Review of Course Math and Formulas
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For this worksheet, we will analyze the properties of recently discovered extrasolar planet Kepler-62f and its host star (Kepler-62). The planet, Kepler-62f, is the most Earth-like (in terms of its size) of any exoplanet known to orbit in its star's habitable zone.

Be careful with units!

Part A: Stellar Properties

1. The star, Kepler-62, is located at a distance of 320 parsecs = 1040 light-years = 9.9×10^{18} m from our Solar System.
   a. The fastest space probe ever launched is NASA's New Horizons, which will fly past Pluto in July 2015 and then eventually leave our Solar System at a speed of 12.5 km/s. How long would it take an object traveling at that speed to reach the Kepler-62 system? (Be careful with the units!)
   b. The USS Enterprise, from Star Trek, has a maximum cruising speed of “warp six”, which is 216 times the speed of light. How long would it take the Enterprise to travel from Earth to the Kepler-62 system? (Don't worry about the effects of relativity here. Note that it is not actually possible to travel faster than the speed of light, but science fiction writers have the luxury of being able to ignore any laws of physics that they find inconvenient.)

2. a. Analysis of Kepler-62's spectrum shows that its effective surface temperature is 4925 K, and its radius is 0.63 solar radii = 4.38×10^{8} m. Use the Stefan-Boltzmann law to calculate the star's luminosity. (Hint: It should be the less than Sun's luminosity, 3.85×10^{26} W, but within a factor of ten.)
b. What color would this star appear to our eyes? (Hint: The Sun's surface is 5800 K.)

3. a. At what wavelength does Kepler-62's blackbody spectrum peak?

   b. What is the frequency corresponding to that wavelength?

   c. What is the energy of a photon at that frequency?

Part B: Stellar Observations

4. a. What is the star's diameter, in meters?

   b. What is the angular diameter of the star (as seen from Earth), in radians and in arcseconds?

5. a. How large a telescope would you need (what diameter) if you wanted to resolve this star at visible wavelengths (say 500 nanometers)? How does this compare to the largest current optical telescopes, which have a diameter of about ten meters?
b. What diameter would your telescope have to be, in order to resolve this star at a wavelength of 1.0 micron?

**Part C: Orbital Mechanics**

6. Stellar astronomers have determined, based on the star Kepler-62's spectrum, that its mass is 0.69 solar masses = 1.37×10^{30} kg. The planet, Kepler-62f, orbits the star with a period of 267.29 days. What is its orbital semimajor axis? (Remember, the star does not have the same mass as the Sun.)

7. a. The planet's orbital eccentricity is estimated to be 0.15, but the actual value of this eccentricity is quite uncertain. Assuming that the eccentricity is exactly 0.15, how far is the planet from the star at perihelion (or, rather, periastron, since it doesn't orbit the Sun)?

b. How far is the planet from the star at aphelion/apastron (most distant point in its orbit)?

c. Does the star appear brightest from Kepler-62f when the planet is at periastron, or apastron? What is the difference in the star's apparent brightness (what percentage) between periastron and apastron?
8. Since this planet's orbital eccentricity is very uncertain, and it could actually be zero, we will assume (for simplicity) in the remaining problems that the eccentricity is zero, so that the orbit is circular. So, for a circular orbit with the semimajor axis you calculated above, what is the planet's orbital speed? (Hint: What distance does the planet travel in each orbit? How long does each orbit take?)

Part D: Planetary Properties

9. a. In order to estimate the planet's average surface temperature, we need to know its albedo and what is in its atmosphere. We don't yet know either of those, but we can at least try some albedo values to estimate Kepler-62f's equilibrium temperature. Calculate the planet's equilibrium temperature, assuming that its albedo is 0.10 (rather low). You will need the stellar luminosity that you found in problem 2.

b. Now calculate Kepler-62f's equilibrium temperature, assuming that its albedo is a more Earth-like value of 0.30.
10. The planet's radius was measured to be 1.41 Earth radii = 9000 km = 9.0×10^6 m, based on how much the star's brightness appears to decrease when the planet passes in front of it. Given this planet radius and the stellar radius from question 2, what fraction (or what percentage) of the star's light is blocked by the planet during a transit?

