), (C), and (D) come from Ingersoll (1990).
zone. The outflows from the two neighboring zones will converge between the two zones at the top of the weather layer. The convergence at the top of the weather layer will induce a large-scale sinking motion within the weather layer. Then a band with large-scale sinking motion and a deficiency of large-scale clouds, which is called a belt, develops in the middle of the two neighboring zones. Therefore, a self-organizing system, which includes the rising motion in zones, the sinking motion in belts, and the meridional motion from zones to belts at the top of weather layer, is developed.

3. Model
The above system is illustrated in Fig. 3 for the four giant planets. The large-scale rising motion from the deep atmosphere and the large-scale sinking motion from the weather layer guarantee the mass conservations of the two layers of the atmospheric system. The alternating westward and eastward jets are located in the interfaces of the zones and the belts. Such a configuration, which is an inevitable consequence of the meridional motion from the zones to the belts, is consistent with observations. We set a rising branch in a zone, a sinking branch in the neighboring belt, and the meridional motion from the zone to the belt, as a meridional cell (Fig. 3). There are two kinds of meridional cells in the cartoon of Fig. 3. The first kind has the rising branch in the relatively low latitudes and the sinking branch in the relatively high latitudes. The second kind has the opposite configuration of the vertical branches. Figure 3 shows that the equatorial cell is of the first kind on Jupiter and Saturn but of the second kind on Uranus and Neptune. The different configuration of the equatorial cell between the gas giants and the ice giants is critical for the direction of the equatorial jet, which is discussed in the following paragraphs.

Figure 3. A simple model of the large-scale atmospheric dynamics on the four giant planets. (A) The model for Jupiter and Saturn. (B) The model for Uranus and Neptune. The simple two-layer model is composed of a thin upper weather layer and a very thick deep atmosphere. The interface between the upper weather layer and the stationary deep atmosphere is set as the base of the water cloud in our simple model. It should be pointed out that the interface is uncertain, and can be much deeper than the pressure-level of the water cloud. The prograde (eastward) jet is represented by a symbol , and the retrograde (westward) jet is represented by a symbol ⊗.

The proposed self-organizing system will drive axisymmetrical multiple jets on the giant planets. The axisymmetric model of planetary atmospheres has been discussed in previous
studies (Hunt, 1975; Gierasch, 1975; Held and Hou, 1980; Read, 1986; Conrath et al., 1990). Here, we focus on the transport of angular momentum in the simple two-layer model. Let us assume that the whole atmosphere (the upper weather layer and the deep atmosphere) on the four giant planets is stationary relative to the rotation of the magnetic field at the initial state. Therefore, the rising branches and the sinking branches between the weather layer and the deep atmosphere have the same angular velocity, which is the same as the rotation period of the magnetic field. In addition, let us assume that the rising branch and the corresponding sinking branch in each cell have the same mass, to satisfy the mass conservation for the upper weather layer and the deep atmosphere. However, the rising branch and the sinking branch have different cylindrical radius so that they have different angular momentum. Therefore, a net vertical transport of angular momentum occurs in each meridional cell. The angular momentum is larger in the rising branch than in the sinking branch for the first kind of cell. The second kind of cell has the opposite configuration of the vertical branches and the related angular momentum. For the first kind of cell, the net vertical transport of the angular momentum to the cell is positive so that the cell will be accelerated. On the other hand, the second kind of cell will be decelerated due to a negative net vertical transport of angular momentum. As a result, a prograde (eastward) jet develops in the first kind of meridional cell and a retrograde (westward) jet develops in the second kind of meridional cell. Therefore, multiple zonal jets with alternating directions are developed on the four giant planets. For the equatorial cell on Jupiter and Saturn, which is the first kind of cell, a prograde jet will develop there by the above reasoning. Likewise, a retrograde jet, which corresponds to the second kind of cell, will develop in the equatorial region of Uranus and Neptune.

