Fireball in Fast Radio Bursts

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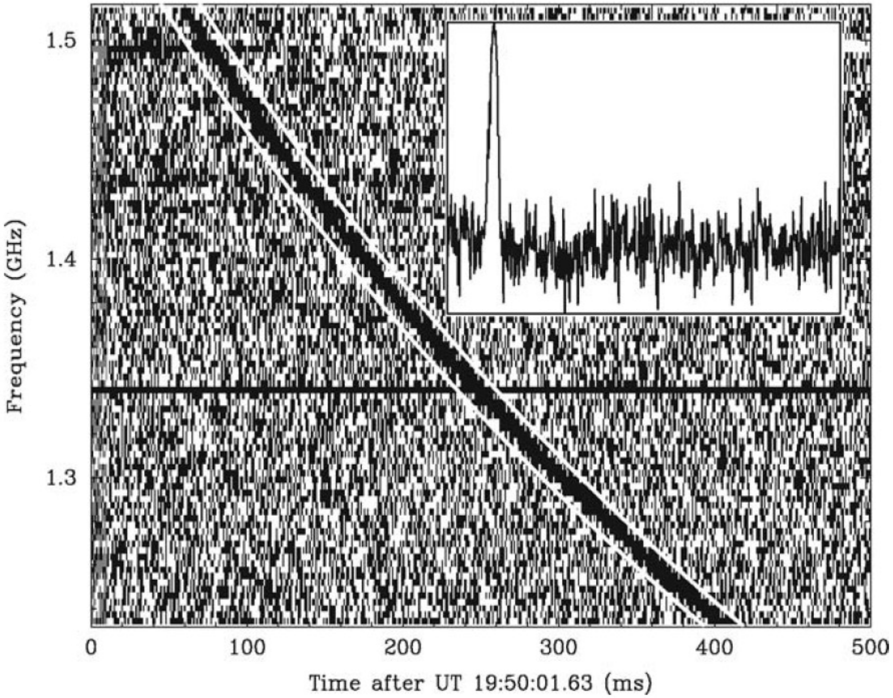
Fast Radio Bursts

Enigmatic transients

Extremely high $T_B \sim 10^{32\text{-}36}$K

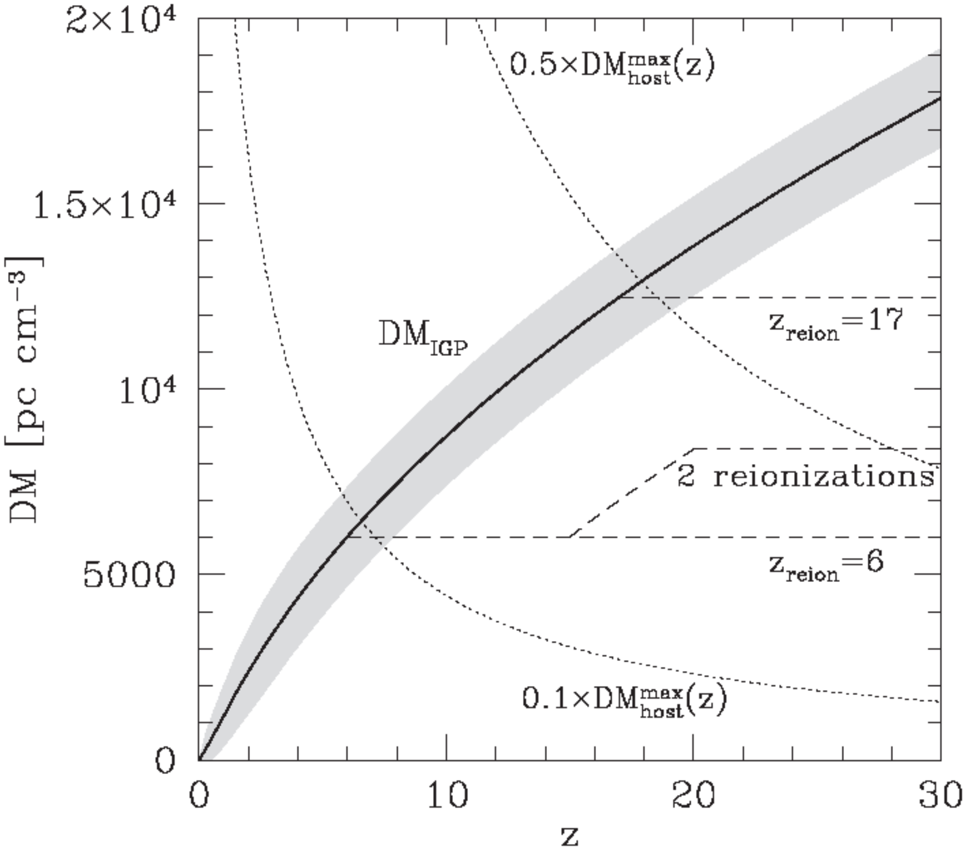


Lorimer+ 07
Thornton+ 13

$W \propto f^{-4.8 \pm 0.4}$

FRB cosmology

Dispersion measure → d



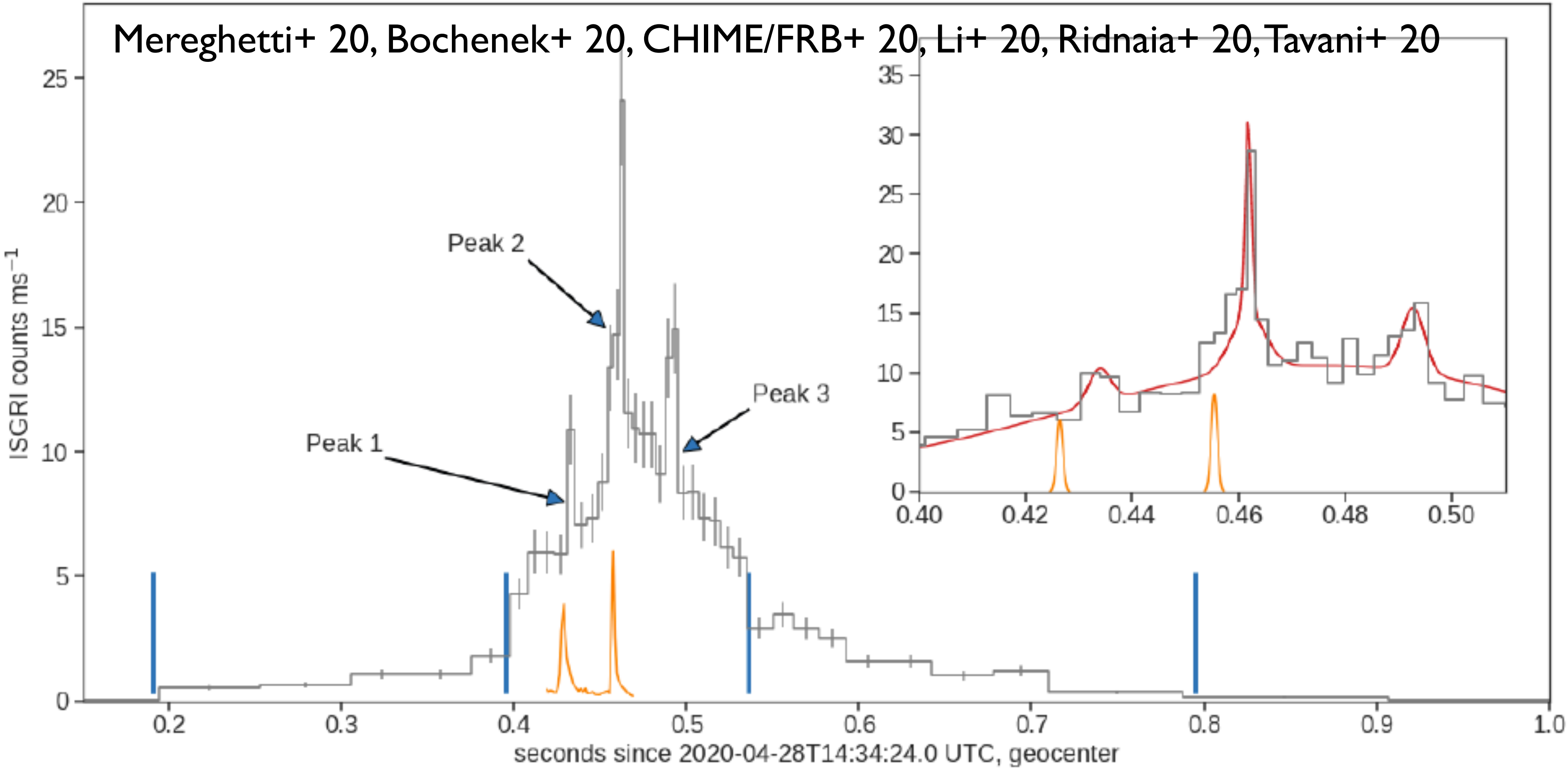
KI 03, Inoue 04, Macquart+ 20, Takahashi+ 21

Galactic FRB from Magnetar Bursts

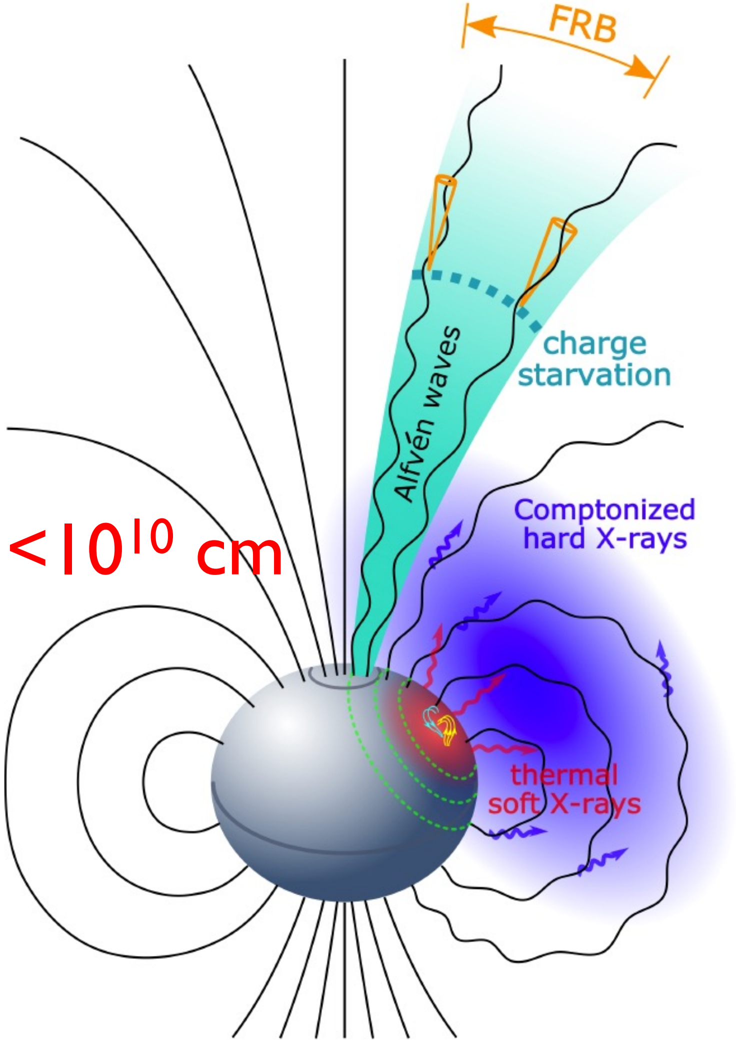
Magnetar: One of the origins

$L_X \sim 10^{41}$ erg/s >> $L_{FRB} \sim 10^{38}$ erg/s

Mereghetti+ 20, Bochenek+ 20, CHIME/FRB+ 20, Li+ 20, Ridnaia+ 20, Tavani+ 20

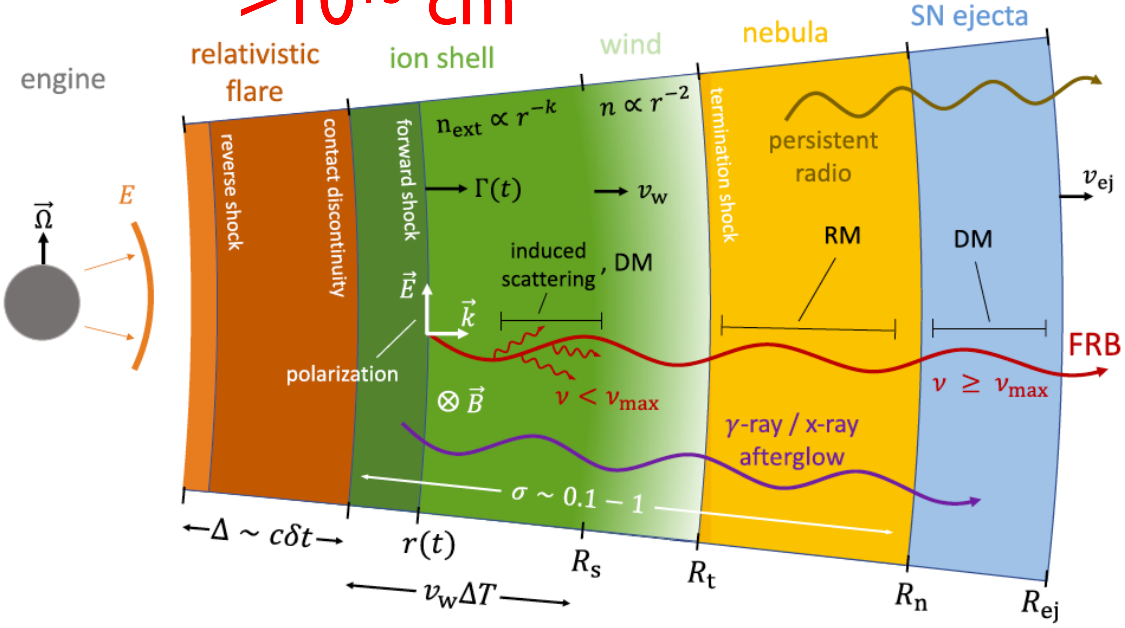


Magnetosphere? Outflow?



$<10^{10}$ cm

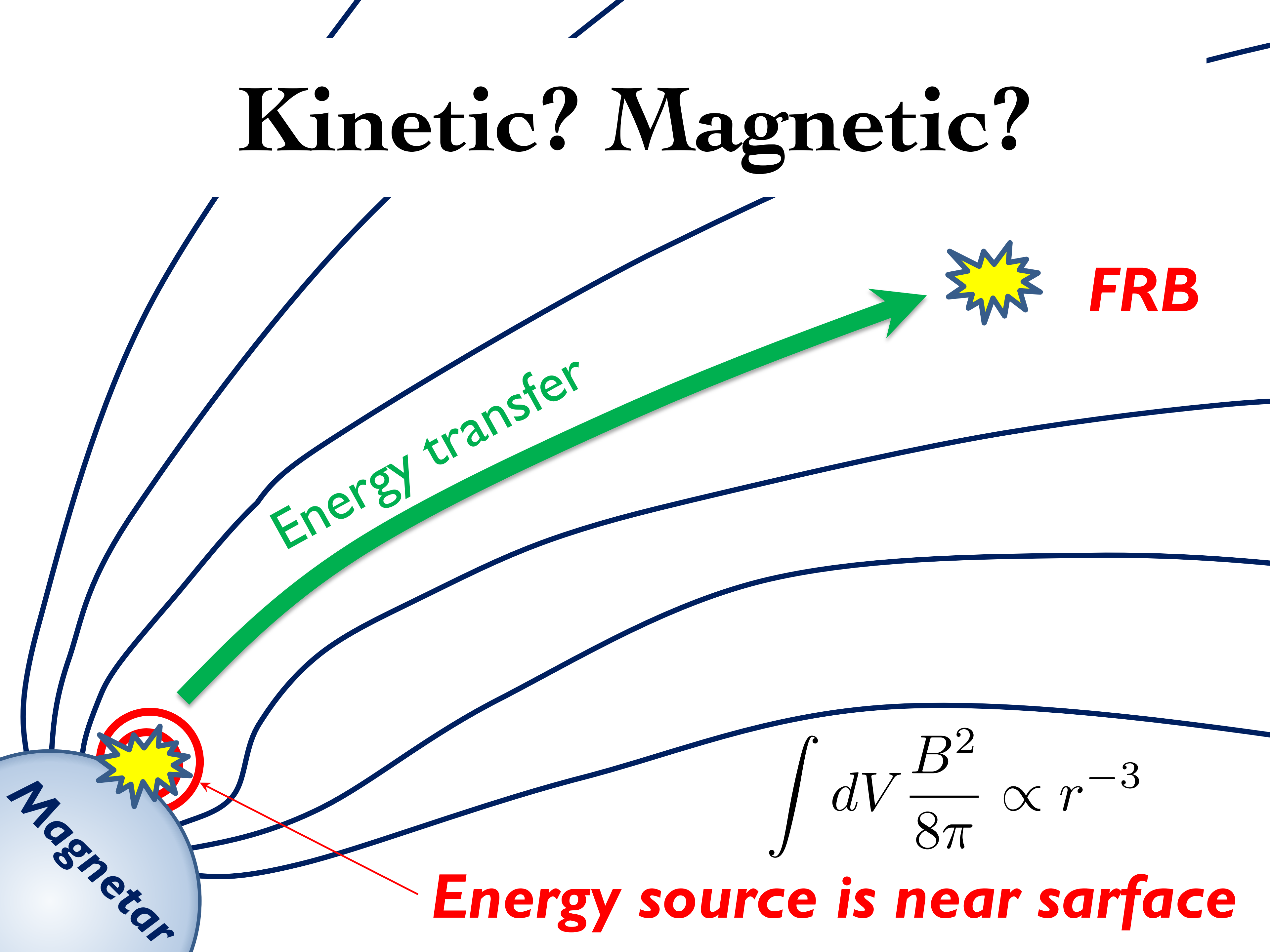
$>10^{13}$ cm



Margalit et al. 20; Yu et al. 20; Yuan et al. 20
Lyubarsky 14; Murase et al. 16; Waxman 17;
Beloborodov 17; Metzger et al. 17

Lu et al. 20; Lyutikov & Popov 20; Katz 20;
Kashiyama et al. 13; Pen & Connor 15;
Cordes & Wasserman 16; Lyutikov et al. 16;
Kumar et al. 17; Zhang 17; Lyubarsky 20;
Kumar & Bošnjak 20; Ioka & Zhang 20

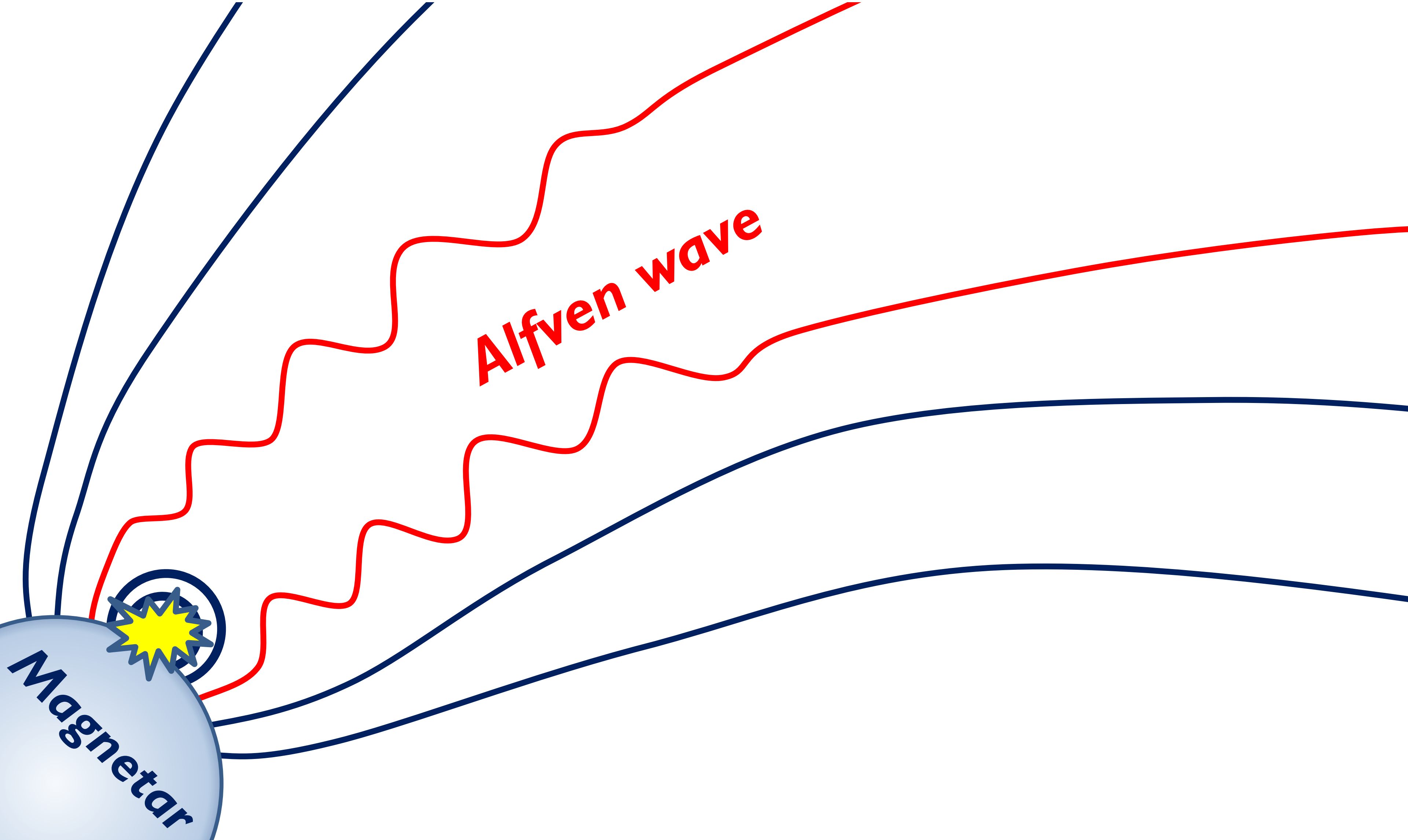
Kinetic? Magnetic?