11. The planet's mass has not yet been measured, so its density is unknown, but we'll assume for this worksheet that it has the same average density as the Earth: 5514 kg/m^3. If that is the planet's density, what is its mass? How does this compare to Earth's mass (which is 5.973×10^{24} kg)?

12. What is the acceleration due to gravity on Kepler-62f? How does this compare to the value on Earth (9.8 m/s^2)?

13. What is the minimum speed at which would a rocket have to be launched from the surface of Kepler-62f to escape its gravity?
Part E: Stellar Observations

14. As the planet goes around the star, it causes a slight periodic wobble in the star's motion. Assuming that the planet's mass is what you calculated above, at what speed does the star move around the system's center of mass, due to the gravitational influence of the planet Kepler-62f? (Hint: Use conservation of momentum. You will need some previous answers.)

15. a. We can observe this stellar wobble as a small periodic Doppler shift in the star's spectral lines. What would be the greatest observed change in wavelength for a spectral line with a rest wavelength of exactly 500 nm?

b. At what wavelength would we observe this spectral line when the star is moving directly toward us?

c. When the star is moving toward us, is the planet moving toward us or away from us?

d. At what wavelength would we observe this spectral line when the star is moving directly away from us?

e. When the star is moving away from us, is the planet moving toward us or away from us?

16. What is the gravitational force between the star Kepler-62 and the planet Kepler-62f?
Part F: Planetary Atmospheres

We don't know anything about Kepler-62f's atmosphere (including whether it even has one), so in the interest of reviewing planetary atmosphere math, let's consider the Earth's atmosphere.

17. The atmospheric pressure at Earth's surface is 1.0 bar = 1.0\times10^5 \text{ Pa}, and the average surface temperature is 15°C.
   a. Near Earth's surface, how many air particles are there in each cubic meter?
   
   b. (extra) What is the total mass of the air molecules above each square meter of Earth's surface? (Hint: Pressure equals the weight of the overlying air molecules. We haven't done this calculation before, but see if you can figure out which formula to use.)
   
   c. (extra) Use your answer from part (b) to calculate the total mass of Earth's atmosphere. (This is something else we haven't done before, but here's a hint: You will need to know that Earth's radius is 6371 km.)

18. a. At a temperature of 15°C, what is the average speed of a molecule of nitrogen? (An N\textsubscript{2} molecule has a mass of 4.65\times10^{-26} \text{ kg}.)
   
   b. What is the average kinetic energy of a CO\textsubscript{2} molecule at 15°C?
   
   c. What is the average kinetic energy of a hydrogen atom at 15°C? (A hydrogen atom has a mass of 1.67\times10^{-27} \text{ kg}.)
Potentially useful unit conversions, physical constants, and equations:

1 arcsecond = $4.848 \times 10^{-6}$ radians
1 day = 86,400 seconds
1 g/cm$^3$ = 1000 kg/m$^3$
1 nm = $10^{-9}$ m
1 micron = $10^{-6}$ m
1 AU = $1.50 \times 10^{11}$ m
1 Hz = 1 s$^{-1}$
1 N = 1 kg m s$^{-2}$
1 J = 1 kg m$^2$ s$^{-2}$ = 1 N m
1 W = 1 kg m$^2$ s$^{-3}$ = 1 J/s
1 Pa = 1 kg m$^{-1}$ s$^{-2}$ = 1 N/m$^2$
1 bar = $10^5$ Pa
0 K = -273°C; 273 K = 0°C; 373 K = 100°C
b = 2.90×$10^6$ nm K
c = 3.00×$10^8$ m/s
G = 6.67×$10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$
h = 6.63×$10^{-34}$ J s
k$_B$ = 1.38×$10^{-23}$ J/K
σ = 5.67×$10^{-8}$ W m$^{-2}$ K$^{-4}$

Summary of the star Kepler-62’s properties:
Radius is 0.63 solar radii = 4.38×$10^8$ m  
Mass is 0.69 solar masses = 1.37×$10^{30}$ kg
Effective surface temperature is 4925 K