However, the process of acceleration/deceleration cannot continue forever. After the net vertical transport of angular momentum differentiates the rotations of the meridional cells in the upper weather layer, the sinking branch from the weather layer will have a different angular momentum from its original value while the rising branch from the deep atmosphere still keeps its original angular momentum. In principal, it is possible that the changed down-welling transport of angular momentum by the sinking branch balances the unchanged upwelling transport of angular momentum by the rising branch. With the possible balance, a steady system with alternating jets in the weather layer seems to be realized after the initial adjustment. The above hypothesis offers a large-scale self-organizing atmospheric system on the four giant planets, which is composed by the meridional cells and the resulting alternating jets. The steady atmospheric system requires that the angular momentum of the rising branch be equal to that of the sinking branch. This conservation of angular momentum without dissipation will result in a dramatically increase of angular velocity when an air parcel moves away from the equator. The following discussion shows that the conservation of angular momentum without dissipation contradicts the observed zonal jets on the four giant planets, which implies that some dissipation mechanisms have to be introduced into the above system.

Referring to the cartoon in Fig. 3, we divide the upper weather layer into a few meridional cells (boxes), which are composed by the rising branches in the zones, the sinking branches in the belts, and the meridional motion from the zones to the belts at the top of weather layer. We set $a$ as radius of planet, $\lambda$ as longitude, $\phi$ as latitude. We also set $\Delta \phi$ as the half-width of the meridional cell (box), $\phi_0$ as the center latitude, and $w_0$ as the vertical velocity of these vertical branches. Assuming that density of atmosphere is constant, the mass flux to each meridional cell must integrate to zero.
\[
\frac{dm}{dt} = \int_{\phi_{n-1}}^{\phi_n} \int_{\phi_n}^{\phi_n+\Delta\phi_n} w_n a^2 \cos \phi \, d\phi \, d\lambda = 0, \quad n = 1, 2, 3, \ldots
\]  

(1)

This is satisfied if we choose

\[
w_n = (-1)^{n+1} W_n \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) / \cos \phi, \quad n = 1, 2, 3, \ldots
\]

(2)

where \( W_n \) is the characteristic vertical velocity. The characteristic vertical velocity \( W_n \) is set to be a positive constant for each cell, but can be different for different cells. The index \( n \) controls the different kind of the meridional cells with the odd \( n \) representing the first kind of cell and the even \( n \) representing the second kind of cell. So the vertical flux of angular momentum transported to each cell in the weather layer can be expressed as

\[
I^M_n = \int_{\phi_{n-1}}^{\phi_n} \int_{\phi_n}^{\phi_n+\Delta\phi_n} M_n w_n a^2 \cos \phi \, d\phi \, d\lambda, \quad n = 1, 2, 3, \ldots
\]

(3)

Let us assume that the meridional cells are symmetric in the longitudinal direction. Then we can simplify eq.(3) into

\[
I^M_n = 2\pi \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} M_n w_n a^2 \cos \phi \, d\phi, \quad n = 1, 2, 3, \ldots
\]

(4)

Substituting \( w_n \) into eq. (4), we have

\[
I^M_n = \left( -1 \right)^{n+1} \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \left( -1 \right)^{n+1} M_n w_n a^2 \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) \cos \phi \, d\phi, \quad n = 1, 2, 3, \ldots
\]

(5)

The sinking branch has the angular velocity \( \omega_n \), which is same as the rotating period of the corresponding meridional cell in the weather layer. The angular velocity \( \omega_n \) is assumed to be constant through each cell in our simple model. Therefore, the angular momentum per unit mass carried by the sinking branch at latitude \( \phi \) can be expressed as \( \omega_n a^2 \cos^2 \phi \) for each cell. Likewise, the rising branch has the angular velocity \( \Omega a^2 \cos^2 \phi \). Then eq. (5) can be rewritten as

\[
I^n_n = \left( -1 \right)^{n+1} 2\pi a^4 W_n \left[ \Omega \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) \cos \phi \, d\phi + \omega_n \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) \cos^2 \phi \, d\phi \right]
\]

(6)

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I^M_n = \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \int_{\phi_n}^{\phi_n+\Delta\phi_n} M_n w_n a^2 \cos \phi \, d\phi \, d\lambda, \quad n = 1, 2, 3, \ldots
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\]

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\[
I^n_n = \left( -1 \right)^{n+1} 2\pi a^4 W_n \left[ \Omega \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) \cos \phi \, d\phi + \omega_n \int_{\phi_{n-1}}^{\phi_n+\Delta\phi_n} \sin \left( \frac{\pi \phi_n - \phi}{2 \Delta\phi_n} \right) \cos^2 \phi \, d\phi \right]
\]

(6)
where we set the integration terms \( C(\phi^-) = \int_{\phi_n}^{\phi_n+\Delta\phi} \sin[\pi(\phi_n - \phi)/(2\Delta\phi)] \cos^2 \phi \, d\phi \) and \( C(\phi^+) = \int_{\phi_n}^{\phi_n+\Delta\phi} \sin[\pi(\phi_n - \phi)/(2\Delta\phi)] \cos^2 \phi \, d\phi \). The above equation works for northern hemispheres of planets. We can mirror the meridional cells in the northern hemisphere to the southern hemisphere.