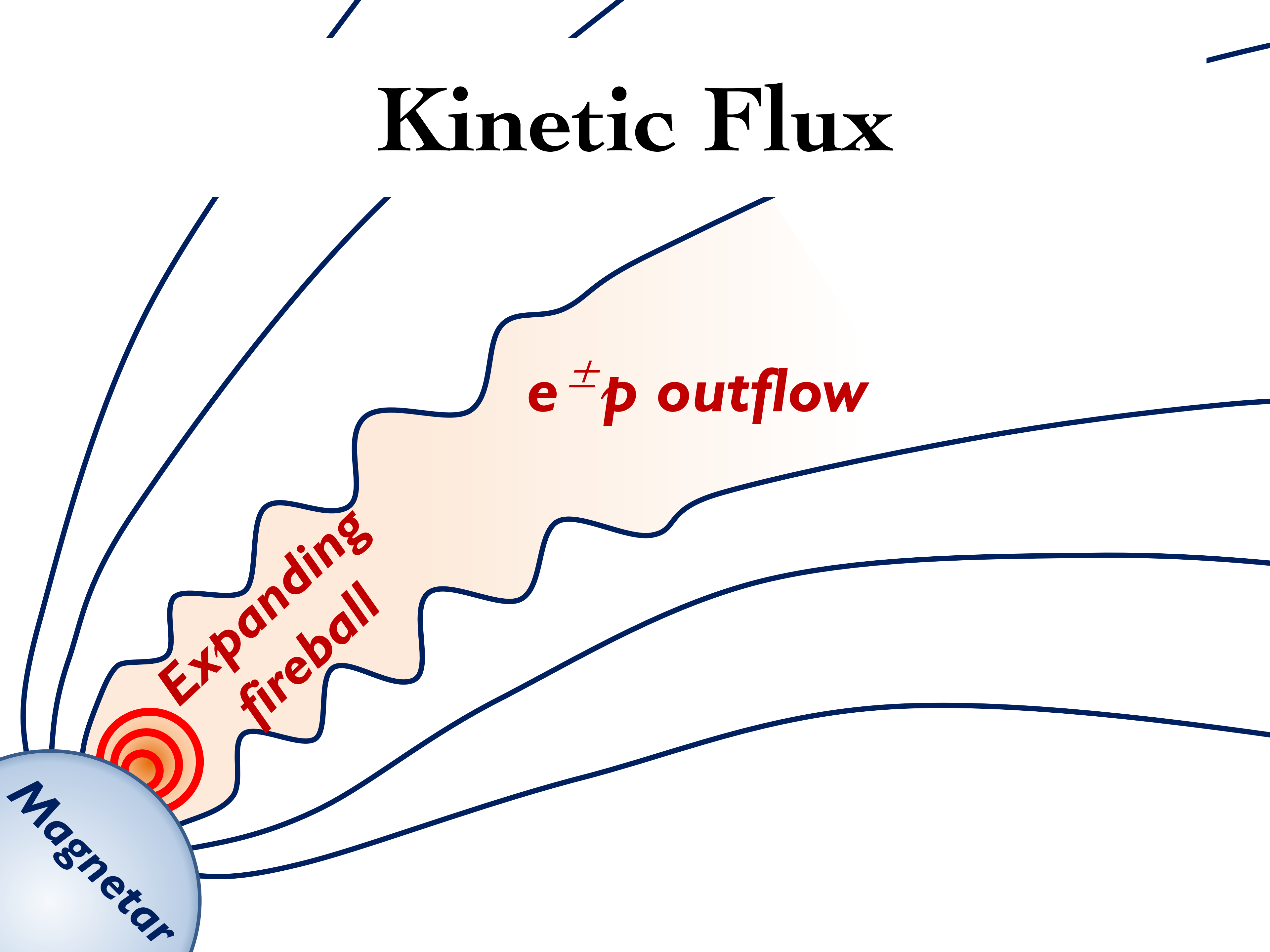
$$\int dV \frac{B^2}{8\pi} \propto r^{-3}$$

Energy source is near sarface

Poynting Flux

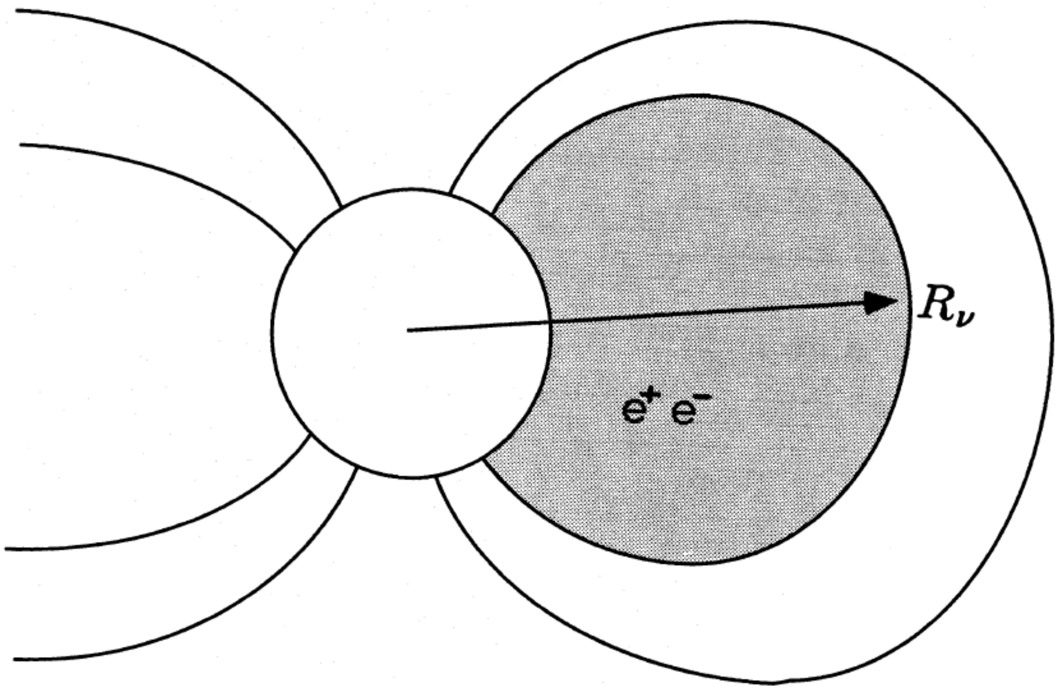


Kinetic Flux



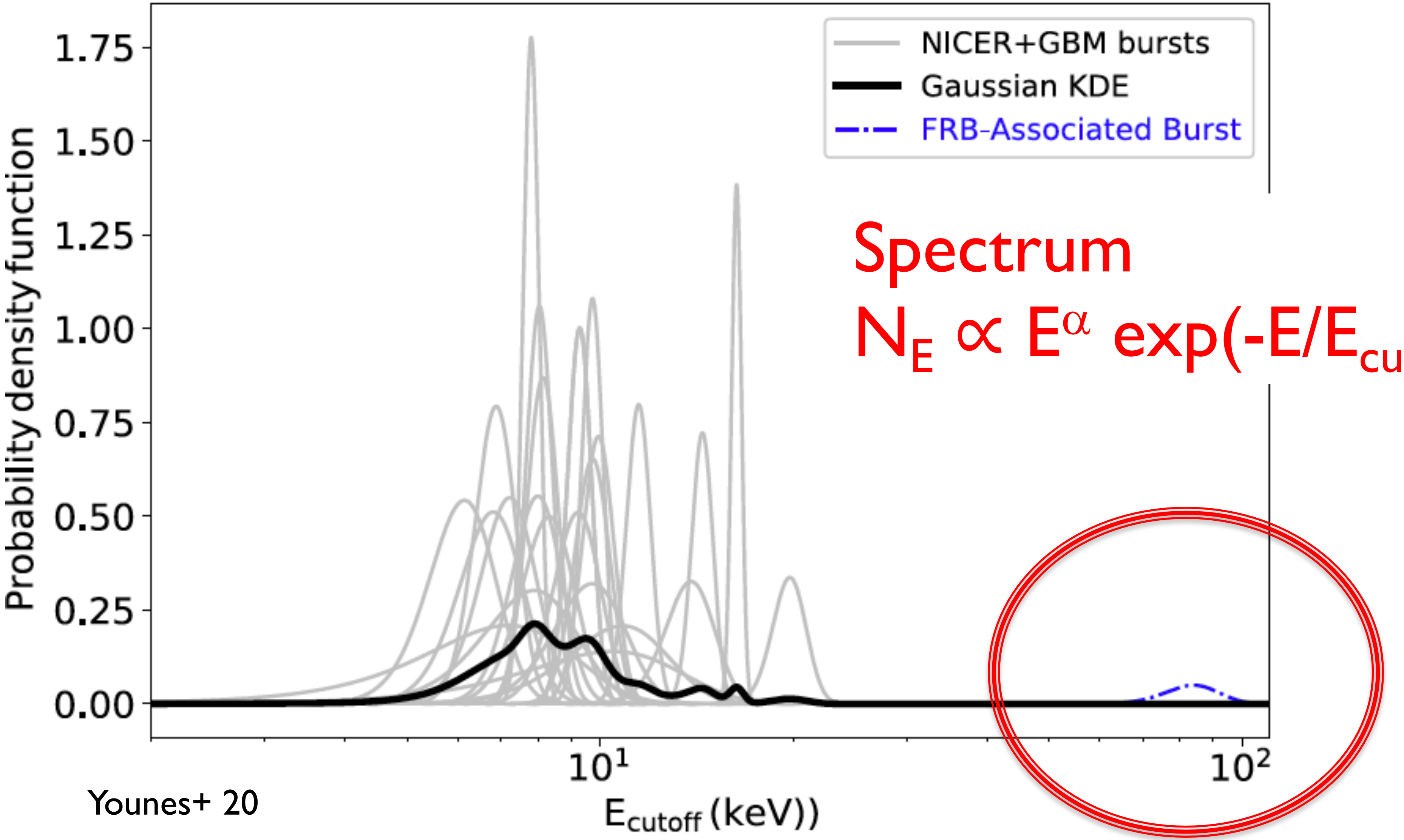
Trapped Fireball

Thompson & Duncan 95



$$\ell_X \sim \left(\frac{L_X}{2\pi c a T^4}\right)^{1/2} \sim 1 \times 10^4 \, \mathrm{cm} \, L_{X,41}^{1/2} T_{1.9}^{-2},$$

High Temperature

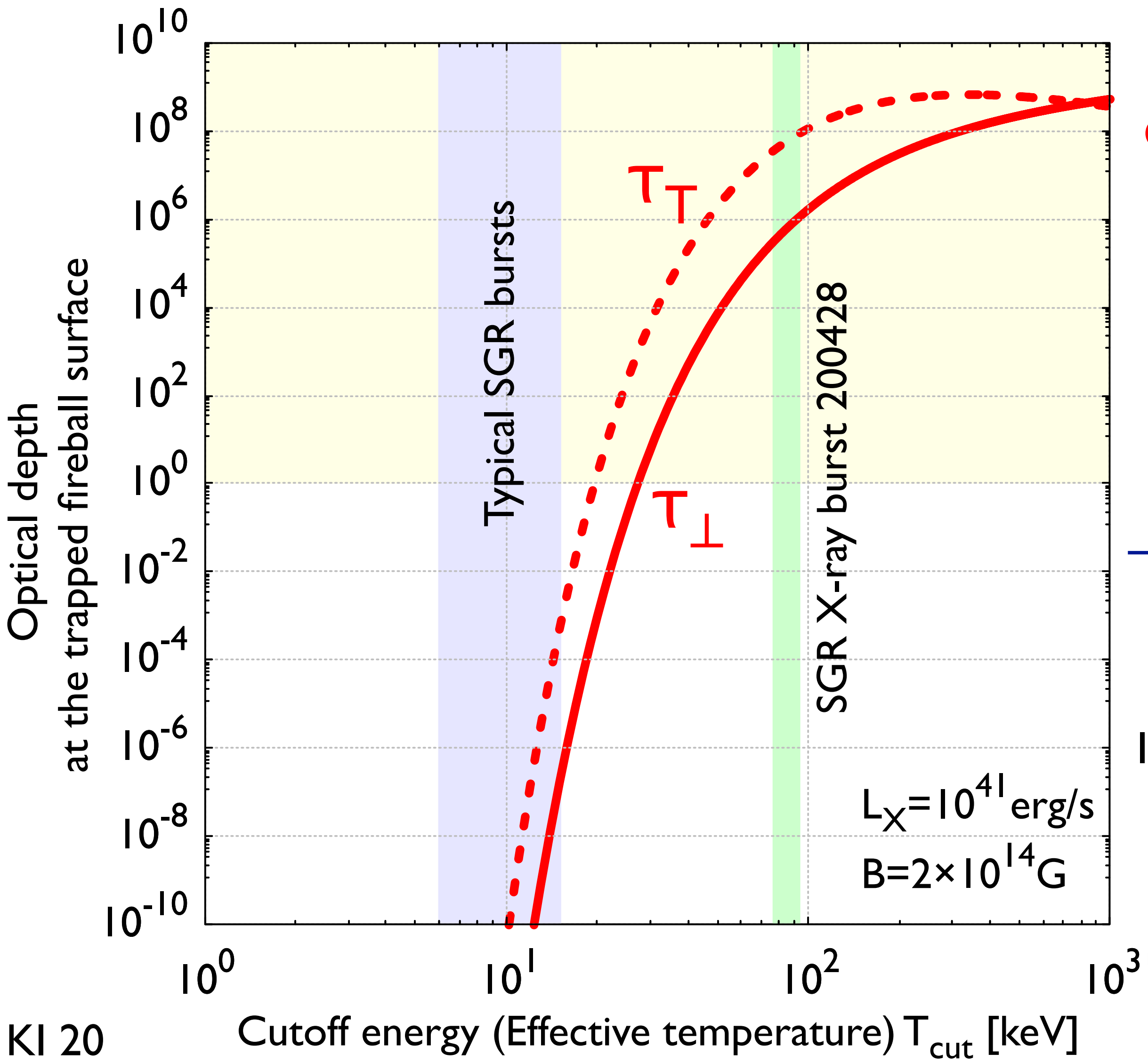


Spectrum

$N_E \propto E^\alpha \exp(-E/E_{cut})$

Younes+ 20

Optical Depth

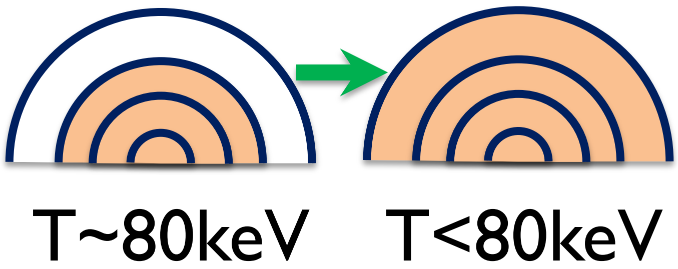


KI 20

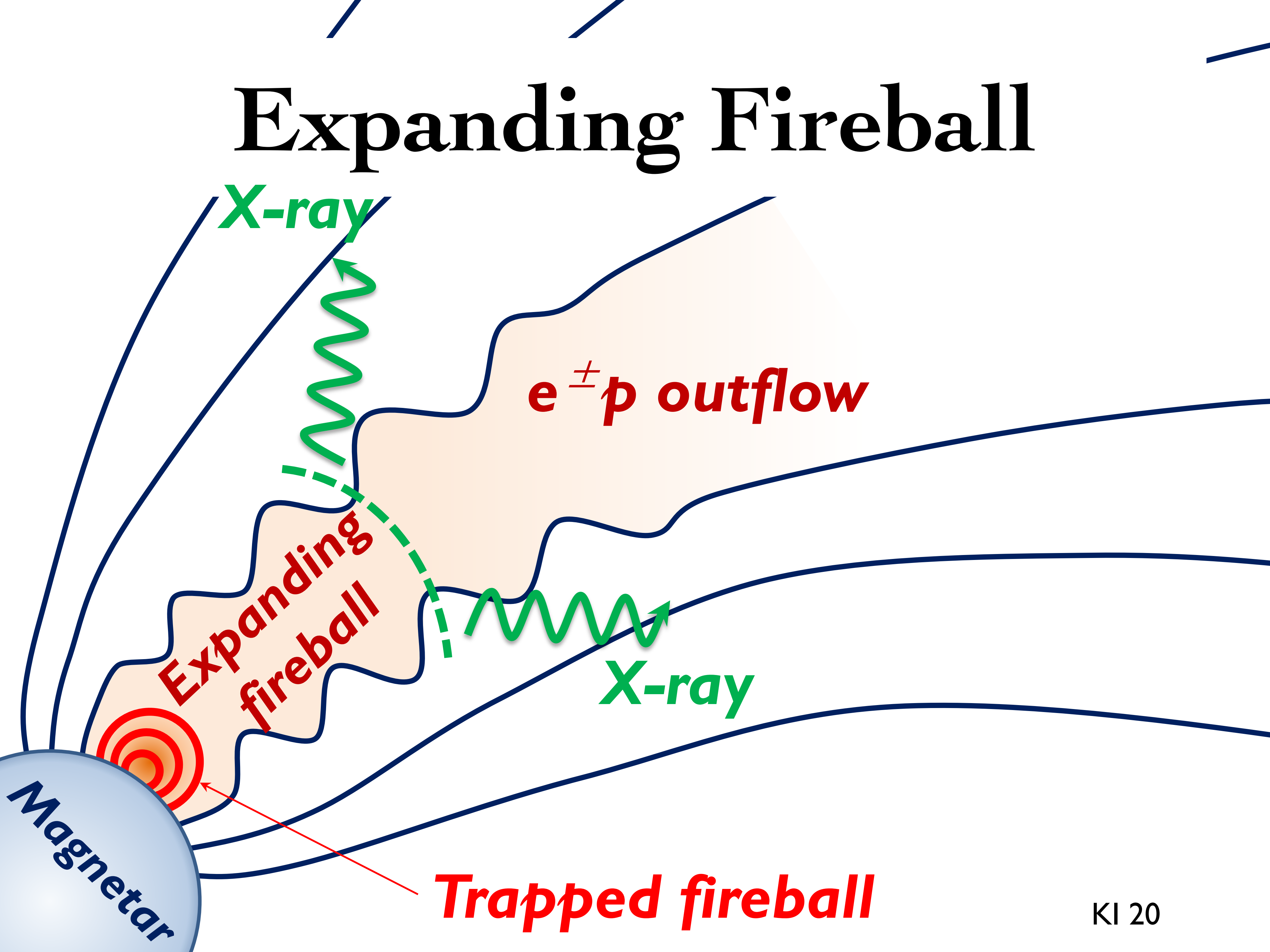
τ**>>1** at the surface of the trapped fireball

X-rays create $e^{\pm}$
→ Surrounding field should be open
→ ***Expanding fireball***

If B were closed, The fireball would get too big



Expanding Fireball



Fireball Temperature

Spherical GRB

$$T \propto r^{-1}$$

$$\Gamma \propto r$$

Dipole flux tube

$$T \propto r^{-3/2}$$

$$\Gamma \propto r^{3/2}$$

fast evolution

$$T_{obs} \sim \Gamma T \sim T_0$$

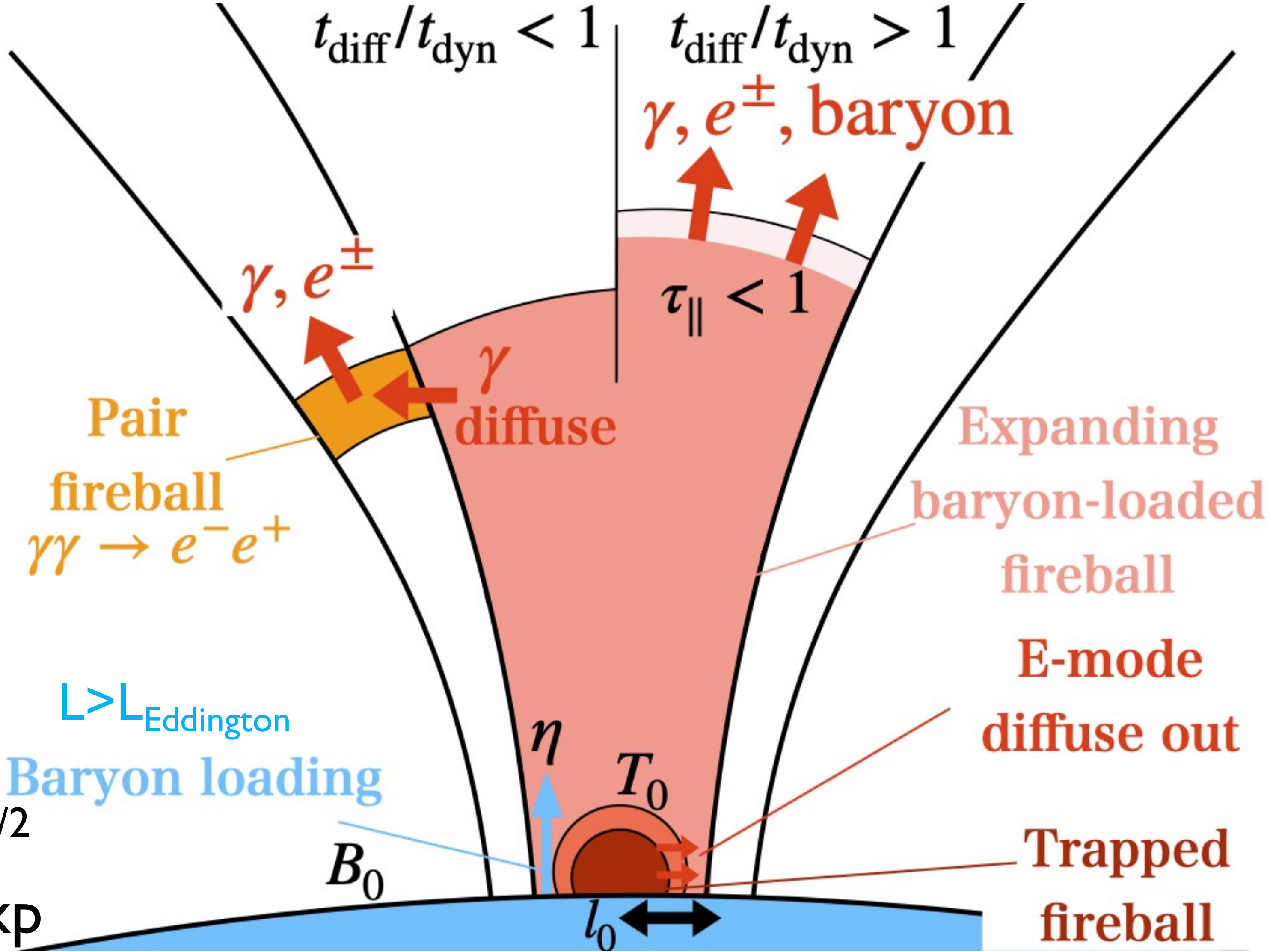
$$r^2 \Delta\Omega \rho \Gamma \beta = \mathrm{const.(number)}$$

$$r^2 \Delta\Omega e^{3/4} \Gamma \beta = \mathrm{const.(entropy)}$$

$$r^2 \Delta\Omega \left(\rho c^2 + \frac{4}{3} e \right) \Gamma^2 \beta^2 = \mathrm{const.(energy)}$$

