Damping of GHz waves in magnetar magnetospheres

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Suppose a GHz burst is emitted near a magnetar. For example, consider a sine radio wave $E(\xi) = E_0 \sin(\omega\xi)$ where $\xi = t - r/c$ and r is the radial distance from the star. We wish to know how the radio wave evolves as it expands to larger radii r through the magnetosphere of the magnetar. This is a well posed physics problem, which can be solved. As long as the magnetospheric particles exposed to the wave remain magnetized (i.e. their Larmor frequency far exceeds the wave frequency ω) the radio wave obeys MHD and can be thought of as a fast magnetosonic wave in the strongly magnetized background. Below the solution to this problem is summarized (details are given in Beloborodov 2022, in preparation).

As the radio wave expands to larger r, it initially experiences no effects from the background magnetosphere — it propagates as in vacuum, with no change in the wave profile apart from the decrease of amplitude $E_0 \propto r^{-1}$. In particular, the wave power $L = cr^2 E_0^2/2$ remains unchanged. This behavior holds as long as E_0 remains small compared with the background magnetic field $B_{\rm bg} = \mu/r^3$, where μ is the magnetic dipole moment of the star. The behavior drastically changes when the wave approaches radius $R_\times$ where $E_0 = B_{\rm bg}/2$,

$$R_\times = \left(\frac{c\mu^2}{8L}\right)^{1/4} \approx 2.5\times 10^8\, \frac{\mu_{33}^{1/2}}{L_{42}^{1/4}}\ \text{cm}. \tag{1}$$

Near this radius, each oscillation of the wave steepens into a shock. Then, the shock dissipation continues to damp the wave oscillations and practically erases them. One example is shown in Figure 1. It assumes a typical magnetar magnetosphere with $\mu = 10^{33}\,\text{G cm}^{-3}$ and $e^\pm$ plasma density $n(r) = 10^{13}\, r_8^{-3}\ \text{cm}^{-3}$. Note that at radii of interest, $r \gtrsim R_\times$, the magnetosphere consists of mildly relativistic particles (before the arrival of the wave), as their motion is limited by drag exerted by radiation flowing from the magnetar (Beloborodov 2020, ApJ).

The damping effect is further demonstrated in Figure 2. It shows the wave power evolution with radius for two GHz bursts, with initial luminosities $L = 10^{42}$ erg/s and 10^{40} erg/s.

The wave evolution in these examples was calculated in the equatorial plane of the magnetosphere, $\theta = \pi/2$, where the background magnetic field $\mathbf{B}_{\rm bg}$ is perpendicular to the wave propagation direction. Similar shock damping will occur at latitudes $\theta \neq \pi/2$ where $\mathbf{B}_{\rm bg}$ is oblique to the radially expanding wave. The vacuum propagation fails where it predicts $B^2 - E^2 \leq 0$, which leads to shock formation and damping of wave oscillations. This condition is met on the surface

$$r(\theta) = \left(\frac{1 + 3\cos^2\theta}{\sin\theta}\right)^{1/2} R_\times. \tag{2}$$

There is a narrow cone near the magnetic axis $\theta < \theta_{\rm esc}$ where the radio burst could escape. A typical $\theta_{\rm esc} \sim 10^{-2}$ implies a small probability $\sim \theta_{\rm esc}^2/2$ for the line of sight to be within the escape cone.

The MHD calculation fails for radio bursts with $L < 10^{40}$ erg/s. Then a kinetic description is required. It also gives strong damping of the wave (Beloborodov 2022, PRL).

Our main conclusion is that observed FRBs can hardly be emitted by a source confined in the inner magnetosphere of a magnetar. FRBs must be emitted outside the magnetosphere by magnetospheric explosions. Possible mechanisms include ejecta modulated by reconnection (Lyubarsky 2020, ApJ) or blast waves in the magnetar wind (Beloborodov 2017, 2020, ApJ).