Without dissipation, the angular momentum transported into each cell must be zero for a steady state. Let us examine the equatorial cell on Jupiter. The equatorial meridional cell on Jupiter extends from the equator to \( \sim 15^\circ \) N/S based on the observed widths of the equatorial bands and zonal jets (also refer to Table 1). So we have \( \phi_n = \Delta\phi_n = 15^\circ/2 = 7.5^\circ \) for the equatorial cell on Jupiter. From \( I_M^n = 0 \), we have

\[
\Omega C(\phi^-) + \omega_n C(\phi^+) = 0
\]

(7)

Substituting \( \phi_n , \Delta\phi_n \) into \( C(\phi^-) \) and \( C(\phi^+) \), we can rewrite eq.(7) as

\[
\frac{\omega_n}{\Omega} = \frac{-\int_{7.5}^{0} \sin\left(\frac{\pi}{7.5} \frac{7.5 - \phi}{\phi}ight) \cos^2 \phi \, d\phi}{\int_{7.5}^{0} \sin\left(\frac{\pi}{7.5} \frac{7.5 - \phi}{\phi}\right) \cos^2 \phi \, d\phi}
\]

(8)

The expression in Eq. (8) gives a ratio \( \omega_n/\Omega = 1.045 \) for the equatorial cell on Jupiter by analytical integration or numerical estimate. The difference of angular velocity between the meridional cell in the weather layer (\( \omega_n \)) and the deep atmosphere (\( \Omega \)) can be converted to the zonal wind relative to the system III. So we have the corresponding zonal jet in the equatorial cell on Jupiter as \( u_n(\phi_n = 7.5^\circ) = a \cos \phi_n \times (\omega_n - \Omega) = a \cos 7.5^\circ \times 0.045\Omega \sim 550 \) m/s, which is much larger than the observed jovian equatorial jet \( \sim 100 \) m/s.

The above discussion suggests that we have to introduce some dissipation mechanisms into the system to decrease the unrealistically large increase of angular velocity away from the equator. One dissipation mechanism is the linear drag due to the meridional shear of the angular velocity between different cells and the vertical shear of the angular velocity between the deep atmosphere and the upper weather layer. The second dissipation mechanism, the mixing of eddies at different scales, is also important to reduce the angular momentum of the meridional cells (Hartmann, 1994). The latter is hard to measure or parameterize. Here, we simplify the two mechanisms into a linear drag, by assuming that the linear drag is the dominant dissipation mechanism or that the dissipation due to eddies can be integrated into the linear drag by varying the drag coefficient. The linear drag force due to the meridional and vertical shears of the angular velocity can be represented as Rayleigh friction (Salby, 1996)

\[
F_D = K_D u_n + K_D (u_n - u_{n-1}) + K_D (u_n - u_{n-2})
\]

with a drag coefficient, \( K_D \). Substituting \( u_{n-1} = a \cos \phi_{n-1} (\omega_n - \Omega) \), \( u_n = a \cos \phi_n (\omega_n - \Omega) \), \( u_{n+1} = a \cos \phi_{n+1} (\omega_n - \Omega) \) into \( F_D \), we have

\[
F_D = K_D a [3 \cos \phi_n (\omega_n - \Omega) - \cos \phi_{n-1} (\omega_n - \Omega) - \cos \phi_{n+1} (\omega_n - \Omega)]
\]