+ radiation-dominated

Photon Escape



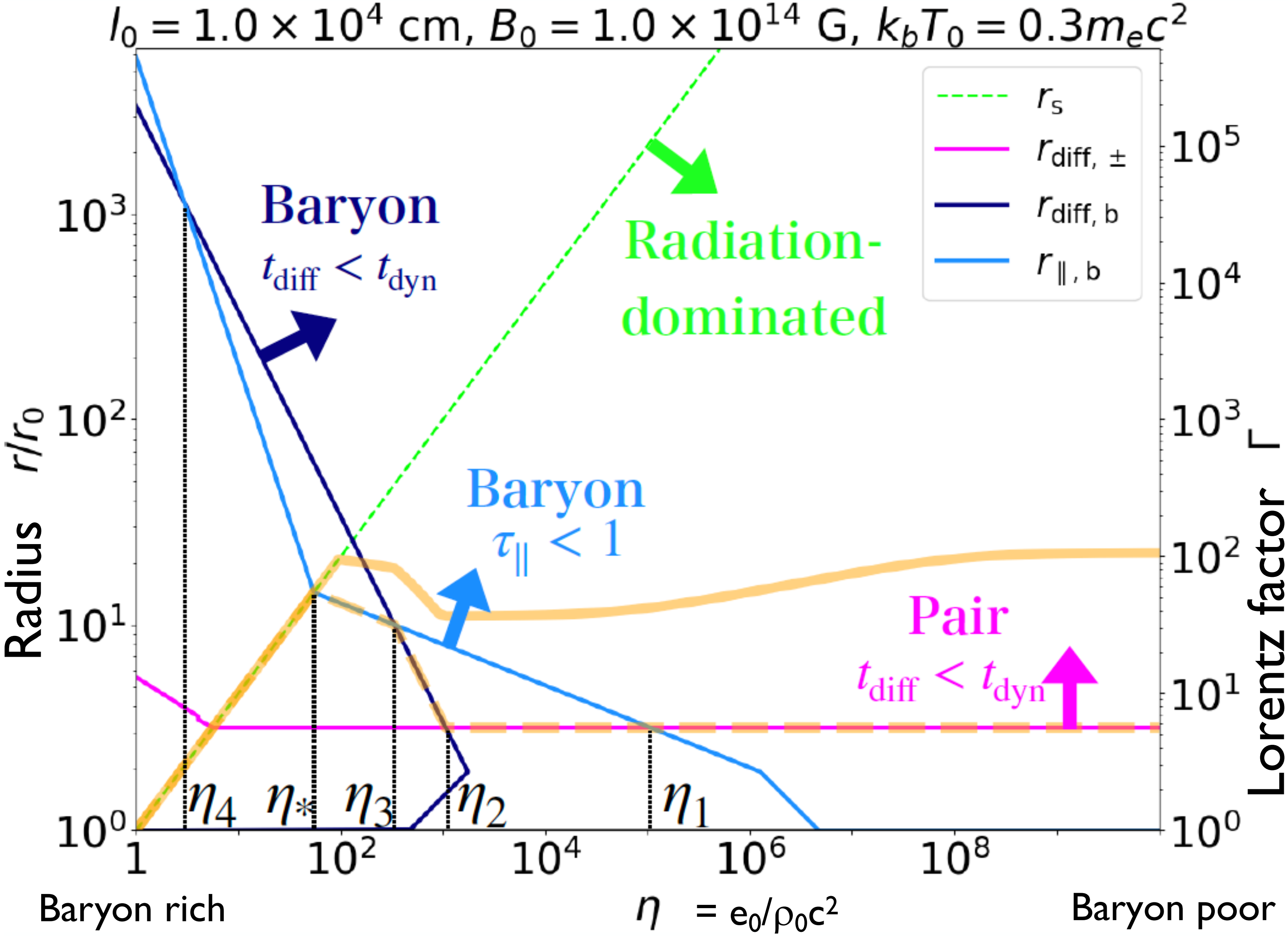
$L > L_{\rm Eddington}$

$n_e \propto r^{-9/2}$

$n_{\pm} \propto \exp$

Wada
& KI 22

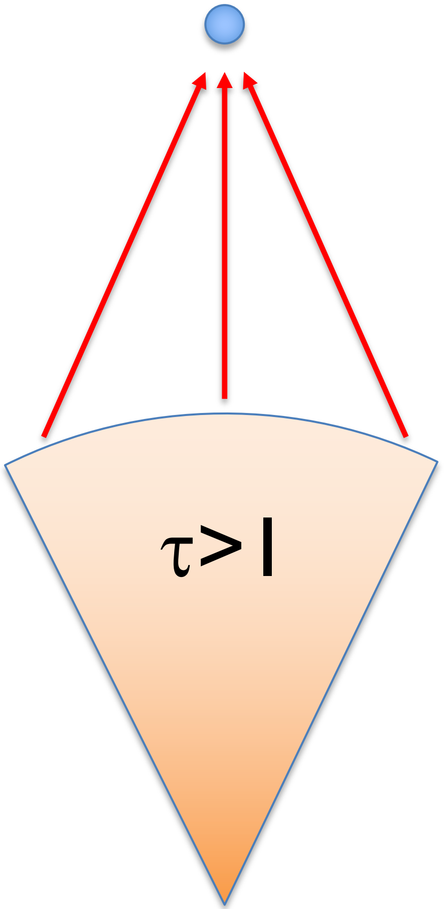
Fireball Evolution



Wada & KI 22

Radiative Acceleration

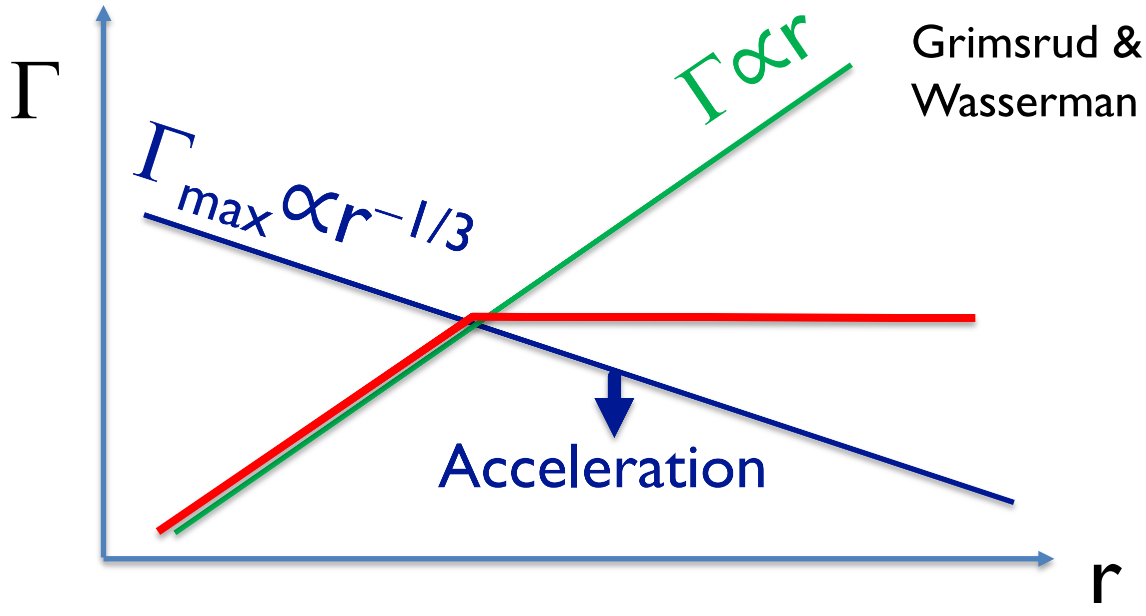
Even above the photosphere



Work by γ > Rest mass

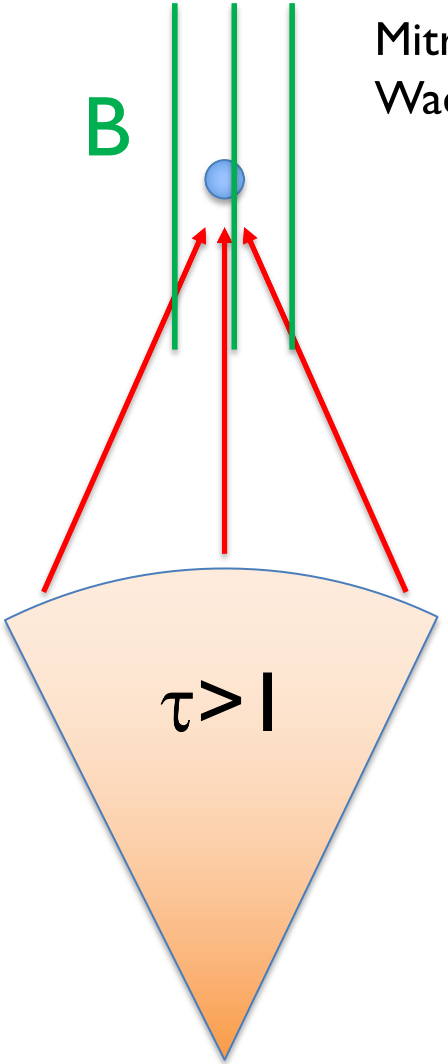
Thomson scattering

$$\frac{\sigma_T L_{iso}}{4\pi r^2\Gamma^2}\frac{r}{c\Gamma} > \bar{m}c^2$$



Meszaros+ 93
Grimsrud & Wasserman 98

Radiative Acceleration

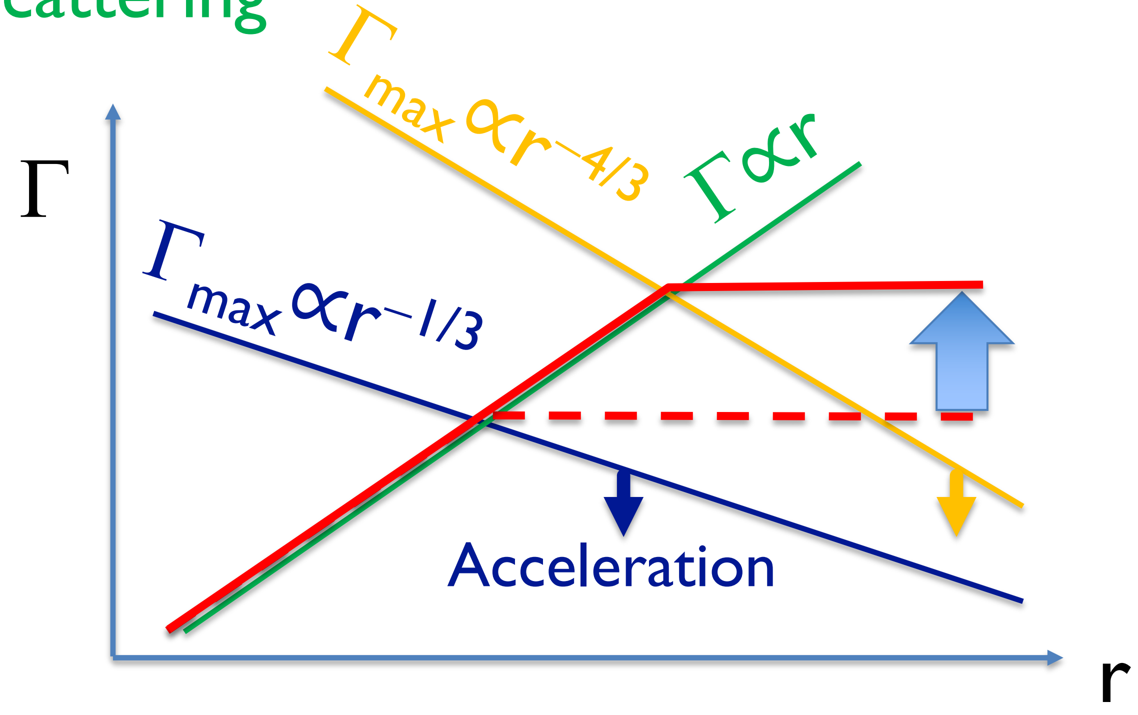


Mitrofanov & Pavlov 82
Wada & KI 22

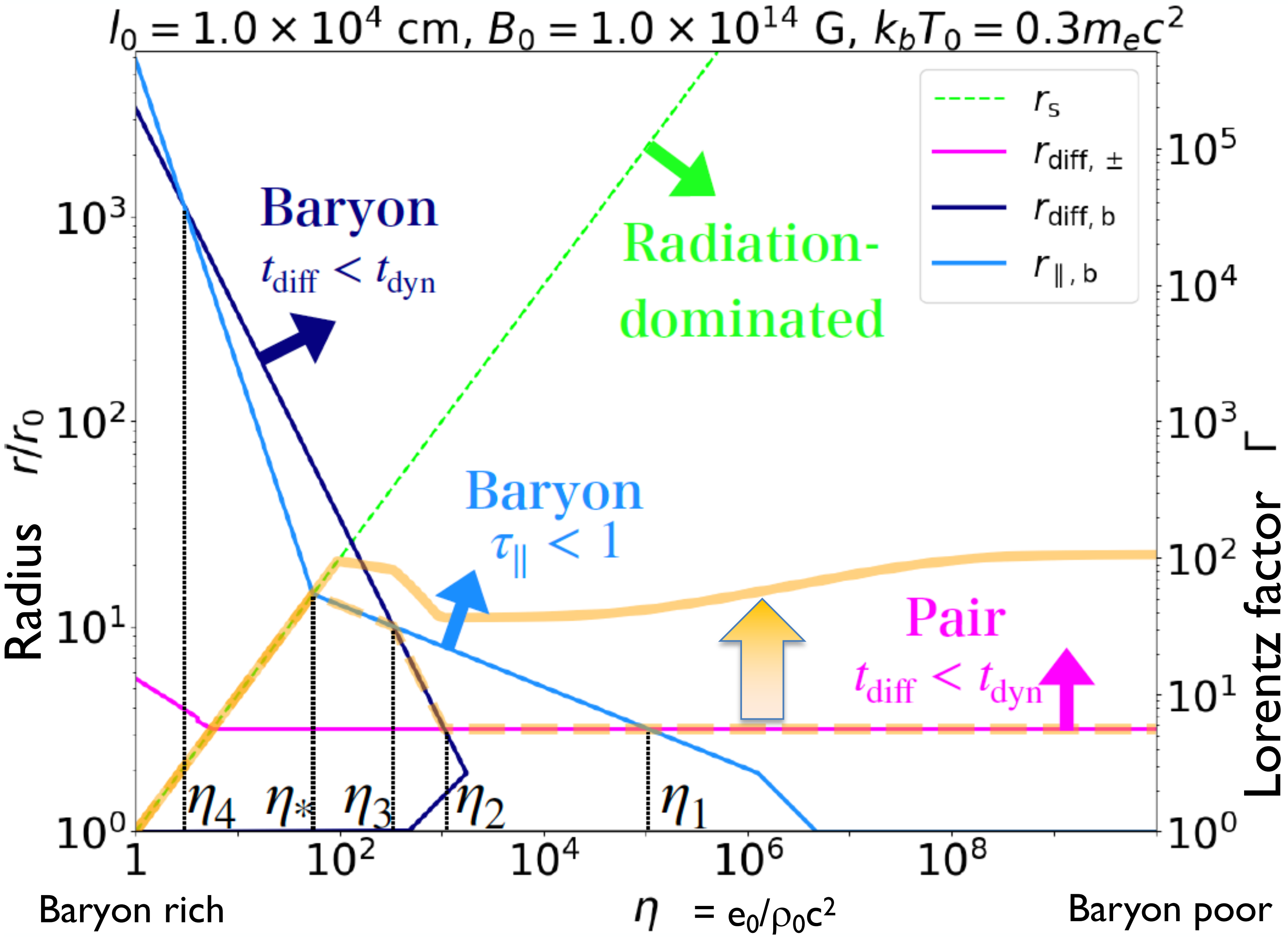
Resonant cyclotron scattering

Work by γ > Rest mass

$$\sigma_{res} \sim \frac{2\pi^2 e^2}{m_e c}\delta(\omega - \omega_c)$$

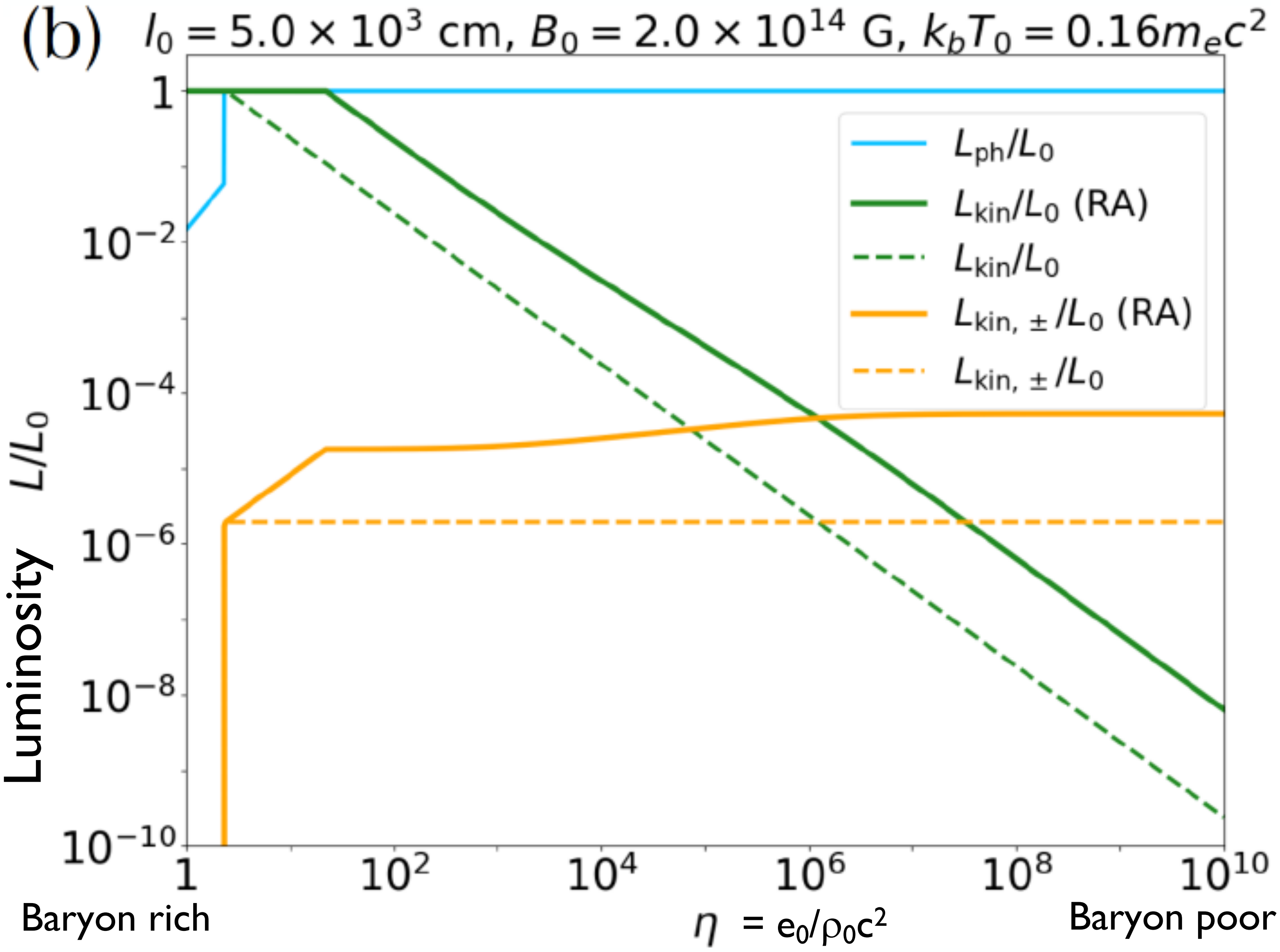


Fireball Evolution



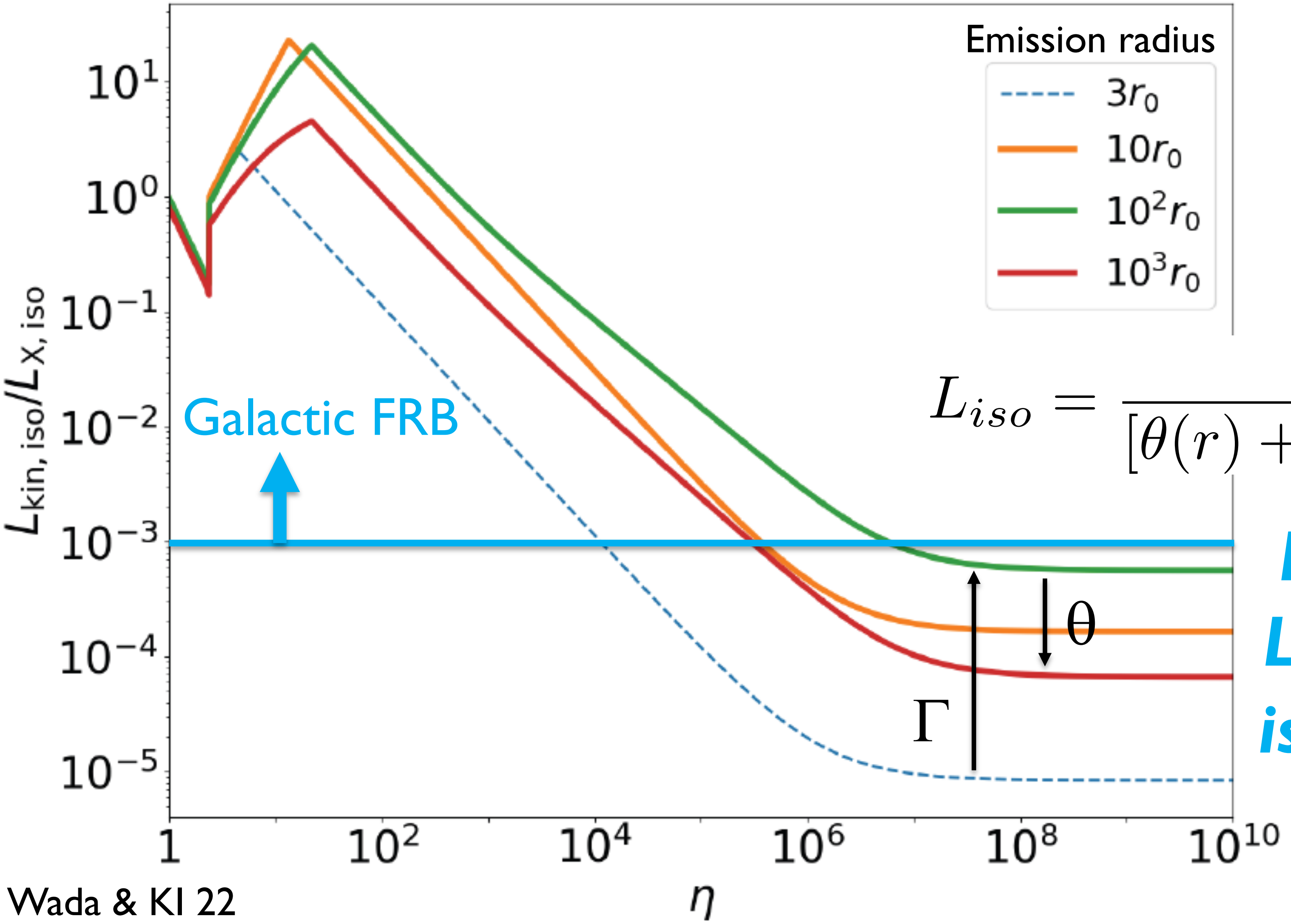
Wada & KI 22

L_{kin} for FRB 20200428A



Wada
& KI 22

Isotropic Luminosity

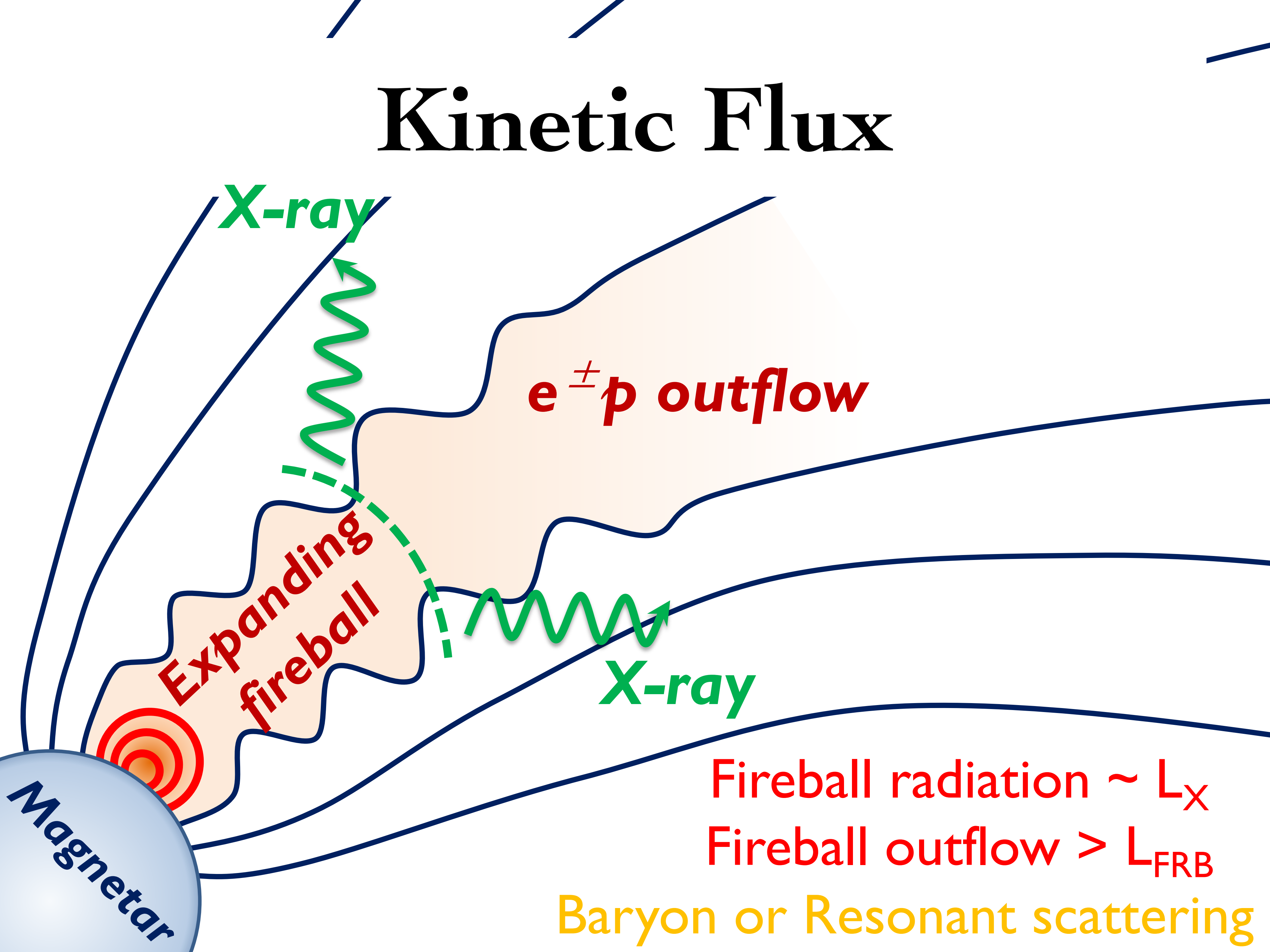


$$L_{iso} = \frac{4}{[\theta(r) + \Gamma^{-1}(r)]^2} L$$

Isotropic $L_{kin} > L_{FRB}$ is possible

Wada & KI 22

Kinetic Flux

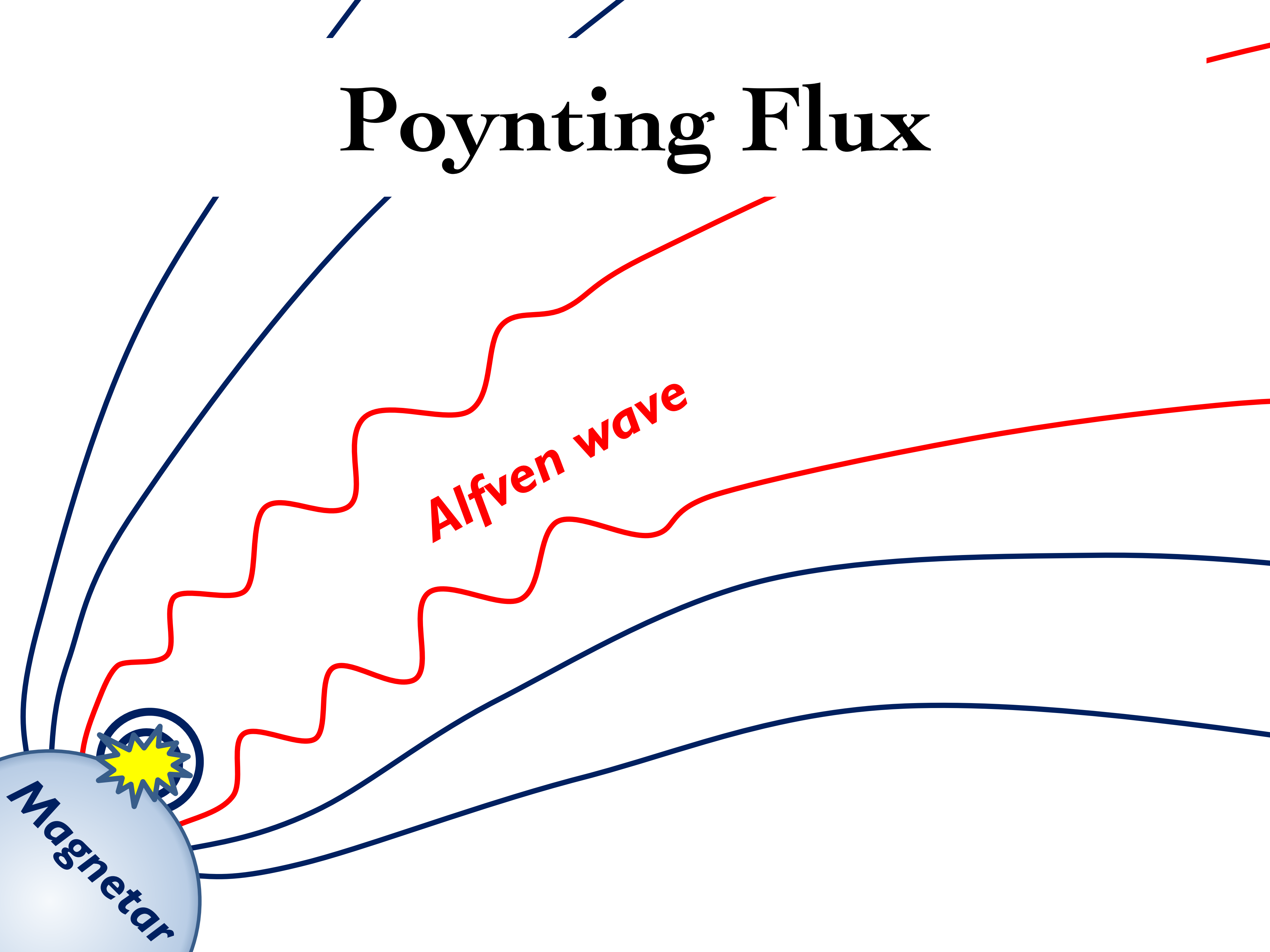


Fireball radiation ~ L_X

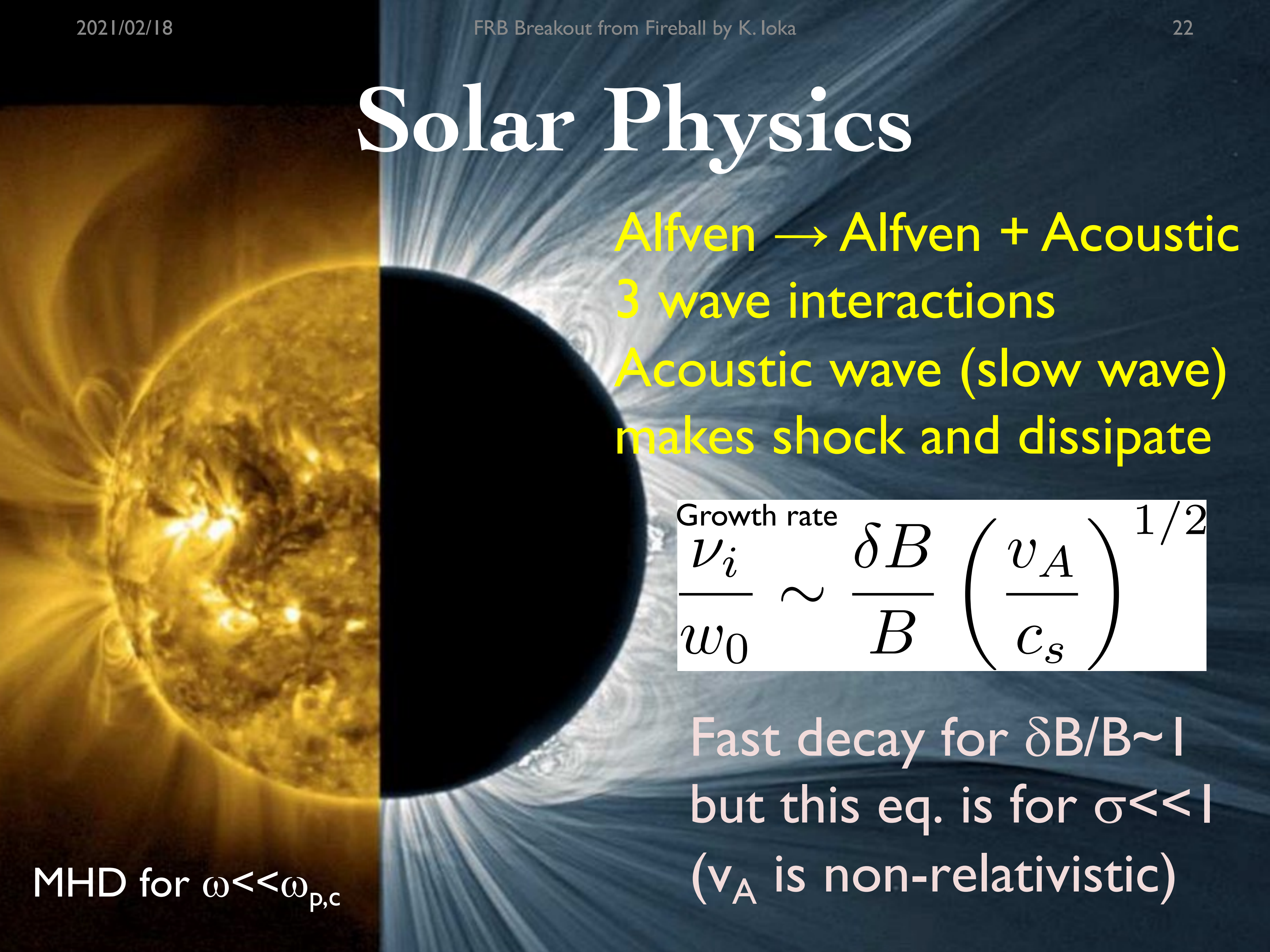
Fireball outflow > L_{FRB}

Baryon or Resonant scattering

Poynting Flux



Solar Physics



Alfven → Alfven + Acoustic

3 wave interactions

Acoustic wave (slow wave) makes shock and dissipate

Growth rate

$$\frac{\nu_i}{w_0} \sim \frac{\delta B}{B}\left(\frac{v_A}{c_s}\right)^{1/2}$$

Fast decay for $\delta B/B \sim 1$ but this eq. is for $\sigma \ll 1$ (v_A is non-relativistic)

MHD for $\omega \ll \omega_{p,c}$

To Relativistic MHD

Energy-momentum conservations & Induction eqs.

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 - p + \frac{1}{8\pi}\left(E^2+B^2\right)\right] + \nabla\cdot\left[(\epsilon+p)\gamma^2 v + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\,\gamma^2\boldsymbol{v} + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})\right] + \nabla\cdot\left[(\epsilon+p)\gamma^2\boldsymbol{v}\otimes\boldsymbol{v} - \frac{c^2}{4\pi}(\boldsymbol{E}\otimes\boldsymbol{E}+\boldsymbol{B}\otimes\boldsymbol{B})\right] + c^2\nabla\left[p + \frac{E^2+B^2}{8\pi}\right] = 0$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B})$$

EOS

$$p_1 = \frac{C_s^2}{c^2}\epsilon_1, \quad C_s^2 = c^2\left(\frac{\partial p}{\partial \epsilon}\right)_s$$

1. Ideal MHD ($\omega << \omega_{p,c}$)
2. Adiabatic EOS
3. B_0 || k
4. Circular polarization
5. In the fluid comoving frame (background v_{fluid} ~ 0)

Previous Research

- Please let us know the relevant papers
- We find papers on ***two fluids (pair fluid)***
  - Matsukiyo & Hada 03, Lopez+ 12
  - But the equations are very complex, calling for simple formulae
- Nonlinear interactions in ***force-free***
  - Thompson & Blaes 98, Lyubarsky 19, Li+ 19
  - Force-free kills the slow mode (a single Alfven wave is stable)

Perturbation

	Background	Parent wave	Daughter waves	
$\boldsymbol{B} =$	$\boldsymbol{B}_0$	$+\,\delta\boldsymbol{B}$	$+\,\boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\delta\boldsymbol{v}$	$+\,\boldsymbol{v}_\perp$	$+\,\boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\,\epsilon_\parallel$
		Alfvén wave	**Alfvén wave**	**Sound wave**
	$O(1)$	$O(\eta)$	$O(\varepsilon)$	

Section 2.2 : Neglecting $O(\varepsilon^2)$ and keeping all order of η

Section 2.3 : Neglecting both $O(\varepsilon^2)$ and $O(\eta^2)$, but keeping $O(\eta\epsilon)$

Perturbed Equations

$$\frac{1}{c}\frac{\partial e_\parallel}{\partial t} + \beta_s \frac{\partial u_\parallel}{\partial z} = -\frac{\sigma}{1+\sigma}\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{u}\cdot\boldsymbol{u}_\perp\right) - \sigma\delta\boldsymbol{u}\cdot\left(\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t} - \beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z}\right) \tag{27}$$