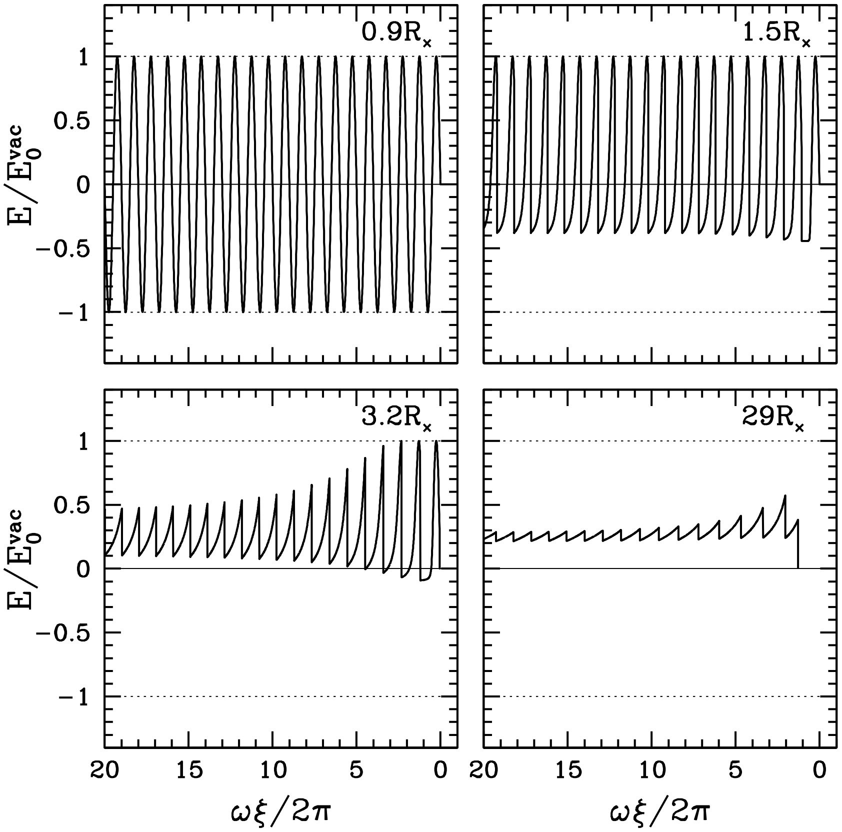

Figure 1: Evolution of the wave profile $E(\xi)$, where $\xi \equiv t - r/c$. In this example, the wave has frequency $\nu = 0.3\,$GHz and initial power $L = 10^{42}\,$erg/s. The snapshots were taken when the wave reached $r/R_\times = 0.9$, 1.5, 2.6, and 29. For clarity only the leading 20 oscillations are shown (the simulated burst has 3×10^4 oscillations). At $r > R_\times$, each oscillation forms a strong shock. The wave electric field E in each panel is normalized to the amplitude $E_0^{\rm vac}(r)$ that the wave would have in vacuum.

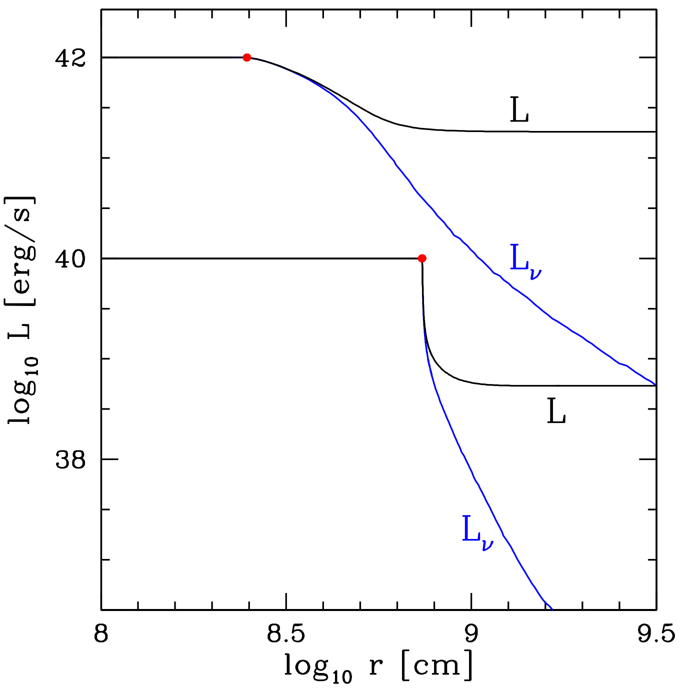

Figure 2: Evolution of wave power calculated for 0.3 GHz bursts with initial $L = 10^{42}\,$erg/s and $L = 10^{40}\,$erg/s. The bursts have 0.1 ms duration. Power of the oscillating component L_ν is shown by the blue curve, and total Poynting flux L is shown by the black curve. The moment of shock formation (near $R_\times$) is indicated by the red dot. One can see that the wave oscillations are damped by more than 3 orders of magnitude. Most of the wave energy is converted to synchrotron X-rays, and part of it converts to a non-oscillating Poynting flux with the 0.1 ms duration.

(Beloborodov 2022, in preparation)