The coefficient \( K_D \) with unit \( s^{-1} \) can be rewritten as \( K_D = 1/t_D \), where \( t_D \) is a drag time scale. The drag time scale \( t_D \)
also represents the time scale of the meridional cell in the weather layer. Setting
\( D(\omega_{n-1}, \omega_n, \omega_{n+1}) = 3 \cos \phi_n (\omega_n - \Omega) - \cos \phi_{n-1} (\omega_{n-1} - \Omega) - \cos \phi_{n+1} (\omega_{n+1} - \Omega) \), We can express the
dissipation of the angular momentum for each meridional cell as

\[
I_n^D = \int_V a^2 \cos \phi_n D(\omega_{n-1}, \omega_n, \omega_{n+1}) / t_D \ dV
\]

\[
= \int_0^Z \int_0^{2\pi} \int_{\phi_n - \Delta \phi_n}^{\phi_n + \Delta \phi_n} a^2 \cos \phi_n D(\omega_{n-1}, \omega_n, \omega_{n+1}) / t_D \ a^2 \cos \phi \ d\phi d\lambda dz
\quad n = 1, 2, 3, \ldots
\]

where \( Z \) is the height of each cell. Approximating the height of each cell by the scale height \( H \),
we change eq. (9) into

\[
I_n^D = \frac{2\pi H a^4 \cos \phi_n}{t_D} D(\omega_{n-1}, \omega_n, \omega_{n+1}) \int_{\phi_n - \Delta \phi_n}^{\phi_n + \Delta \phi_n} \cos \phi \ d\phi
\]

\[
= 4\pi H a^4 \cos^2 \phi_n \sin \Delta \phi_n D(\omega_{n-1}, \omega_n, \omega_{n+1}) / t_D \quad n = 1, 2, 3, \ldots
\]

Therefore, for a steady state the balance of flux between the vertical transport and the linear drag
\( (I_n^L = I_n^D) \) can be written as

\[
(-1)^{n+1} 2\pi a^4 W_n \left[ \Omega C(\phi_n^+) + \omega_n C(\phi_n^-) \right]
\]

\[
= 4\pi H a^4 \cos^2 \phi_n \sin \Delta \phi_n D(\omega_{n-1}, \omega_n, \omega_{n+1}) / t_D \quad n = 1, 2, 3, \ldots
\]

The above equation shows that the rotation periods of the neighboring three cells \( (\omega_{n-1}, \omega_n, \) and
\( \omega_{n+1}) \) are coupled together, so we can expressed the coupled equations as

\[
\cos \phi_{n-1} - \omega_{n-1} + (E_n C(\phi_n^+) - 3 \cos \phi_n) \omega_n + \cos \phi_{n+1} \omega_{n+1}
\]

\[
= - (E_n C(\phi_n^-) + 3 \cos \phi_n - \cos \phi_{n-1} - \cos \phi_{n+1}) \Omega \quad n = 1, 2, 3, \ldots
\]

where \( E_n = (-1)^{n+1} W_n t_D / (2H \cos^3 \phi_n \sin \Delta \phi_n) \). The coupled linear-equation group (12) with a
series unknown variables \( \omega_n (n = 1, 2, 3, \ldots) \) can be solved by the Gaussian elimination with
suitable boundary condition in the meridional direction if the parameter \( E_n \) and the boundary of
each cell \( [\phi_n - \Delta \phi_n, \phi_n - \Delta \phi_n] \) are known. On the other hand, we can estimate the parameters \( W_n \)
and \( t_D \) in \( E_n \) if we know the boundaries and rotation of each cell. Setting \( F_n = W_n t_D \), we have

\[
F_n = \frac{(-1)^{n+1} (2H \cos^2 \phi_n \sin \Delta \phi_n) D(\omega_{n-1}, \omega_n, \omega_{n+1})}{C(\phi_n^-) + C(\phi_n^+)} \quad n = 1, 2, 3, \ldots
\]
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<td>[49°N, 55°N]</td>
<td>52°N</td>
<td>20</td>
<td>1.00264</td>
<td>1.79×10^3</td>
</tr>
<tr>
<td>Cell 10</td>
<td>[55°N, 61°N]</td>
<td>58°N</td>
<td>-3</td>
<td>0.99954</td>
<td>6.66×10^2</td>
</tr>
<tr>
<td>Cell 11</td>
<td>[61°N, 65°N]</td>
<td>63°N</td>
<td>10</td>
<td>1.00179</td>
<td>5.97×10^2</td>
</tr>
</tbody>
</table>