$$\frac{1}{c}\frac{\partial u_\parallel}{\partial t} + \beta_s \frac{\partial e_\parallel}{\partial z} = -\theta^{-1}\beta_A\frac{\partial}{\partial z}\left(\delta\boldsymbol{e}\cdot\boldsymbol{e}_\perp\right) + \sigma\theta^{-1}\delta\boldsymbol{e}\cdot\left(\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t} - \beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z}\right) \tag{28}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t} - \beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z} = \theta\beta_A^2\frac{1}{c}\frac{\partial}{\partial t}\left(u_\parallel\delta\boldsymbol{e}\right) - \frac{1}{1+\sigma}\left[\beta_s u_\parallel\frac{\partial}{\partial z}(\delta\boldsymbol{u}) + \beta_A e_\parallel\frac{\partial}{\partial z}(\delta\boldsymbol{e}) + \beta_s^2\frac{1}{c}\frac{\partial}{\partial t}\left(e_\parallel\delta\boldsymbol{u}\right)\right] \tag{29}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{e}_\perp}{\partial t} - \beta_A\frac{\partial \boldsymbol{u}_\perp}{\partial z} = -\theta\beta_A\frac{\partial}{\partial z}\left(u_\parallel\delta\boldsymbol{e}\right) \tag{30}$$

For the normalized quantities

$$\delta\boldsymbol{u} \equiv \frac{\delta\boldsymbol{\beta}}{\beta_A}, \quad \boldsymbol{u}_\perp \equiv \frac{\boldsymbol{\beta}_\perp}{\beta_A}$$

$$\delta\boldsymbol{e} = \frac{\delta\boldsymbol{B}}{B_0}, \quad \boldsymbol{e}_\perp = \frac{\boldsymbol{b}_\perp}{B_0}$$

$$u_\parallel \equiv \frac{\beta_\parallel}{\beta_s}, \quad e_\parallel \equiv \frac{\epsilon_\parallel}{w_0}$$

Alfven velocity

$$\beta_A^2 = \sigma/(1+\sigma)$$

Enthalpy

$$w_0 \equiv \epsilon_0 + p_0$$

Dimensionless parameters

$$\sigma \equiv \frac{B_0^2}{4\pi\left(\epsilon_0 + p_0\right)}$$

$$\theta \equiv \frac{\beta_s}{\beta_A}$$

Dispersion Relation

Fourier mode expansion

Parent

$$\delta \boldsymbol{e} = \frac{1}{\sqrt{2}} \left(\delta e_0 \exp\left(i\phi_0\right) \mathbf{e}_j + \text{ c.c. } \right)$$

$$\delta \boldsymbol{u} = -\delta \boldsymbol{e}$$

Sound

$$e_\parallel = \frac{1}{2} \left(e_k \exp(i\phi) + \text{ c.c. } \right), \quad u_\parallel = \frac{1}{2} \left(u_k \exp(i\phi) + \text{ c.c. } \right)$$

Alfven

$$\boldsymbol{e}_\perp = \frac{1}{\sqrt{2}} \left(e_+ \exp\left(i\phi_+\right) \mathbf{e}_j + \text{ c.c. } \right) + \frac{1}{\sqrt{2}} \left(e_- \exp\left(i\phi_-\right) \mathbf{e}_j + \text{ c.c. } \right)$$

$$\boldsymbol{u}_\perp = \frac{1}{\sqrt{2}} \left(u_+ \exp\left(i\phi_+\right) \mathbf{e}_j + \text{ c.c. } \right) + \frac{1}{\sqrt{2}} \left(u_- \exp\left(i\phi_-\right) \mathbf{e}_j + \text{ c.c. } \right)$$

$$\phi_0 \equiv k_0 z - \omega_0 t$$

$$\phi \equiv kz - \omega t$$

$$\phi_+ \equiv \phi_0 + \phi, \quad \phi_- \equiv \phi_0 - \phi$$

(satisfying resonance conditions)

Det (6 ×6 matrix) = 0 → Dispersion relation

$$\boxed{(\omega - k)^2 \left(\omega^2 - \theta^2 k^2\right) \left\{(\omega + k)^2 - 4\right\} = \frac{1}{(1+\sigma)^4} \eta^2 (\omega - k) \left(S_0 + S_1 \sigma + S_2 \sigma^2 + S_3 \sigma^3 + S_4 \sigma^4\right)}$$

(in the unit of k_0=1, w_0=1)

$$S_0 = k^2 \left(\omega^3 + k\omega^2 - 3\omega + k\right)$$

S_1=…, S_2=…, S_3=…, S_4=…

Non-relativistic Limit

Neglecting O($\beta_A{}^2$) $\sigma = \beta_A^2/(1-\beta_A^2)$ is small

$$(\omega - k)^2 \left(\omega^2 - \theta^2 k^2\right) \left\{(\omega + k)^2 - 4\right\} = \eta^2 k^2 (\omega - k) \left(\omega^3 + k\omega^2 - 3\omega + k\right)$$

the same as Goldstein (1978) & Derby (1978)

In the limit $\beta = c_s{}^2/v_A{}^2 \rightarrow 0$, the decay instability is recovered: forward Alfven → forward sound + backward Alfven

$$\frac{\omega_i}{\omega_0} = \frac{\delta B}{B} \left(\frac{v_A}{c_s}\right)^{1/2}$$

High σ Limit

$$(\omega-k)^2\left(\omega^2-\theta^2k^2\right)\left\{(\omega+k)^2-4\right\}=\frac{1}{(1+\sigma)^4}\eta^2(\omega-k)\left(S_0+S_1\sigma+S_2\sigma^2+S_3\sigma^3+S_4\sigma^4\right)$$

$\sigma \to \infty$

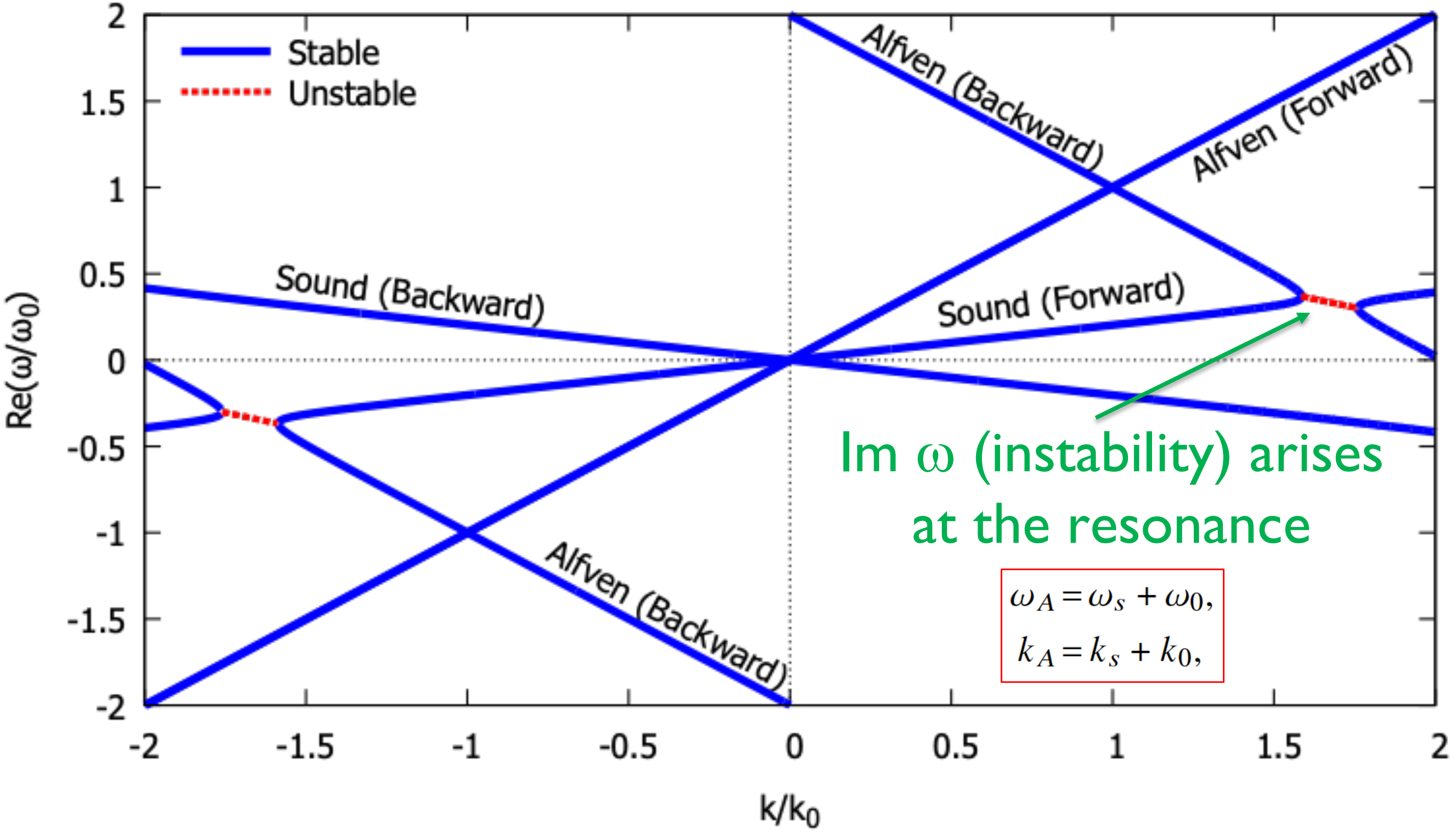
$$(\omega-k)^2\left\{(\omega+k)^2-4\right\}\left(\omega^2-\eta^2\theta^2k\omega-\theta^2k^2\right)=0$$

No instability

consistent with force-free

(a single Alfven wave is stable in force-free)

Dispersion Relation



Im ω (instability) arises at the resonance

$$\omega_A = \omega_s + \omega_0,$$
$$k_A = k_s + k_0,$$

1/σ Expansion

$$(\omega-k)^2\left(\omega^2-\theta^2k^2\right)\left\{(\omega+k)^2-4\right\}=\frac{1}{(1+\sigma)^4}\eta^2(\omega-k)\left(S_0+S_1\sigma+S_2\sigma^2+S_3\sigma^3+S_4\sigma^4\right)$$

Intersection point of backward Alfven and forward sound

$$\omega_1=\frac{2\theta}{1+\theta},k_1=\frac{2}{1+\theta}$$

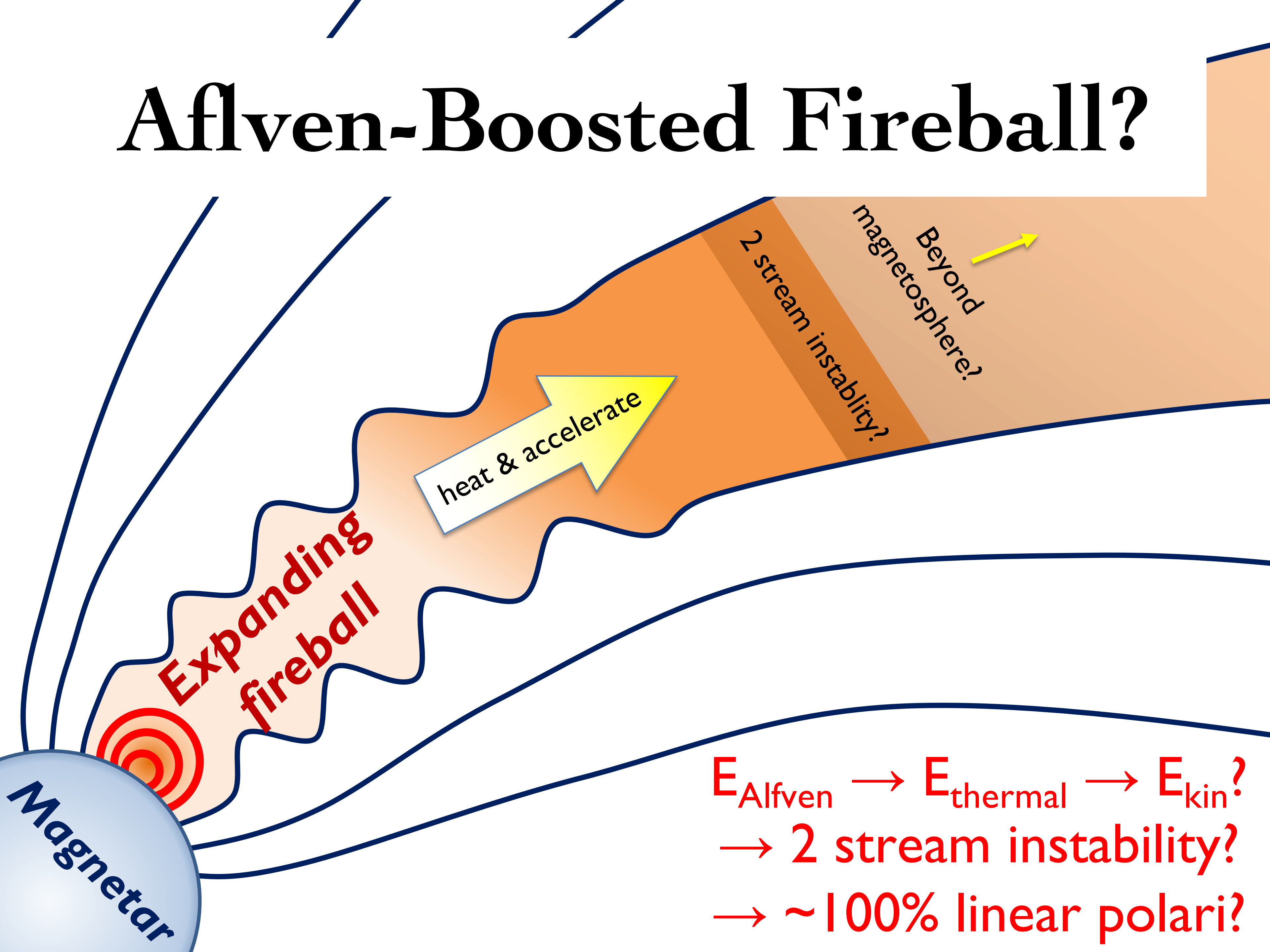
Expansion around this point

$$\omega=\omega_1+\delta\omega\quad(|\delta\omega|\ll\omega_1),\quad k=k_1$$

σ >> 1, θ << 1

$$\frac{\operatorname{Im}\delta\omega}{\omega_0}\sim\frac{1}{2}\eta\theta^{-1/2}\sigma^{-1/2}\sim\underset{\text{Non-rela}}{\underline{\frac{1}{2}\frac{\delta B}{B}\left(\frac{v_A}{c_s}\right)^{1/2}}}\;\underset{\textbf{Rela}}{\underline{\sigma^{-1/2}}}$$

Aflven-Boosted Fireball?



$E_{\text{Alfven}} \rightarrow E_{\text{thermal}} \rightarrow E_{\text{kin}}$?
→ 2 stream instability?
→ ~100% linear polari?

Plasma Freq. in Fireball

Fireball density

$$n'_e = \frac{L_m}{4\pi r^2 m c^3 \Gamma^2}$$

$$\sim 1 \times 10^{17} \frac{L_{m,42}}{P^2\Gamma^2} \left(\frac{r}{r_L}\right)^{-2} \mathrm{cm}^{-3}$$

$$r_L = \frac{cP}{2\pi} \sim 5 \times 10^9 \, P \, \mathrm{cm}$$

Plasma frequency

$$\nu_p = \Gamma \left(\frac{q^2 n'_e}{\pi m}\right)^{1/2}$$

$$\sim 3 \times 10^3 \frac{L_{m,42}^{1/2}}{P} \left(\frac{r}{r_L}\right)^{-1} \mathrm{GHz}$$

Need more studies

Summary

- ***Expanding fireball***
  - $E_\gamma = E_X$ & $\boldsymbol{E_{kin} > E_{FRB}}$
  - Baryon or Radiative acceleration via resonance
- ***Decay of Alfven waves***
  - Relativistic MHD: Alfven → Alfven + Acoustic
  - ***Alfven waves could boost fireball***
- ***Emission mechanism is not yet***
  - ***2-stream instability? Far-away outflow?***

Thank You

Alfven Wave Amplitude

- At the emission region

$$L \sim 4\pi r^2 \frac{\delta B^2}{8\pi} c$$

$$\frac{\delta B}{B} \sim 10^{-6} B_{14}^{-1} L_{38}^{1/2} r_6^2$$ ~1 @r~10^9cm

  – Perturbation is good at r<~10^9 cm

- Below it, if B is dipole, the solid angle is ∝ r

$$\delta B \propto r^{-3/2}, B \propto r^{-3}, \frac{\delta B}{B} \propto r^{3/2}$$

Nonlinear Interaction

$$\mathbf{B}=\underbrace{\mathbf{B}_0}_{\text{BG}}+\underbrace{\delta\mathbf{B}_\perp(z,t)}_{\text{Parent Alfven}}+\underbrace{\mathbf{b}_\perp(z,t)}_{\text{Daughter Alfven}},$$
$$\mathbf{V}=\underbrace{\delta\mathbf{V}_\perp(z,t)}_{\text{Parent Alfven}}+\underbrace{\mathbf{v}_\perp(z,t)}_{\text{Daughter Alfven}}+\underbrace{\mathbf{v}_\parallel(z,t)}_{\text{Daughter sound wave}},$$
$$\rho=\rho_0+\underbrace{\rho(z,t)}_{\text{Daughter sound wave}}.$$

Parent Alfven: Circular

$$\delta\mathbf{B}_\perp(z,t)=\delta\mathbf{B}_\perp\exp[i(k_0z-\omega_0t)]+c.c.,$$
$$\delta\mathbf{V}_\perp=-\frac{B_0}{4\pi\rho_0}\frac{k_0}{\omega_0}\delta\mathbf{B}_\perp(z,t),$$
$$\omega_0^2=(B_0^2/4\pi\rho_0)k_0^2=V_A^2k_0^2$$

Ideal MHD, B_0 || k

$$\frac{\partial v_\parallel}{\partial t}+\frac{c_s^2}{\rho_0}\frac{\partial\rho}{\partial z}=\underline{-\frac{\partial}{\partial z}\left(\frac{\mathbf{b}_\perp\cdot\delta\mathbf{B}_\perp}{4\pi}\right)\frac{1}{\rho_0}},$$

nonlinear dominant term

$$\frac{\partial\rho}{\partial t}+\rho_0\frac{\partial v_\parallel}{\partial z}=0,$$
$$\frac{\partial\mathbf{v}_\perp}{\partial t}-\frac{B_0}{\rho_0}\frac{\partial}{\partial z}\left(\frac{b_\perp}{4\pi}\right)=-v_\parallel\frac{\partial}{\partial z}(\delta V_\perp)\underline{-\frac{B_0\rho}{4\pi\rho_0^2}\frac{\partial}{\partial z}(\delta\mathbf{B}_\perp)}$$

nonlinear dominant term

$$\frac{\partial\mathbf{b}_\perp}{\partial t}-B_0\frac{\partial}{\partial z}\mathbf{v}_\perp=-\frac{\partial}{\partial z}(v_\parallel\delta\mathbf{B}_\perp).$$

Resonant conditions

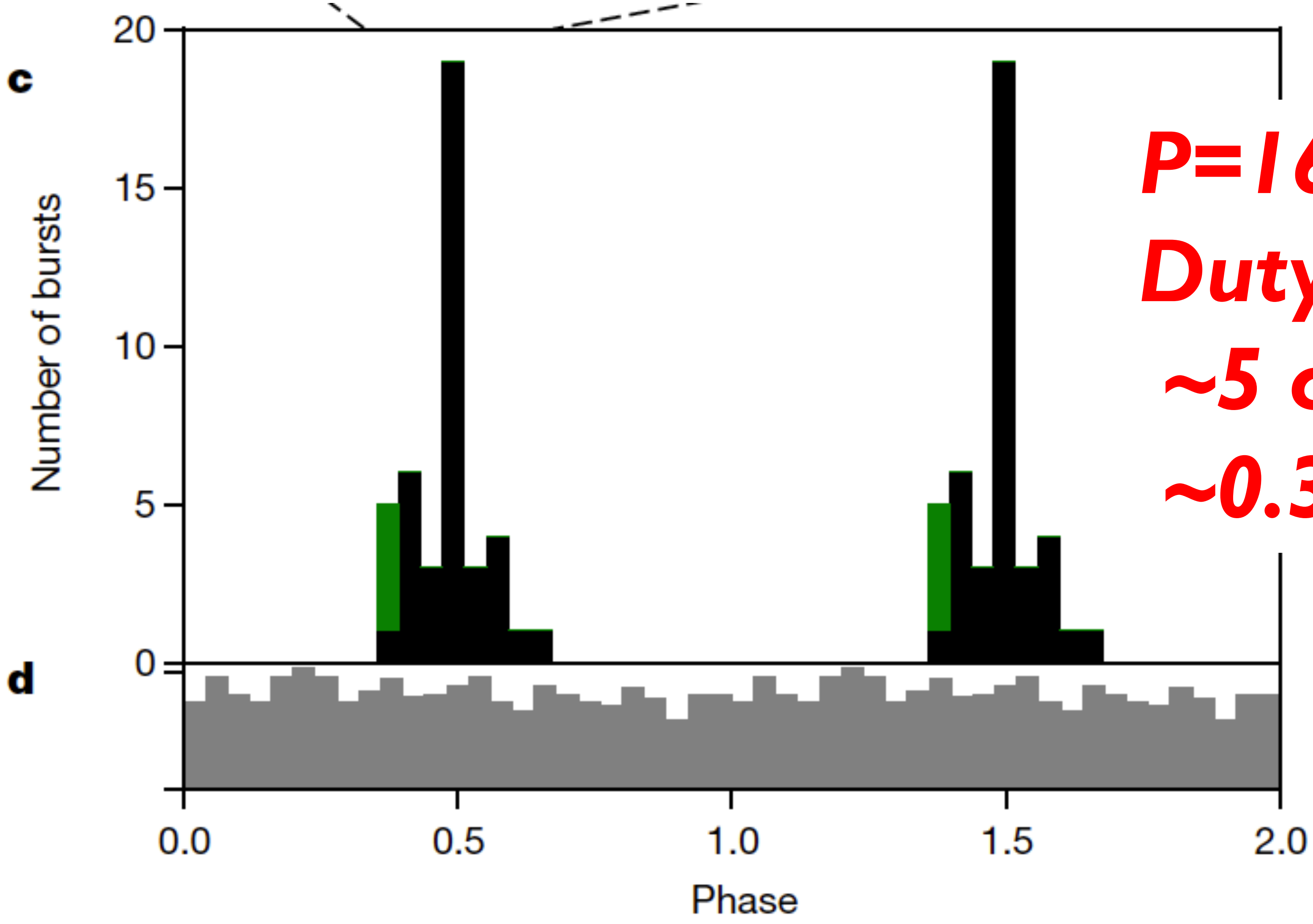
$$\boxed{\begin{aligned}\omega_A&=\omega_s+\omega_0,\\ k_A&=k_s+k_0,\end{aligned}}$$

$$\frac{\partial v_\parallel}{\partial t}=i(k_A-k_0)\frac{B_0k_A}{\rho_0\omega_A}\left(\frac{\delta\mathbf{B}_\perp^*\mathbf{v}_\perp}{4\pi}\right),$$
$$\frac{\partial\mathbf{v}_\perp}{\partial t}=-\frac{iB_0k_sk_0}{4\pi\rho_0\omega_s}v_\parallel\delta\mathbf{B}_\perp,$$

$$v_\parallel,v_\perp\sim e^{i\nu t}\quad\rightarrow\ \mathbf{Im}\ \nu$$

Galeev & Oraevskii 63; Sagdeev & Galeev 69

Periodic FRB
180916.J0158+65

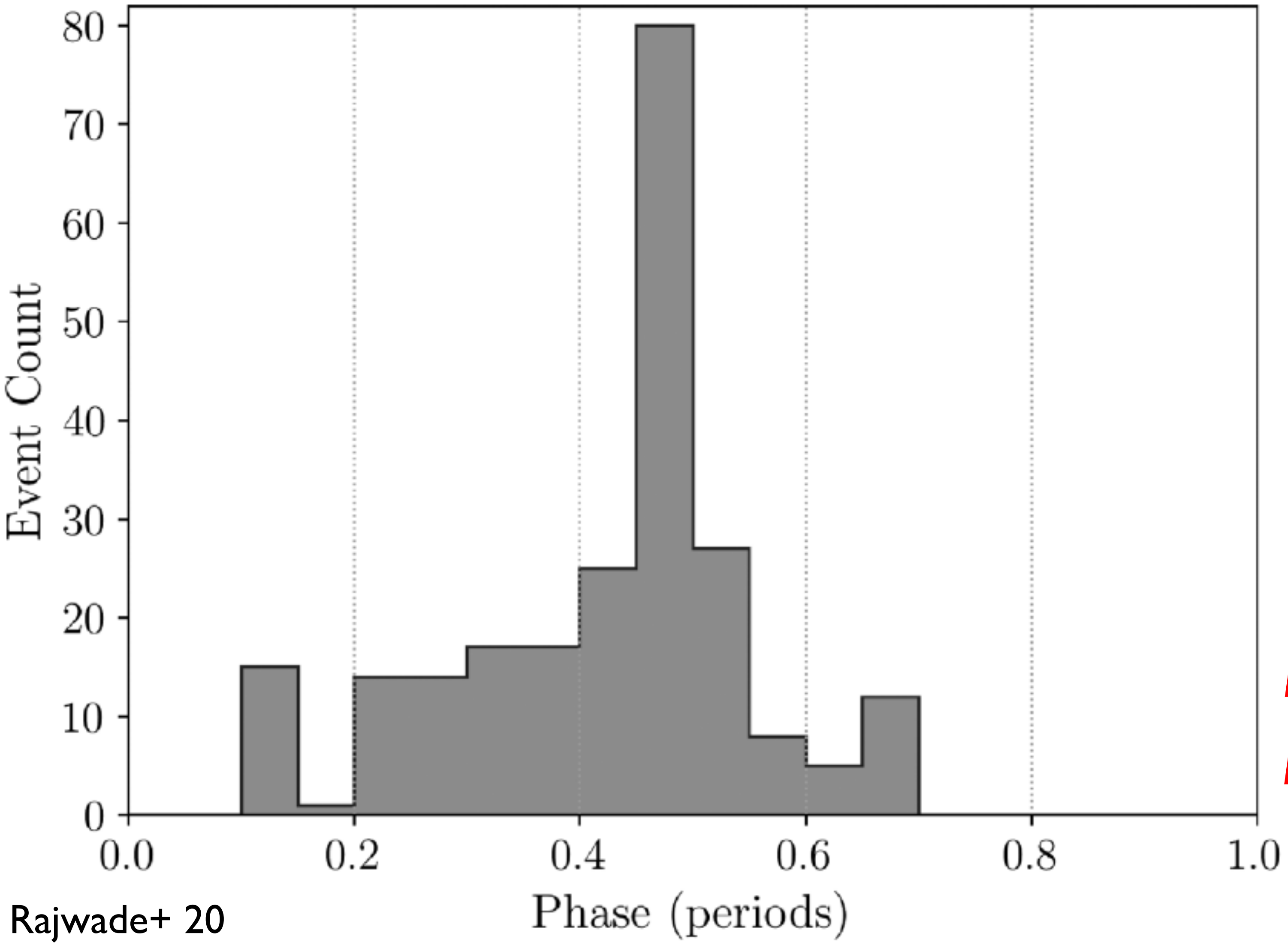


P=16.35 d

Duty cycle
~5 d/16 d
~0.3

CHIME/FRB 20

FRB 121102 also Periodic



Rajwade+ 20

1^{st} repeater
L_R~3 ×10^{41} erg/s

76 m Lovell telescope
5 yr data

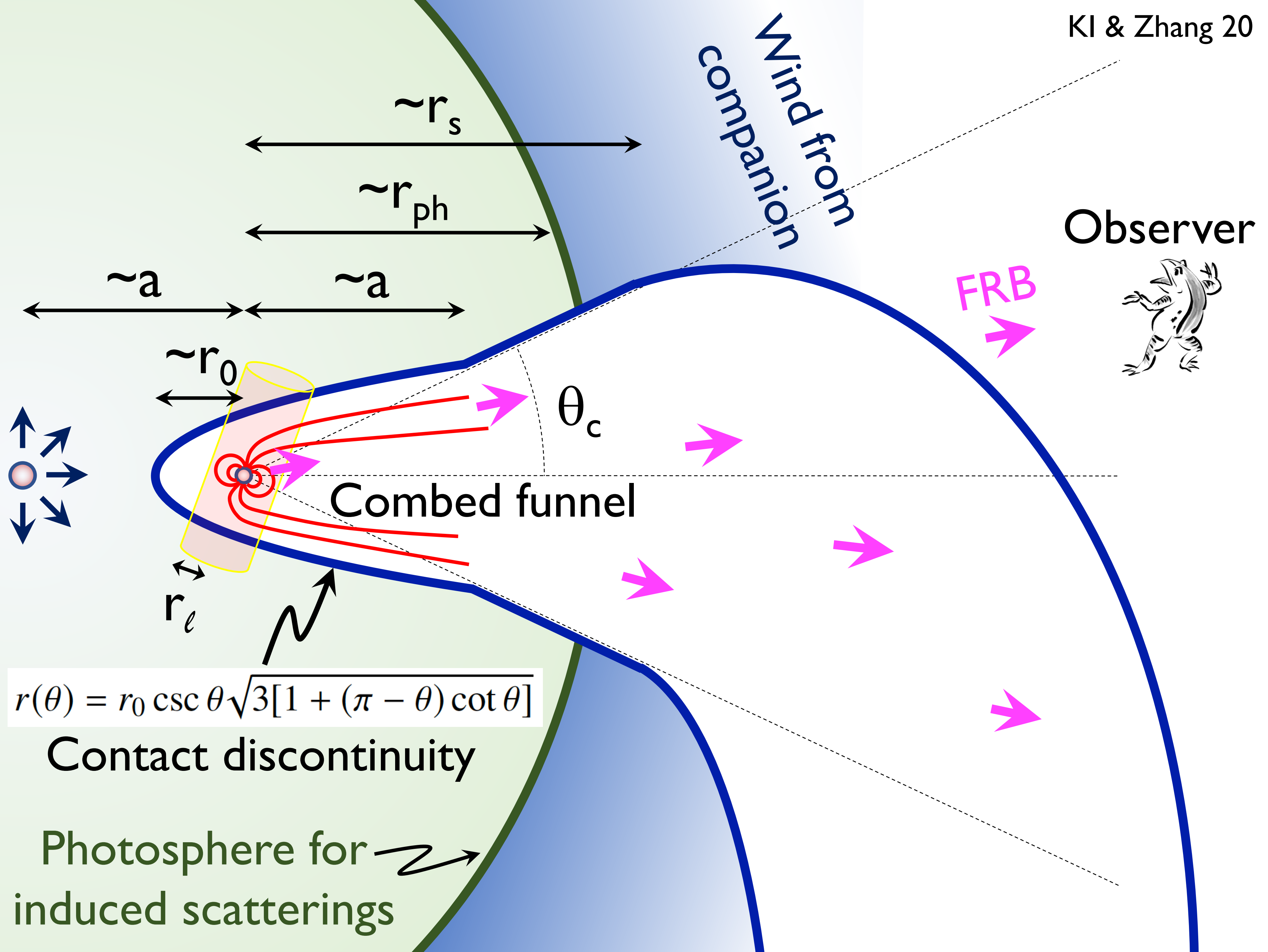
P~157 day
Duty cycle ~0.5

Binary Scenario

$$
\begin{aligned}
a &= (GM)^{1/3}(P_{\rm orb}/2\pi)^{2/3} \\
&\sim 4\times10^{12}\,{\rm cm}\left(\frac{M}{10\,M_\odot}\right)^{1/3}\left(\frac{P_{\rm orb}}{16\,{\rm day}}\right)^{2/3}
\end{aligned}
$$

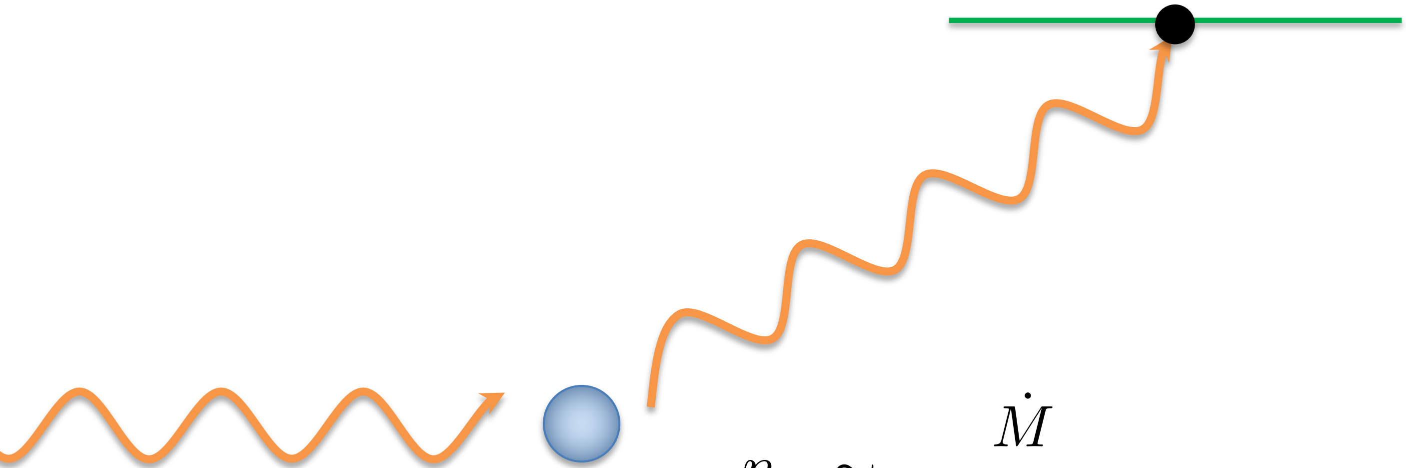
KI & Zhang 20

KI & Zhang 20



$$r(\theta) = r_0 \csc\theta \sqrt{3[1 + (\pi - \theta)\cot\theta]}$$

Opacity



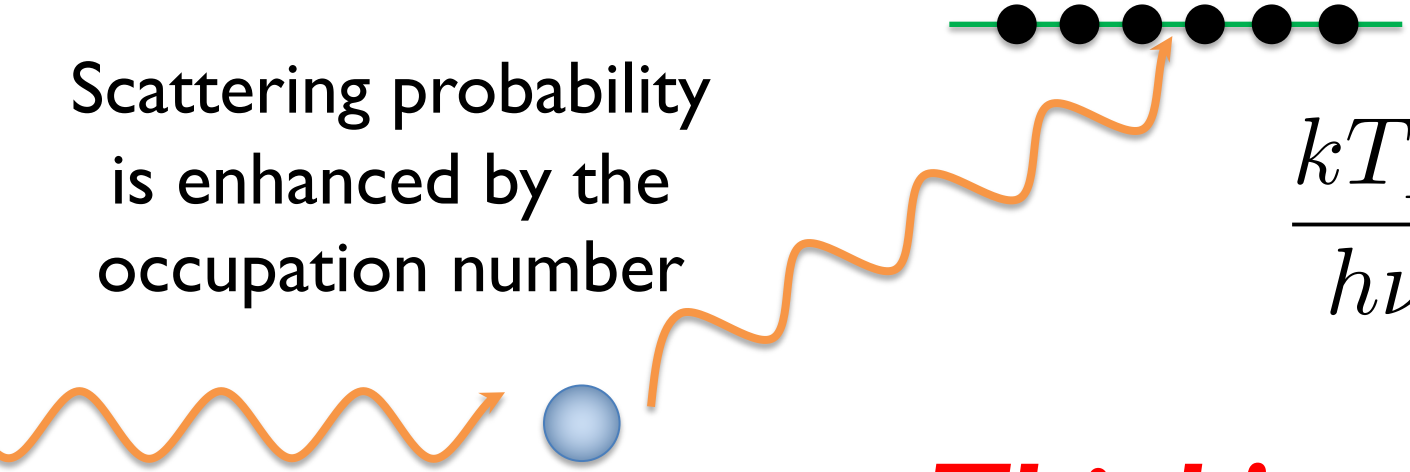
$$n_w \sim \frac{\dot{M}}{4\pi a^2 m_p V}$$

$$\sim 9 \times 10^5 \mathrm{cm}^{-3} \left(\frac{\dot{M}}{10^{-9}\, M_\odot\, \mathrm{yr}^{-1}} \right)$$

$$\times \left(\frac{a}{4 \times 10^{12}\, \mathrm{cm}} \right)^{-2} \left(\frac{V}{2 \times 10^3\, \mathrm{kms}^{-1}} \right)^{-1}$$