SATURN

| Cell 1 | [0°N, 22°N]   | 11°N   | 361               | 1.03870           | 1.03×10^5 |
| Cell 2 | [36°N, 46°N]  | 41°N   | -90               | 0.98760           | 4.08×10^4 |
| Cell 3 | [46°N, 53°N]  | 49°N   | 79                | 1.01265           | 2.23×10^4 |
| Cell 4 | [53°N, 58°N]  | 55°N   | -50               | 0.99082           | 1.50×10^4 |
| Cell 5 | [58°N, 64°N]  | 61°N   | 60                | 1.01287           | 7.57×10^3 |

URANUS

| Cell 1 | [0°N, 22°N]   | 11°N   | 81                | 0.96723           | 7.72×10^2 |
| Cell 2 | [22°N, 90°N]  | 56°N   | 195               | 1.13802           | 5.02×10^3 |

NEPTUNE

| Cell 1 | [0°N, 48°N]   | 24°N   | 368               | 0.84837           | 3.65×10^4 |
| Cell 2 | [48°N, 90°N]  | 69°N   | 224               | 1.23540           | 3.81×10^3 |

Table 1. The division of meridional cells based on Fig. 1 and Fig. 2 for the four giant planets. The boundaries of meridional cells in the meridional direction are chosen as the locations where the zonal winds are zeros, which are shown in the second column. The center of each cell, which determines the location of the zonal jet in the corresponding cell, is shown in the third column. The observed zonal jet at the center of each cell, the ratio of angular velocity between the weather layer and the deep atmosphere, and the estimated parameter $F_n$ are shown in columns 4, 5, and 6, respectively. Only the meridional cells in the northern hemisphere are shown in this table. The meridional cells in the southern hemisphere can be projected by axisymmetry.

The division of meridional cells on the four giant planets is shown in Table 1. As we discussed before, the banded structures and the profiles of zonal winds on the four giant planets are basically axisymmetric. In the high latitudes of Jupiter, the axisymmetry of zonal winds is not perfect, so we average the zonal winds in the two hemispheres to get an axisymmetric profile. Table 1 only shows the division of meridional cells in the northern hemisphere. The division in the southern hemisphere can be projected by the axisymmetry. The boundary of each meridional cell (column 2) is chosen as the latitudinal locations where the observed zonal winds are zero except for the equatorial cells on Jupiter and Saturn, in which the equator-ward boundary of the cell is set as 0°. Column 3 of Table 1 shows the center of each meridional cell ($\phi_n$), which is set as the location of the peak of the corresponding zonal jet velocity $u_n$ (column 4). Based on the location $\phi_n$ the corresponding zonal jet velocity $u_n$, we plot the axisymmetrical wind profiles in Fig.1 (red stars and lines), which are basically consistent with the observed wind profiles. Column 5 of Table 1 shows the ratio between the angular velocity of the interior and the angular velocity of each cell, which is based on the observed jet velocity $u_n$. From the ratio
\(\omega_n/\Omega\) (column 5) and latitude range (column 2) in Table 1, we can estimate the corresponding \(F_n^*\) for each meridional cell by eq. (13) with a suitable meridional boundary condition (the angular velocity of the polar region is set as \(\Omega\)). The estimated \(F_n^*\) for each cell is shown in the last column of Table 1. The latitudinal distribution of \(F_n^*\) is also illustrated in Fig. 4. Figure 4 implies that the observed wind profiles on the four giant planets can be generated with a proposed large-scale upwelling concentrated at some particular latitudes and one related parameter \(F_n^*\). The figure also shows that the parameter \(F_n^*\) basically decreases with latitude, which suggests that either one of the characteristic vertical velocity \(W_n\) and the time scale \(\tau_n\) or both of them decrease with latitude.

Figure 4. The meridional distribution of the parameter \(F_n^*\) for the four giant planets. The estimate of \(F_n^*\) is based on the equation (13), the division of meridional cell \([\phi_n - \Delta \phi_n, \phi_n + \Delta \phi_n]\), and the ratio \(\omega_n/\Omega\) (Table 1).