Thomson thin $\sigma_T n_w a << 1$

Induced Compton



Scattering probability is enhanced by the occupation number

$$\frac{kT_B}{h\nu} \sim 10^{33}$$

Wilson & Rees 78,
Thompson+ 94,
Lyubarsky 08

Thick!

$$\tau_C \sim \frac{3\sigma_T}{32\pi^2} \frac{n_w(r) L c \Delta t}{r^2 m_e \nu^3} \sim 30\, \dot{M}_{-9} V_{3.3}^{-1} r_{13}^{-4} (L\Delta t)_{38} \nu_9^{-3}$$

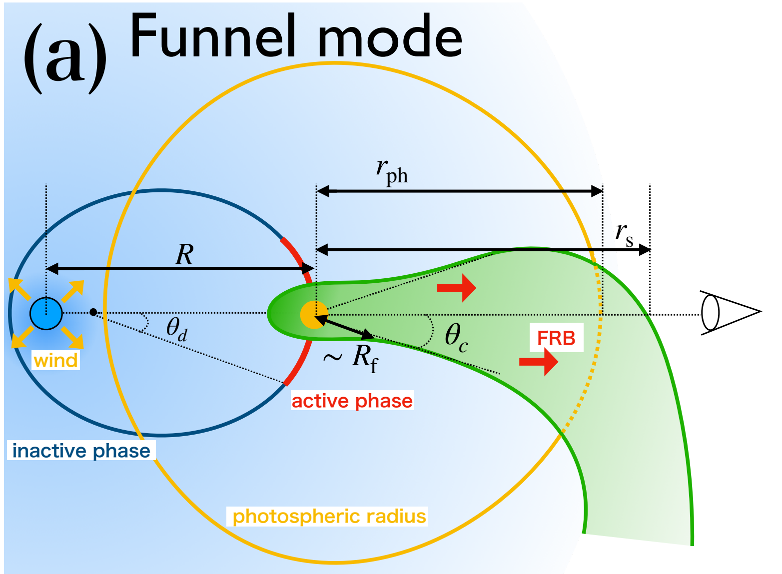
Photospheric radius for the induced Compton scatterings

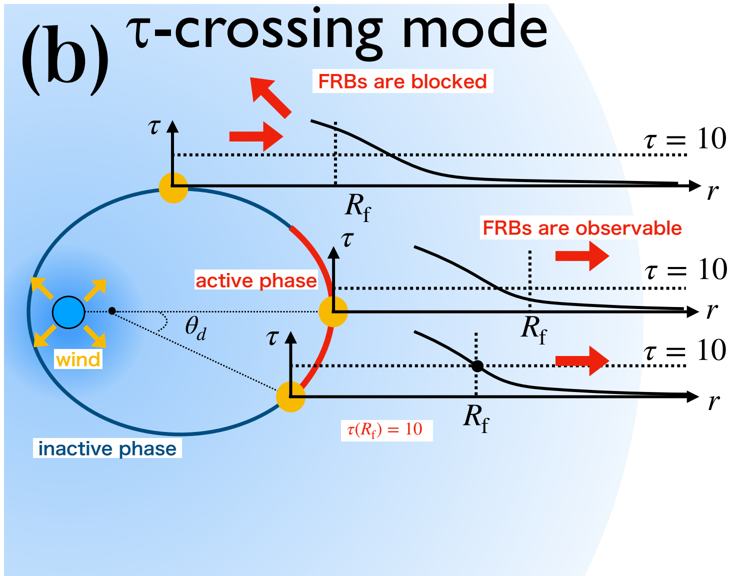
$$r_{\rm ph}^{C} \sim 1 \times 10^{13}\,{\rm cm}\, (L\Delta t)_{38}^{1/4} \dot{M}_{-9}^{1/4} V_{3.3}^{-1/4} \nu_9^{-3/4}$$

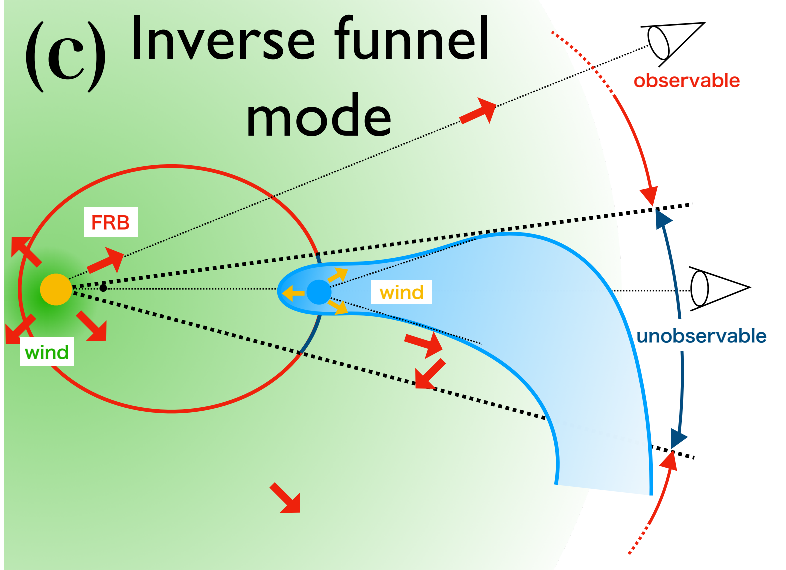
KI & Zhang 20

Free-free is subdominant Lyutikov+ 20

Additional Modes





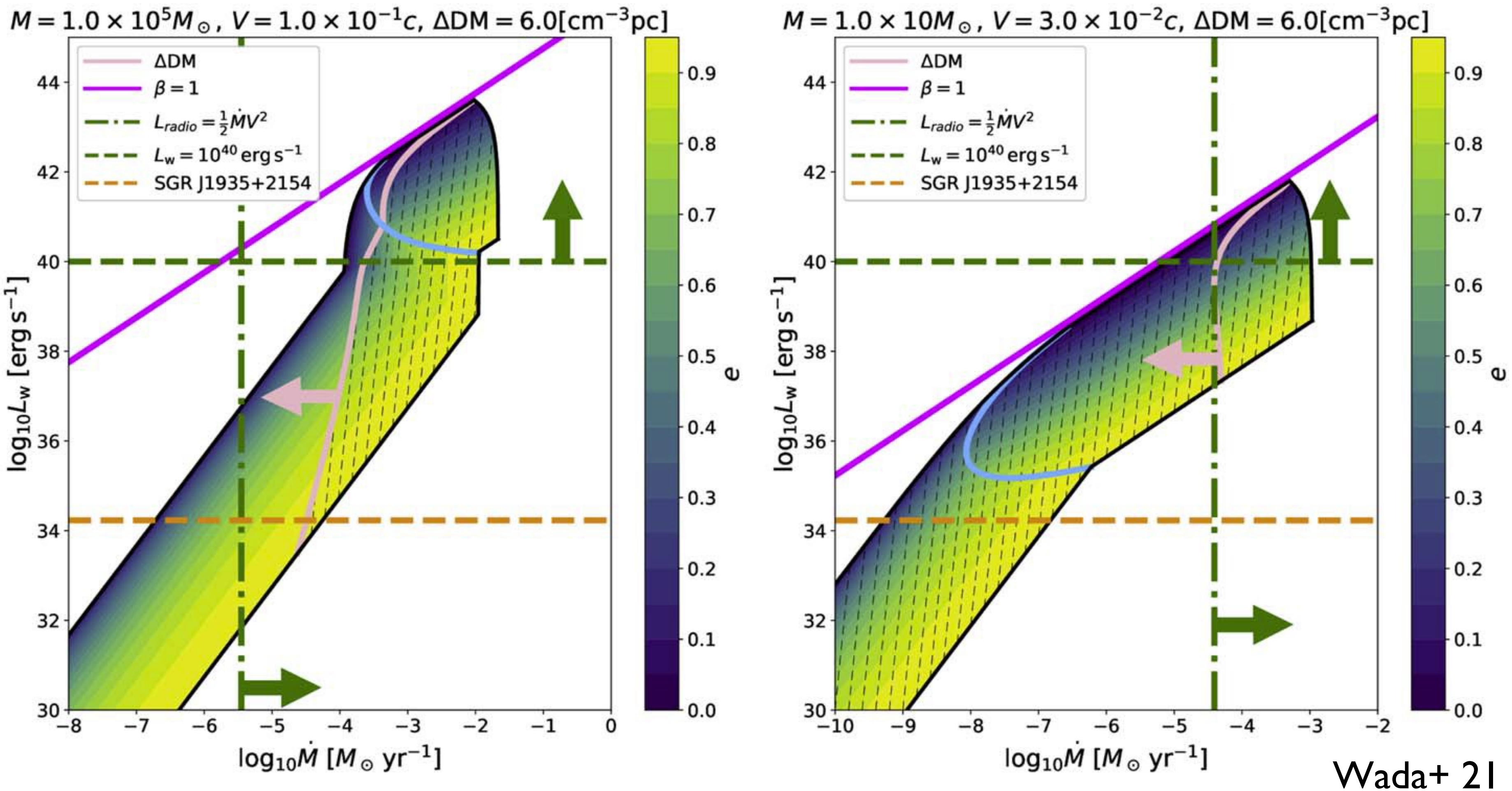


+ Eccentricity

→ A large space of parameters is allowed

Wada+ 21

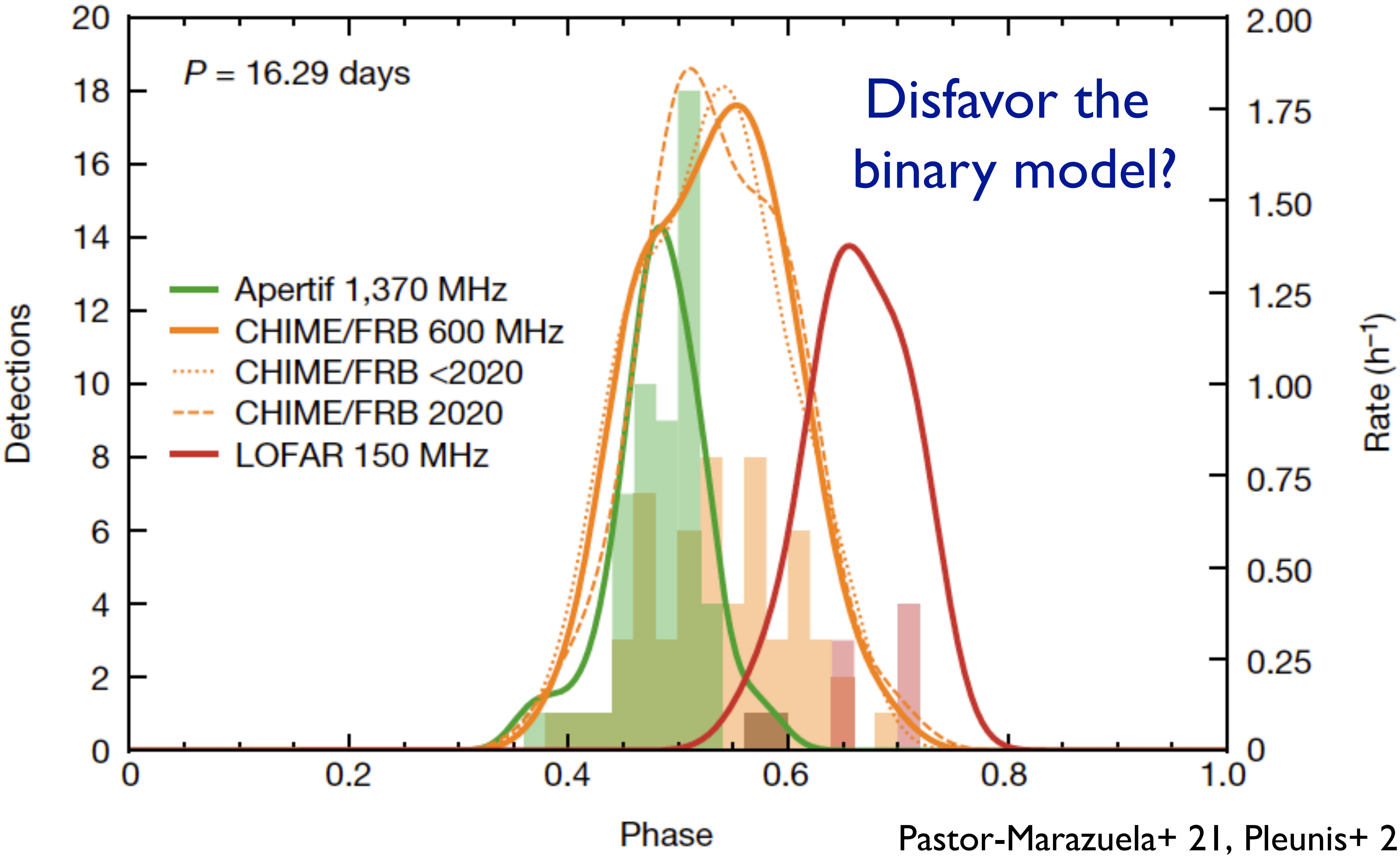
Parameter Space



Wada+ 21

Periodicity + Persistent radio counterpart ~10^{39} erg/s + ΔDM

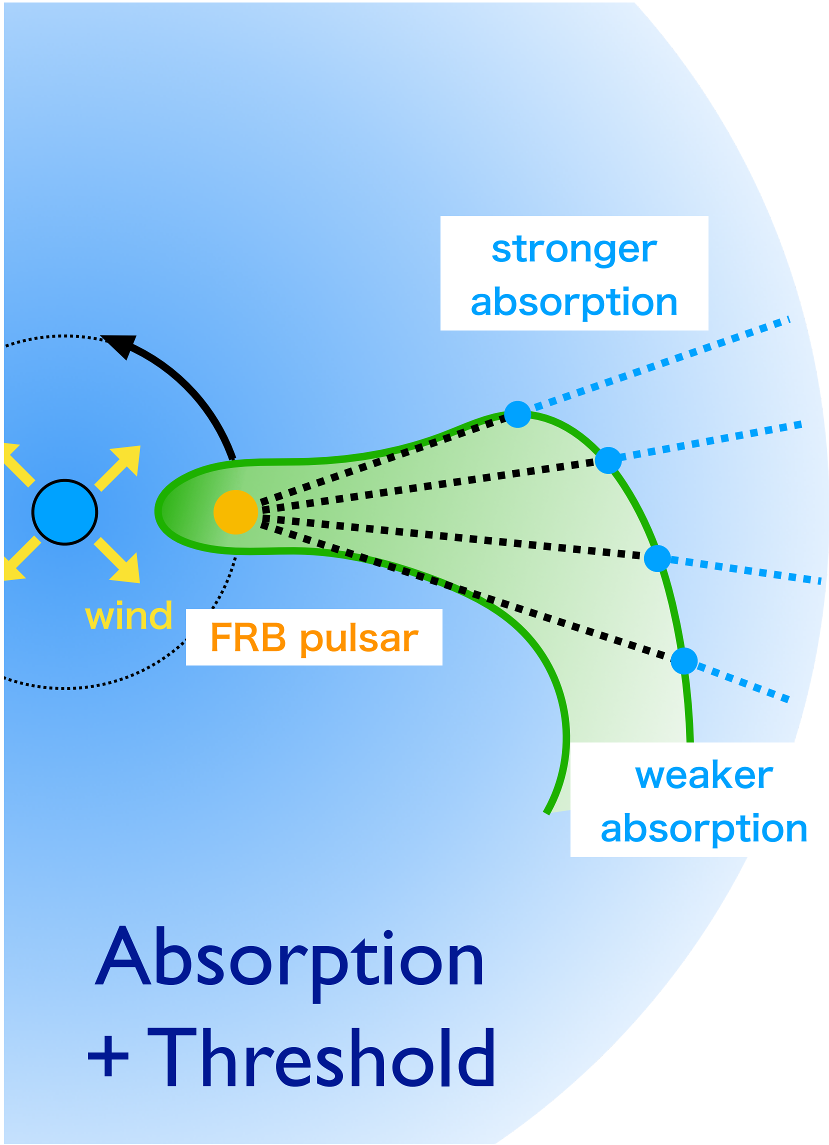
Frequency Dependence



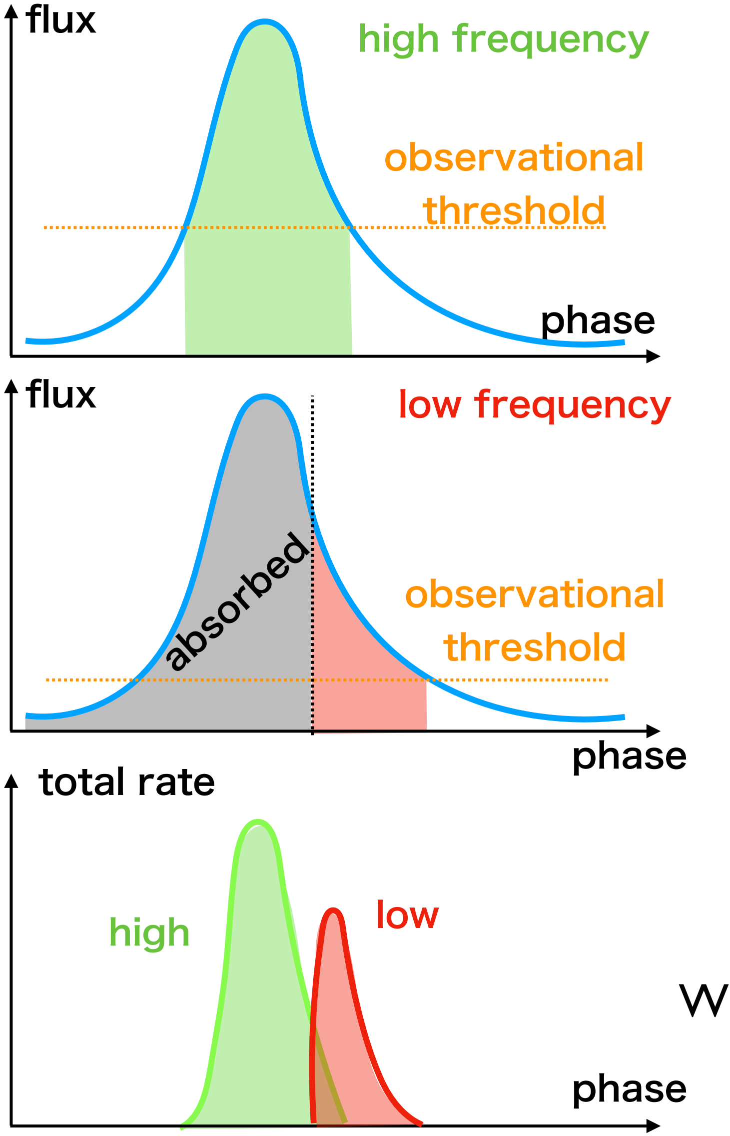
Disfavor the binary model?

Pastor-Marazuela+ 21, Pleunis+ 21

A Possible Solution

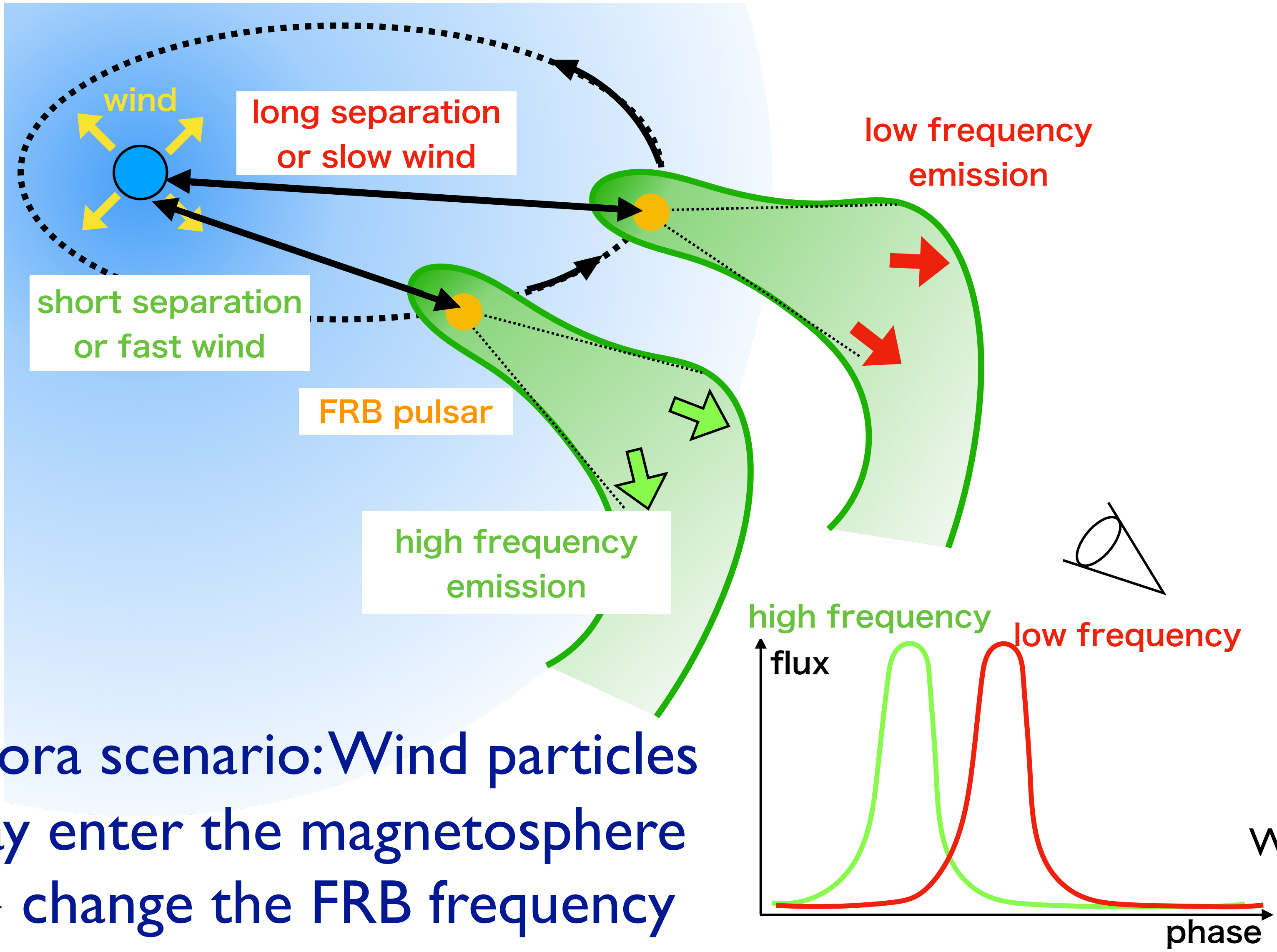


Absorption
+ Threshold



Wada+ 21

Another Solution



Aurora scenario: Wind particles may enter the magnetosphere → change the FRB frequency

Wada+ 21

$e^{\pm}$ Creation

Equilibrium number density of $e^{\pm}$

$$n_{\pm} = \frac{eBm_e}{(2\pi^3)^{1/2}\hbar^2}\left(\frac{T}{m_ec^2}\right)^{1/2}\exp\left(-\frac{m_ec^2}{T}\right),$$

Optical depth

$$\tau_{\perp} = \frac{4\pi^2}{5}\sigma_T\left(\frac{T}{m_ec^2}\frac{B_Q}{B}\right)^2 n_{\pm}\ell_X,$$

$$B_Q = m_e^2c^3/\hbar e = 4.4\times10^{13}\,\mathrm{G}.$$

Timescales

$$\tau_{\|} = (n_{+} + n_{-})\sigma(T, B)\frac{r}{\Gamma}$$

$$\tau_{\perp} = (n_{+} + n_{-})\sigma(T, B)\ell$$

$$t_{\mathrm{dyn}} = \frac{r}{c\Gamma}$$

Dynamical timescale

$$t_{\mathrm{diff}} = \frac{\ell\tau_{\perp}}{c}$$

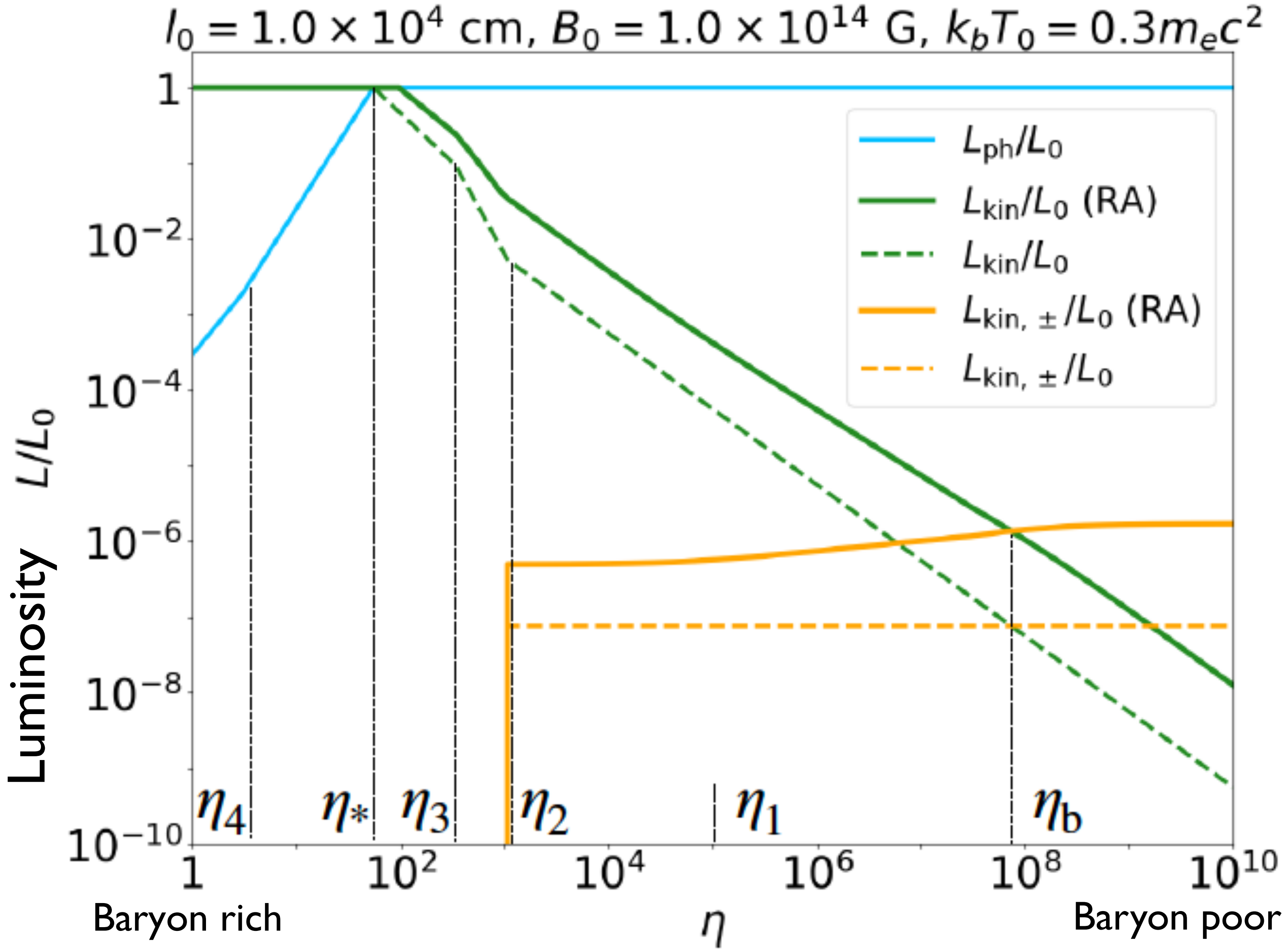
Diffusion timescale

Terminal Lorentz Factor

$$
\begin{aligned}
\Gamma_{\mathrm{RA}}(r_{\parallel}) \simeq & \left(\frac{45}{4\pi^{3}\alpha}\right)^{1/7}\left(\frac{\sigma_{\mathrm{T}} L_{\mathrm{ph,iso}}}{4\pi\bar{m}c^{3}r_{\parallel}}\right)^{1/7} \\
& \times \Gamma(r_{\parallel})^{4/7} B(r_{\parallel})^{2/7} T(r_{\parallel})^{-3/7} \\
& =: f_{\mathrm{RA}}\tilde{r}_{\parallel}^{1/2},
\end{aligned}
$$

$$
\begin{aligned}
f_{\mathrm{RA}} = & \left(\frac{45}{4\pi^{3}\alpha}\right)^{1/7}\left(\frac{\sigma_{\mathrm{T}} L_{\mathrm{ph,iso}}}{4\pi\bar{m}c^{3}r_{0}}\right)^{1/7} B_{0}^{2/7} T_{0}^{-3/7} \\
& \sim 26\, L_{\mathrm{ph,iso},41}^{1/7}\left(\frac{\bar{m}}{m_{\mathrm{e}}}\right)^{-1/7} B_{0,14}^{2/7} T_{0,100\,\mathrm{keV}}^{-3/7},
\end{aligned}
$$

Kinetic Energy



Wada & KI 22

Typical Frequencies

Goldreich-Julian density

$$n_{\rm GJ} \sim \frac{\Omega B_z}{2\pi q c}$$
$$\sim 7 \times 10^{-2}\, B_z P^{-1}\, {\rm cm}^{-3}$$
$$\sim 6 \times 10\, B_{p,14} P^{-4} \left(\frac{r}{r_L}\right)^{-3} {\rm cm}^{-3}$$

Fireball density

$$n'_e = \frac{L_m}{4\pi r^2 m c^3 \Gamma^2}$$
$$\sim 1 \times 10^{17} \frac{L_{m,42}}{P^2 \Gamma^2} \left(\frac{r}{r_L}\right)^{-2} {\rm cm}^{-3}$$

$$r_L = \frac{cP}{2\pi} \sim 5 \times 10^9\, P\, {\rm cm}$$

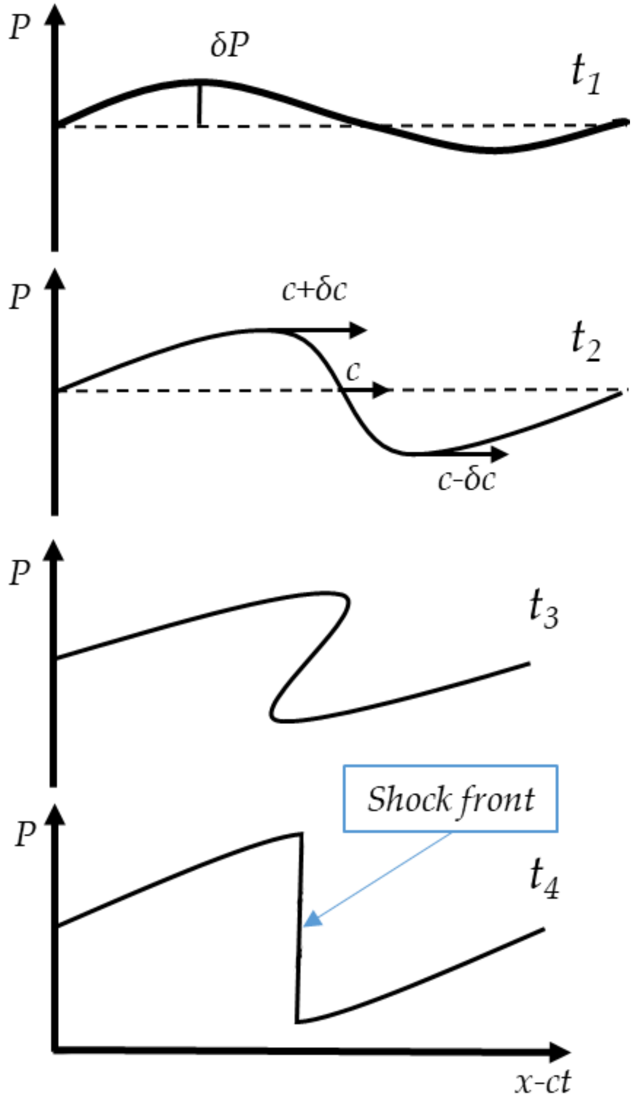
Plasma frequency

$$\nu_{\rm p} = \left(\frac{q^2 n_{\rm e}}{\pi m}\right)^{1/2}$$
$$\sim 9 \times 10^3\, n_{\rm e}^{1/2}\, {\rm Hz}$$
$$\sim 9 \times 10^8\, n_{{\rm e},10} e^{1/2}\, {\rm Hz}$$

Cyclotron frequency

$$\nu_c = \frac{qB}{2\pi m c}$$
$$\sim 3\, \frac{B_{p,14}}{P} \left(\frac{r}{r_L}\right)^{-3} {\rm GHz}$$

Acoustic Nonlinearity



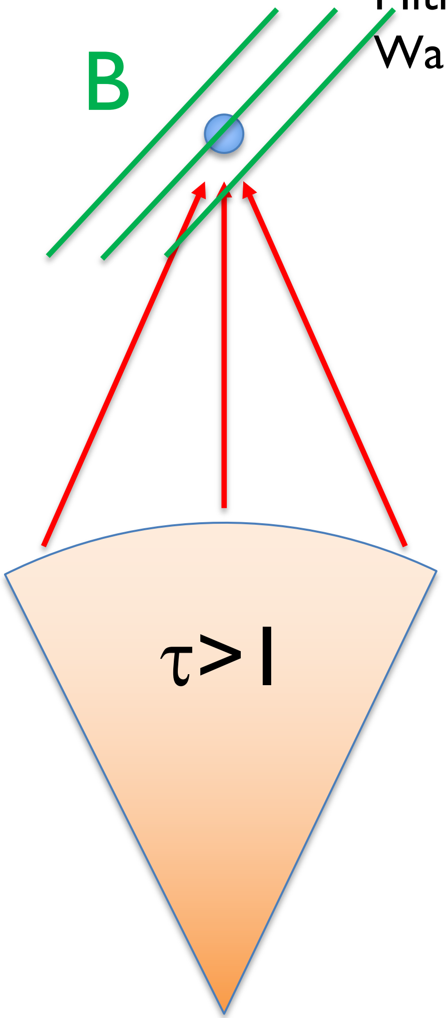
Yevheniia Chernukha 19

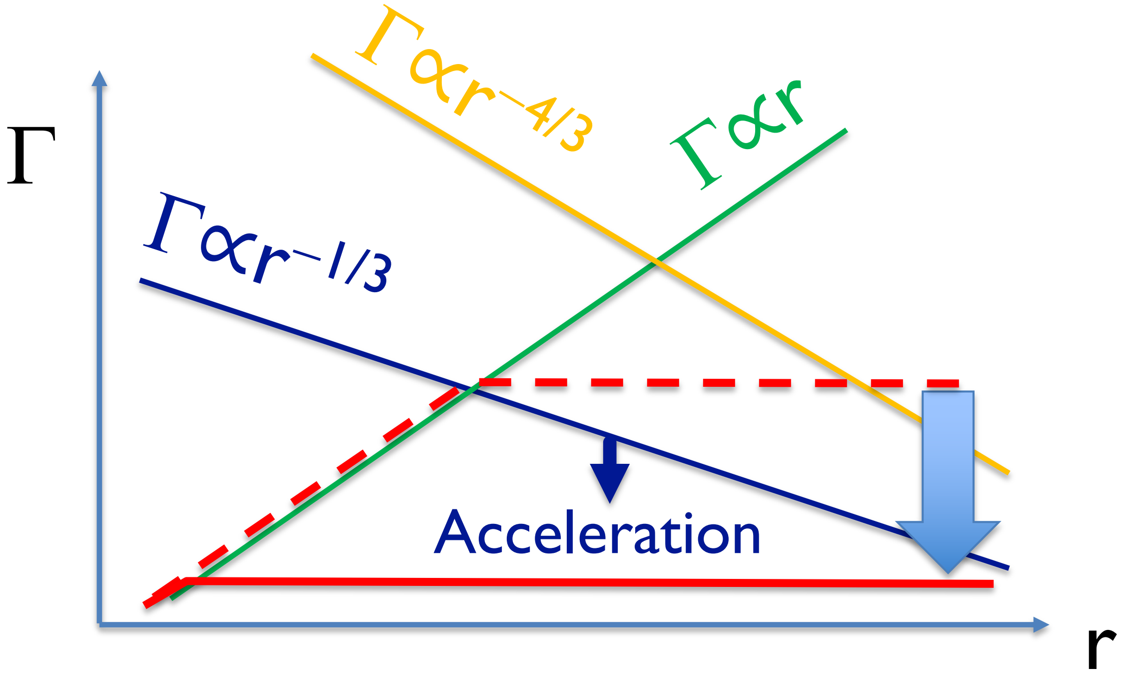
Radiative Deceleration

Mitrofanov & Pavlov 82
Wada & KI 22

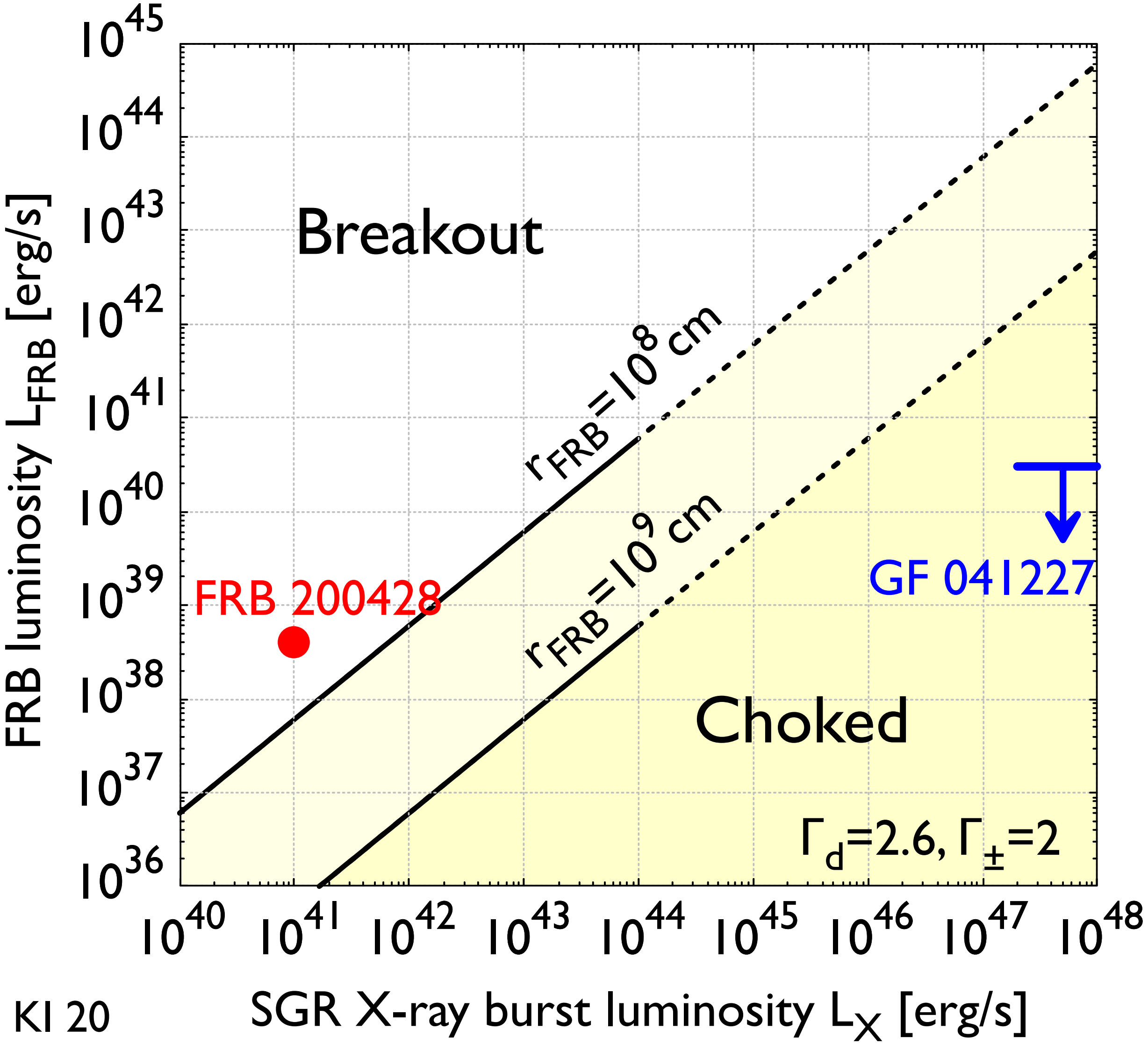
Work by γ > Rest mass

$$\beta = \cos\Theta_{kB}$$





Breakout



KI 20

$e^\pm$ outflow is optically thick

FRB could break out the $e^\pm$ outflow

No X-ray burst with weak FRBs

No FRB with bright X-ray bursts

Compton Drag is Strong

$e^{\pm}$ rest mass energy < Compton cooling energy

$$m_e c^2 < c \sigma_T u'_X t'_{dyn}$$

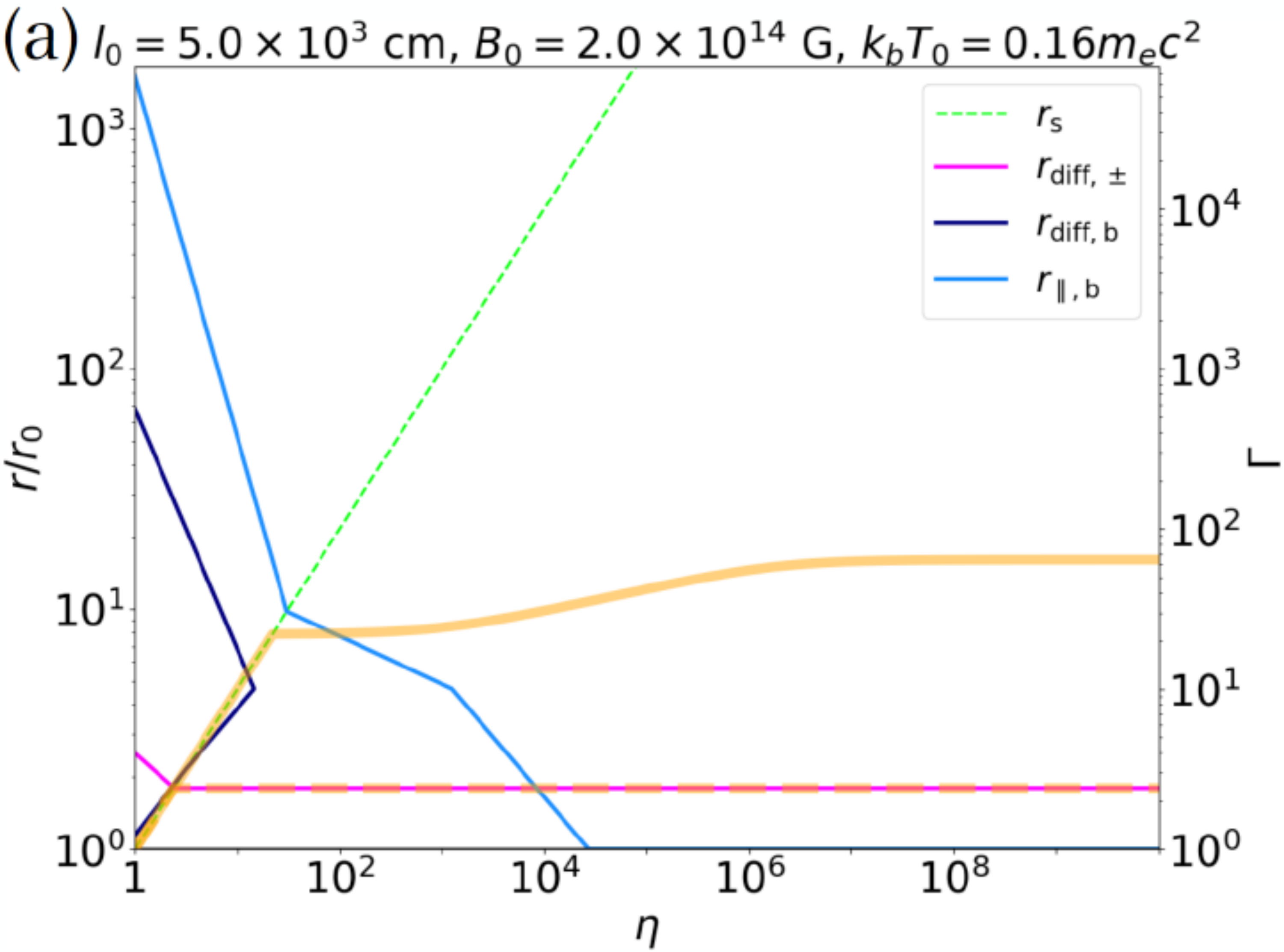
$$t'_{dyn} = r / c\Gamma_{\pm}$$

Breakout condition

$$u'_{\rm FRB} > n'_{\pm} c t'_{dyn} \sigma_T u'_X = \tau_T u'_X$$

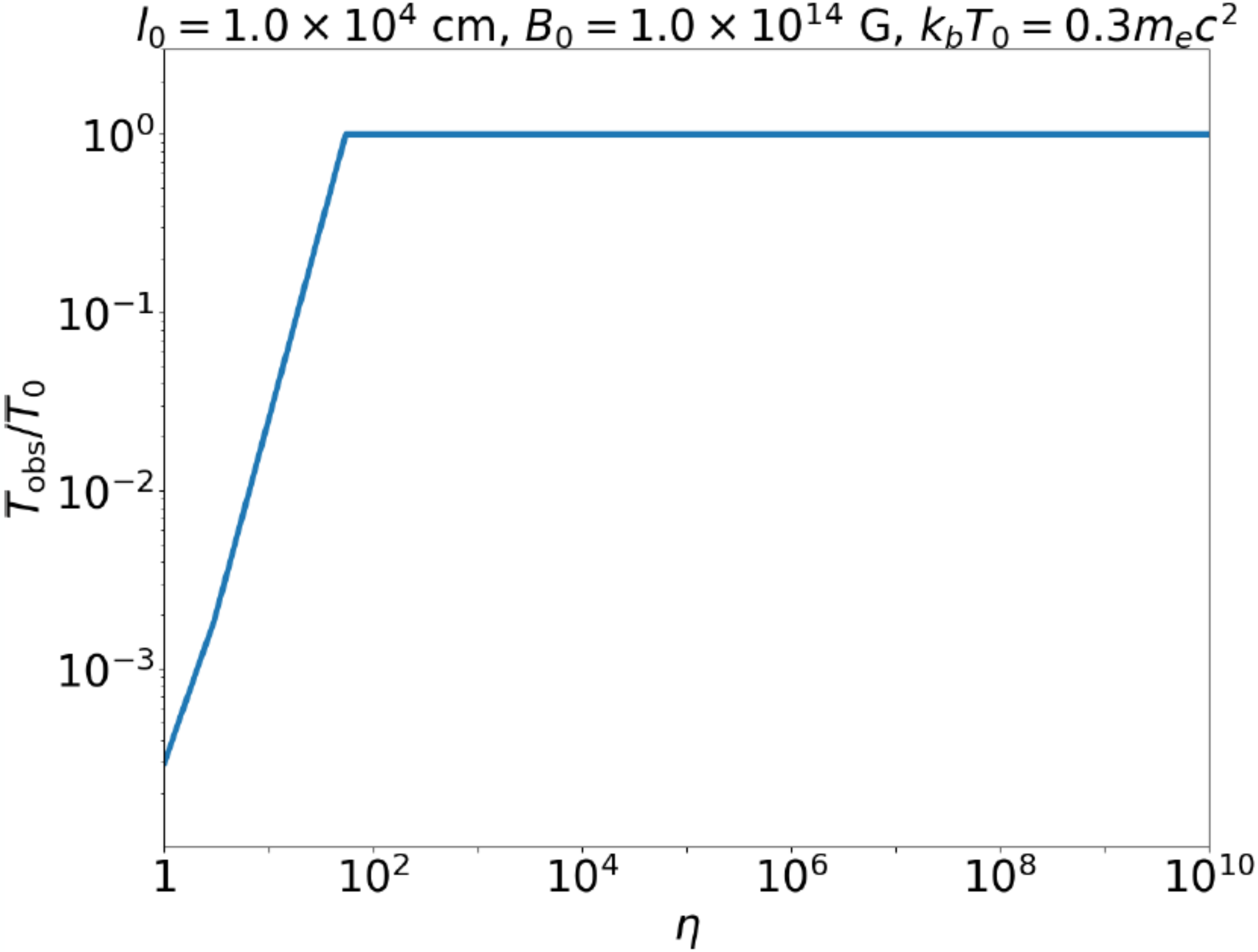
$$1 < \frac{L_{\rm FRB}}{\tau_T L_X} \sim 2 \times 10^{-2} L_{{\rm FRB},38.6} L_{X,41}^{-1} \Gamma_{d,0.4}^{-10/3} \Gamma_{\pm}^{2} r_7^{2},$$