The parameter \(F_n^*\) can be used to estimate the time scale of the meridional cell if the characteristic vertical velocity \(W_n\) is known. The observations from Galileo (Gierasch et al., 2000) show that the divergence of moist convections, which have a typical area of 1000 km×1000 km and a typical depth of 50 km, is \(~20\ \text{km}^2/\text{s}\) on Jupiter. Such divergence rate requires an upwelling velocity \(~1\ \text{m/s}\) within moist convections by the continuity equation. The recent observations from Cassini suggest that the typical vertical velocity within convective storms on Saturn is also in the order of 1 m/s (Del Genio et al., 2007). The estimated meso-scale vertical velocities within convective storms on Jupiter and Saturn are comparable to those within the convective storms on Earth (Futyan and Del Genio, 2007). The typical large-scale vertical velocity of the thermally-direct Hadley cell has the order of \(10^{-3}\ \text{m/s}\) on Earth (Peixoto and Oort, 1992), which suggests that there is a three order of magnitude difference in the vertical velocity between the Hadley cell and the meso-scale processes. Assuming that the magnitude difference of the vertical velocity between the meridional cell and the convective processes is applicable for the four giant planets and that the four giant planets all have typical vertical velocities \(~1\ \text{m/s}\) within convective storms, the characteristic vertical velocity of equatorial cells is on the order of \(10^{-3}\ \text{m/s}\) for the four giant planets. Figure 3 and Table 1 show that the equatorial cell of Jupiter,
Saturn, Uranus, and Neptune has the value of $F_n$ as $5.79 \times 10^4 \text{m}$, $10.32 \times 10^4 \text{m}$, $7.72 \times 10^4 \text{m}$, and $3.65 \times 10^4 \text{m}$, respectively. Combining the value of $F_n$ and the typical vertical velocity $W$ ($10^{-3}$ m/s), we have the time scale $t_D$, which also represents the time scale of the meridional cell, is 670 days, 1194 days, 894 days, and 422 days for Jupiter, Saturn, Uranus, and Neptune, respectively. The meridional velocity $V$ of the Hadley cell on Earth is on the order of 1 m/s (Peixoto and Oort, 1992). This typical meridional velocity suggests that the time for an air parcel to move across the Hadley Cell (from the equator to $\sim 30^\circ$) is $\sim 38$ days on Earth. Therefore, the time scale of the equatorial cell on the giant planets seems to be one order of magnitude longer than the time-scale of the Hadley cell on Earth. If we assume the 1 m/s typical meridional velocity on Earth is applicable to the giant planets, we can estimate the time of an air parcel moving across the equatorial cell as 212 days, 258 days, 111 days, and 238 days for Jupiter, Saturn, Uranus, and Neptune, respectively. The estimated time scales based on the assumed meridional velocity are shorter than the estimated $t_D$, which suggests that large-scale meridional velocity on the four giant planets is probably smaller than 1 m/s. That may explain why the large-scale meridional motion on the four giant planets is difficult to detect from observations.

4. Discussions and Conclusions
In the simple model shown in Fig. 3, we emphasize the large-scale motions. The small-scale moist convection, vortices, turbulence, and waves are neglected. The possible meridional motion from belts to zones, which is forced by the convergence of the large-scale sinking and the small-scale rising within the weather layer (Ingersoll et al., 2000), is also neglected in our simple model. The meridional motion suggested in the previous study (Ingersoll et al., 2000) is within the weather layer, which is below the meridional motion at the top of weather layer in our model. Considering that most of wind measurements are conducted at the top of the weather layer, knowledge of meridional motion from the zones to the belts at the top of weather layer is critical to understand the observed zonal jets on the giant planets. In addition, the existence of the possible meridional motion within the weather layer (Ingersoll et al., 2000) does not affect the discussion regarding the vertical transport of angular momentum.

The other issue we neglect in our simple model is the variations of angular momentum due to the vertical displacement when discussing the conservation of angular momentum. Such variations can be important for the equatorial retrograde jets on Uranus and Neptune if the vertical displacement is in the order of magnitude $10^3$ km or larger (Suomi et al., 1991). In our simple model, the vertical displacement is $\sim 50$-120 km within the weather layer on the four giant planets. At the equator, such a vertical displacement will induce a change of angular velocity $\Delta \omega/\Omega \sim 1.5 \times 10^{-3}$, $3.5 \times 10^{-3}$, $9.1 \times 10^{-3}$, and $7.3 \times 10^{-3}$ for Jupiter, Saturn, Uranus, and Neptune, respectively. Table 1 implies that the observed change of angular velocity $\Delta \omega/\Omega$ for the equatorial cell is $\sim 1.0 \times 10^{-2}$, $3.9 \times 10^{-2}$, $3.3 \times 10^{-2}$, and $1.5 \times 10^{-1}$ for Jupiter, Saturn, Uranus, and Neptune, respectively. Such values are from the conservation of angular momentum along the meridional direction with a linear drag. Therefore, in our simple model the variation of angular velocity due to vertical displacement is one order of magnitude smaller than the variation due to the meridional displacement for the equatorial cell on the four giant planets.

The large-scale upwelling concentrated in some particular latitudes plays a critical role in our simple model. So a question has to be asked: what is the physics behind the large-scale upwelling? We think that the internal heat can be a candidate for the driving force behind the large-scale upwelling. The emitted power (Fig.2) can be utilized to estimate the internal heat in
the four giant planets by comparing with the solar radiance. The comparison between the emitted power and the incident radiance suggests that Jupiter, Saturn, and Neptune have a comparable internal heat to the solar radiance. The case of Uranus is less clear. Estimates based on Voyager observations suggest a significantly smaller internal heat in Uranus compared to the other giant planets. However, a detailed modeling suggests that the internal heat in Uranus is substantially larger than the estimates from Voyager (Marley and McKay, 1999). With this caveat, it is probable that all four giant planets have significant internal heat (Guillot, 2005). As we mentioned before, the banded structures and the zonal jets on the four giant planets further display good axisymmetry in the meridional direction even though they have largely varying obliquities. If the proposed axisymmetric meridional cells (Fig.3) exist in the four giant planets, then uneven solar radiation cannot be the dominant driving force on Saturn, Uranus, and Neptune because of their large obliquities. Instead, the internal heat is a likely candidate driving the meridional circulations on the four giant planets from below.

The next question is the spatial distribution of the internal heat if we think that the internal heat is the driving force in the deep atmosphere. In general, the internal heat is thought to be associated with continued gravitational contraction and differentiation, which suggests the internal heat has a relatively uniform distribution. However, the fact that the emitted power is approximately uniform with latitude (Fig. 2) while the solar radiance changes dramatically with the solar angle suggests that the internal heat has to change on a global scale, or that a meridional circulation exists in the interior (Ingersoll and Porco, 1977) or the atmosphere (Conrath and Gierasch, 1984) to compensate for the non-uniform solar radiance. Regarding the meridional distribution of the internal heat on smaller scales (zones and belts), the observations by Pioneer (Ingersoll, 1976) suggest that the internal heat is the same under the belts and zones on Jupiter. On the other hand, the Jupiter flyby observations by Voyager (Pirraglia, 1984) imply different internal heat under the belts and zones of Jupiter. The internal heat will directly drive the large-scale upwelling if it is concentrated in the zones on the four giant planets. On the other hand, it is also possible that the internal heat, combining with the rotation of planet and the atmospheric processes, drives the upwelling in some particular latitudes even though the internal heat is concentrated somewhere else.

There are some possible long-term changes of the belt-zone structures taking place on the giant planets (Smith and Hunt, 1976), which suggest that there should be corresponding changes in the profile of zonal winds on the giant planets if our proposed relationship between the banded structures and the zonal jets is correct. However, there is no the corresponding changes of the zonal jets on the giant planets. The standard division of belt and zone on the giant planets, which is based the color of zonal bands, is affected by the chaotic atmospheric appearance due to turbulence and small-scale features. The new division of zone and belt mainly based on the meridional distribution of emitted power, which is used for the Uranus and Neptune in this note, offers an alternate criterion for the division. It is possible that the belt-zone structure based on the new criterion is stable with time on the four giant planets. The comparison of the distribution of emitted power on Saturn and Jupiter between the data sets from the Composite Infrared Spectrometer (CIRS) on Cassini and the Infrared Interferometer Spectrometer (IRIS) on Voyager will help us further understand this issue.

In summary, our discussion in this note offers a unified simple model to explain the large-scale atmospheric structures on the four giant planets. The simple model is consistent with most of the observational facts of winds, clouds, and emission. However, the physics behind the large-scale upwelling, which plays a critical role in our model, needs further exploration. It should be
emphasized again that the turbulence, convection, eddies, waves, and vortices that contribute to the active and chaotic meteorology and play important roles in the large-scale processes are neglected in our simple model. Previous theories and models resolve these neglected processes to differing degrees. Therefore, it is possible to combine our simple model and the previous studies to offer a more complete picture of the atmospheric dynamics on the four giant planets.

References