For FRB 20200428A



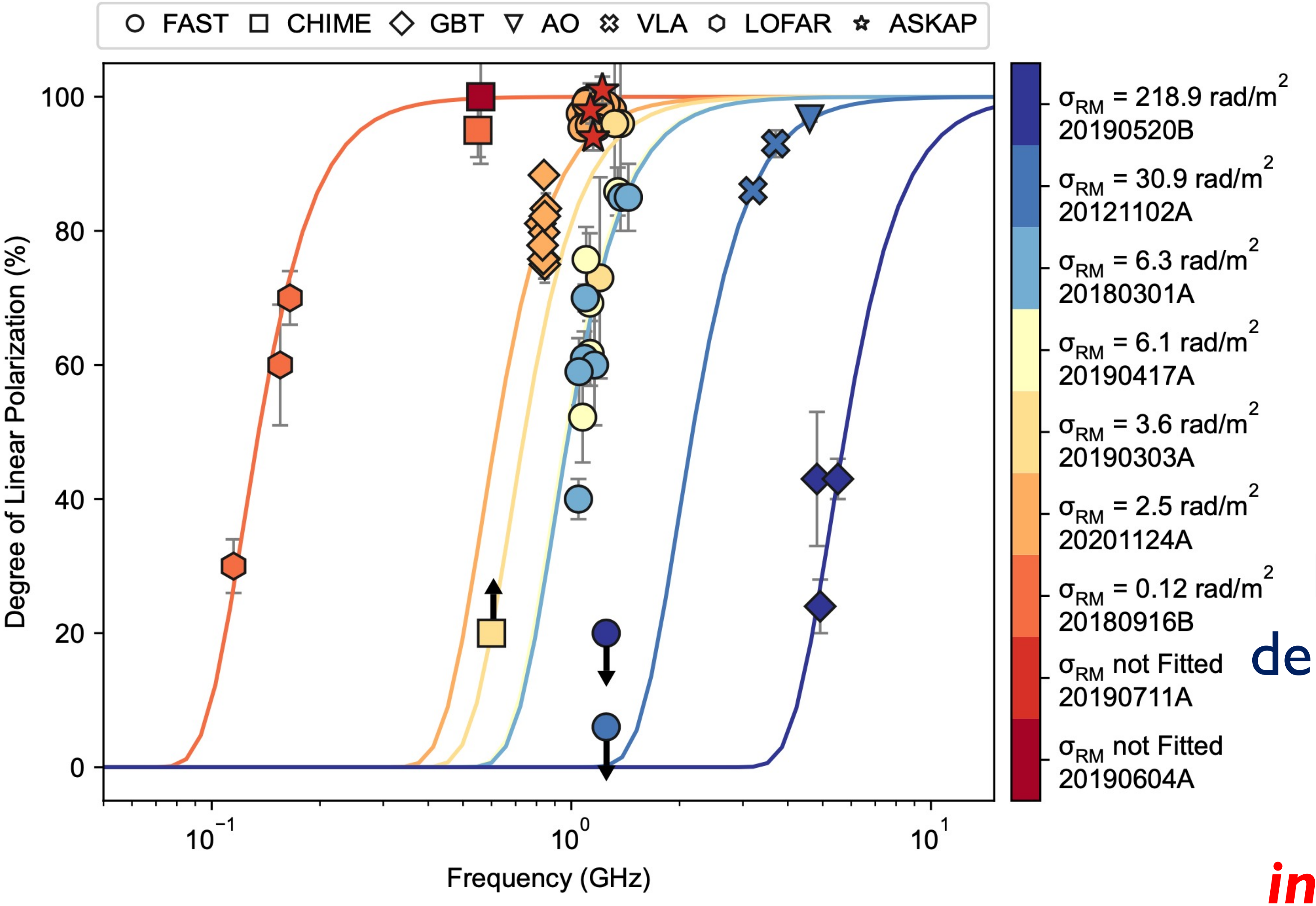
Wada
& KI 22

Observed Temperature



Wada & KI 22

Linear Polarization

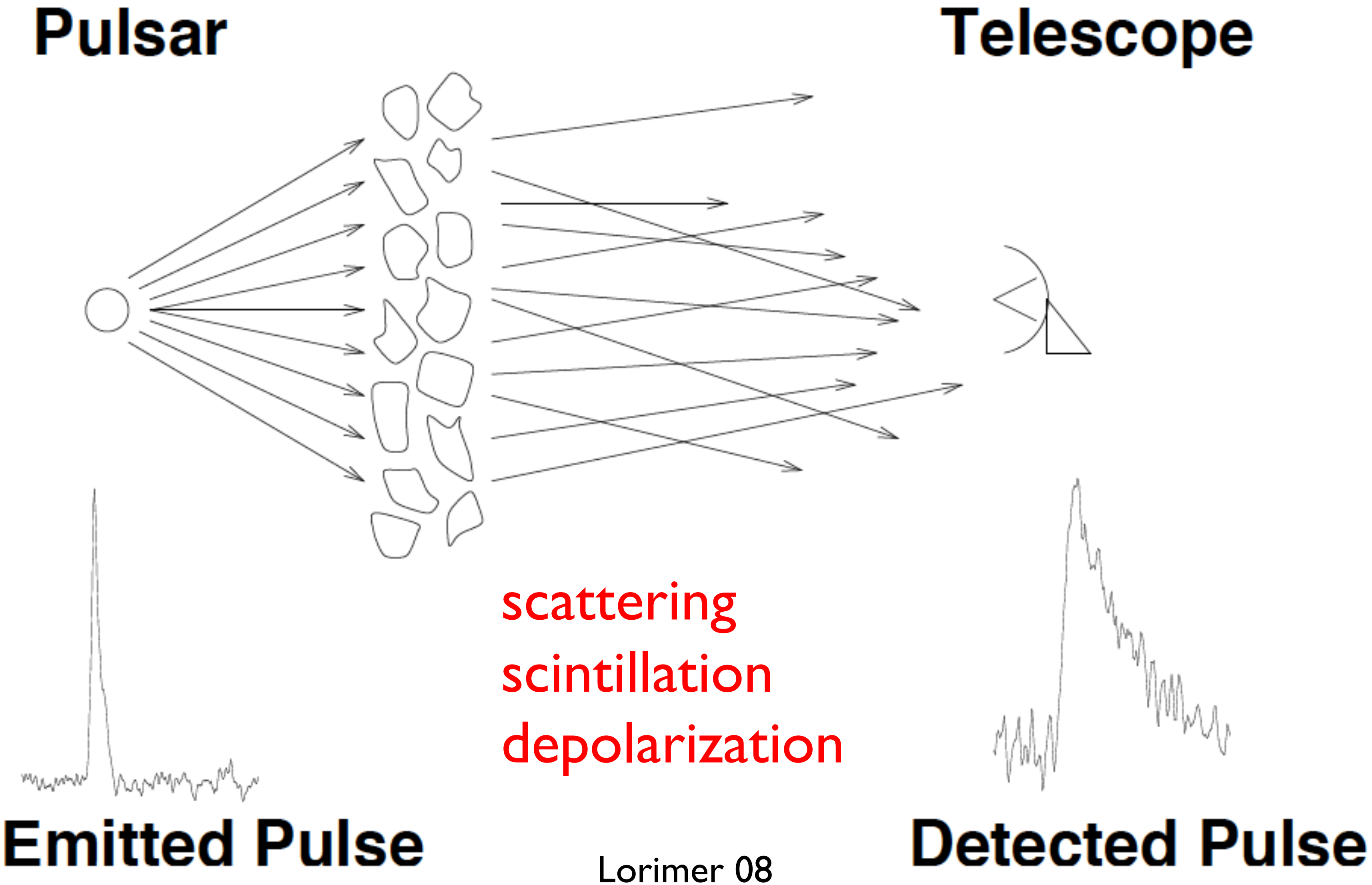


Multipath depolarization

~100% intrinsic LC!

Feng+ 22

Plasma Screen



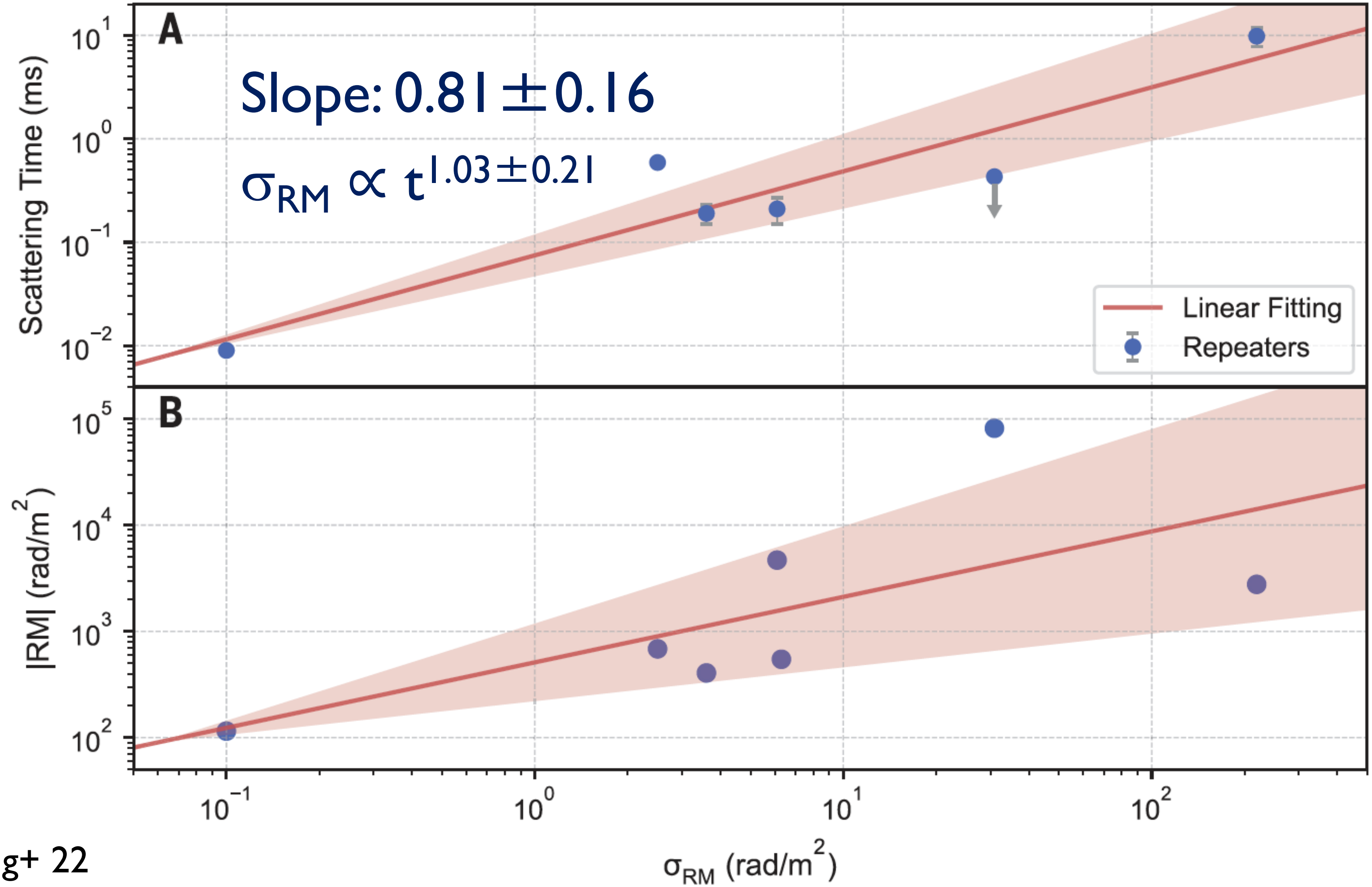
scattering

scintillation

depolarization

Lorimer 08

Correlations

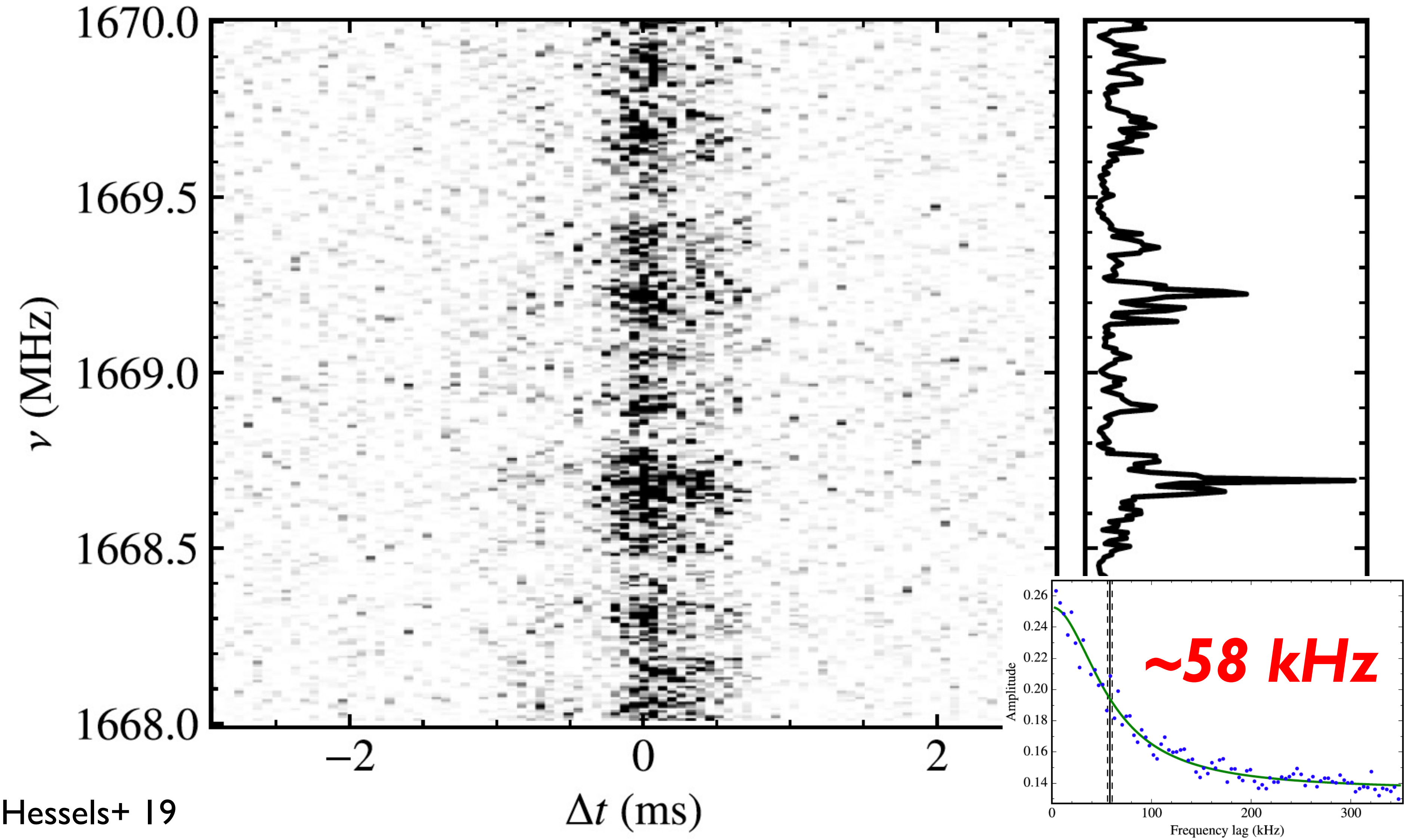


Slope: 0.81 ± 0.16

$\sigma_{RM} \propto t^{1.03 \pm 0.21}$

Feng+ 22

Spectral Fine Structure



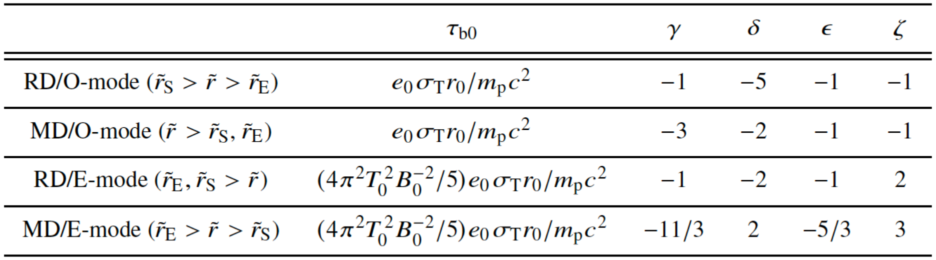
Hessels+ 19

Case Classification

Baryonic electrons

$$\tau_{||} = \tau_{\mathrm{b}0}\eta^{\gamma}\tilde{r}^{\delta},$$

$$\frac{t_{\mathrm{diff}}}{t_{\mathrm{dyn}}} = \tau_{\mathrm{b}0}\theta_0^2\eta^{\epsilon}\tilde{r}^{\zeta},$$

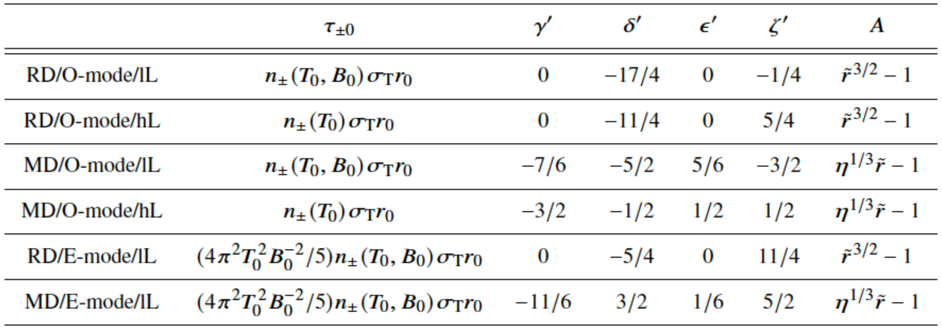


	$\tau_{\mathrm{b}0}$	γ	δ	ϵ	ζ
RD/O-mode ($\tilde{r}_{\mathrm{S}} > \tilde{r} > \tilde{r}_{\mathrm{E}}$)	$e_0\sigma_{\mathrm{T}}r_0/m_{\mathrm{p}}c^2$	-1	-5	-1	-1
MD/O-mode ($\tilde{r} > \tilde{r}_{\mathrm{S}}, \tilde{r}_{\mathrm{E}}$)	$e_0\sigma_{\mathrm{T}}r_0/m_{\mathrm{p}}c^2$	-3	-2	-1	-1
RD/E-mode ($\tilde{r}_{\mathrm{E}}, \tilde{r}_{\mathrm{S}} > \tilde{r}$)	$(4\pi^2T_0^2B_0^{-2}/5)e_0\sigma_{\mathrm{T}}r_0/m_{\mathrm{p}}c^2$	-1	-2	-1	2
MD/E-mode ($\tilde{r}_{\mathrm{E}} > \tilde{r} > \tilde{r}_{\mathrm{S}}$)	$(4\pi^2T_0^2B_0^{-2}/5)e_0\sigma_{\mathrm{T}}r_0/m_{\mathrm{p}}c^2$	$-11/3$	2	$-5/3$	3

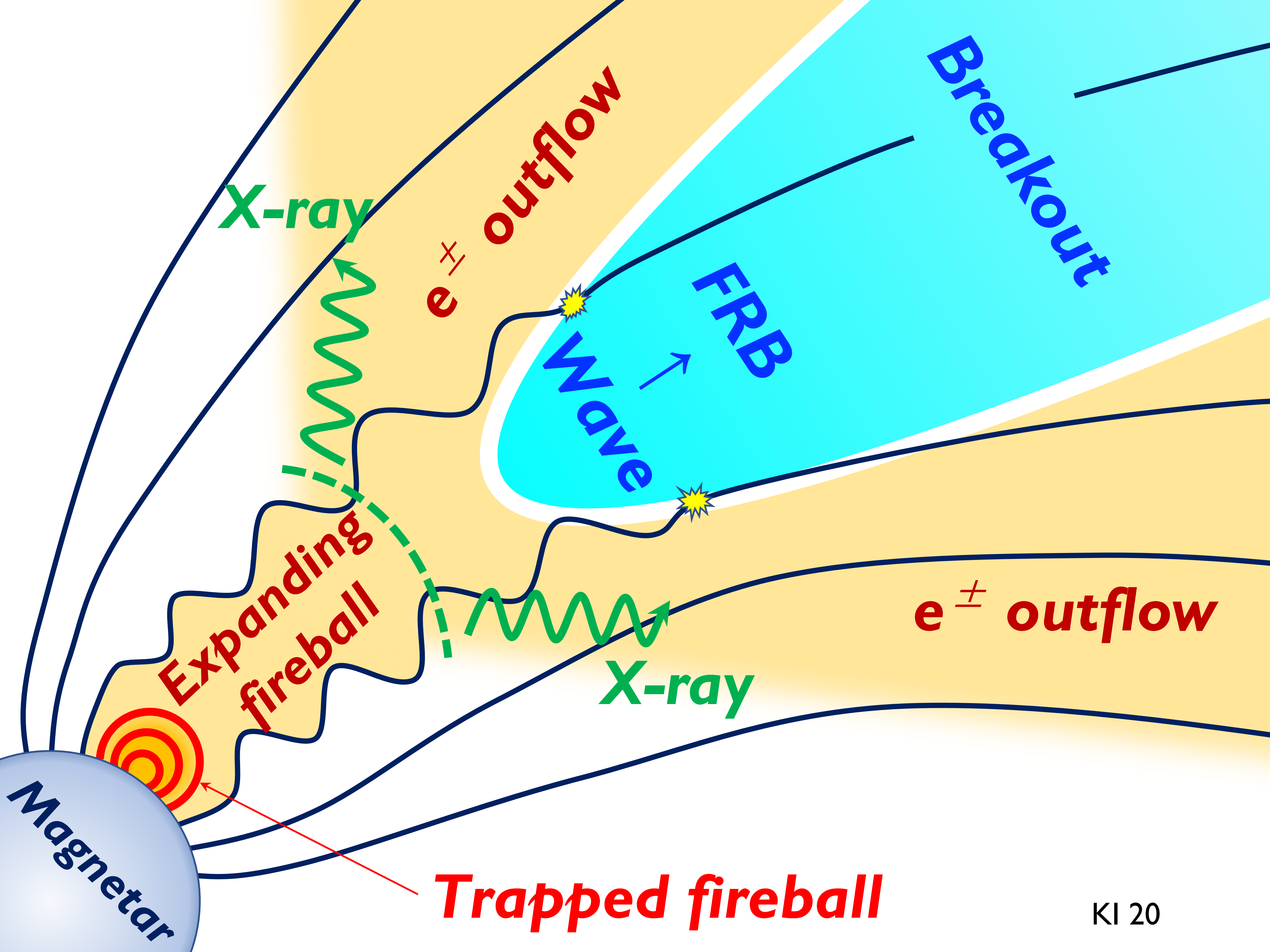
$e^{\pm}$ pairs

$$\tau_{||} = \tau_{\pm 0}\eta^{\gamma'}\tilde{r}^{\delta'}\exp\left(-T_0^{-1}A\right),$$

$$\frac{t_{\mathrm{diff}}}{t_{\mathrm{dyn}}} = \tau_{\pm 0}\theta_0^2\eta^{\epsilon'}\tilde{r}^{\zeta'}\exp\left(-T_0^{-1}A\right),$$

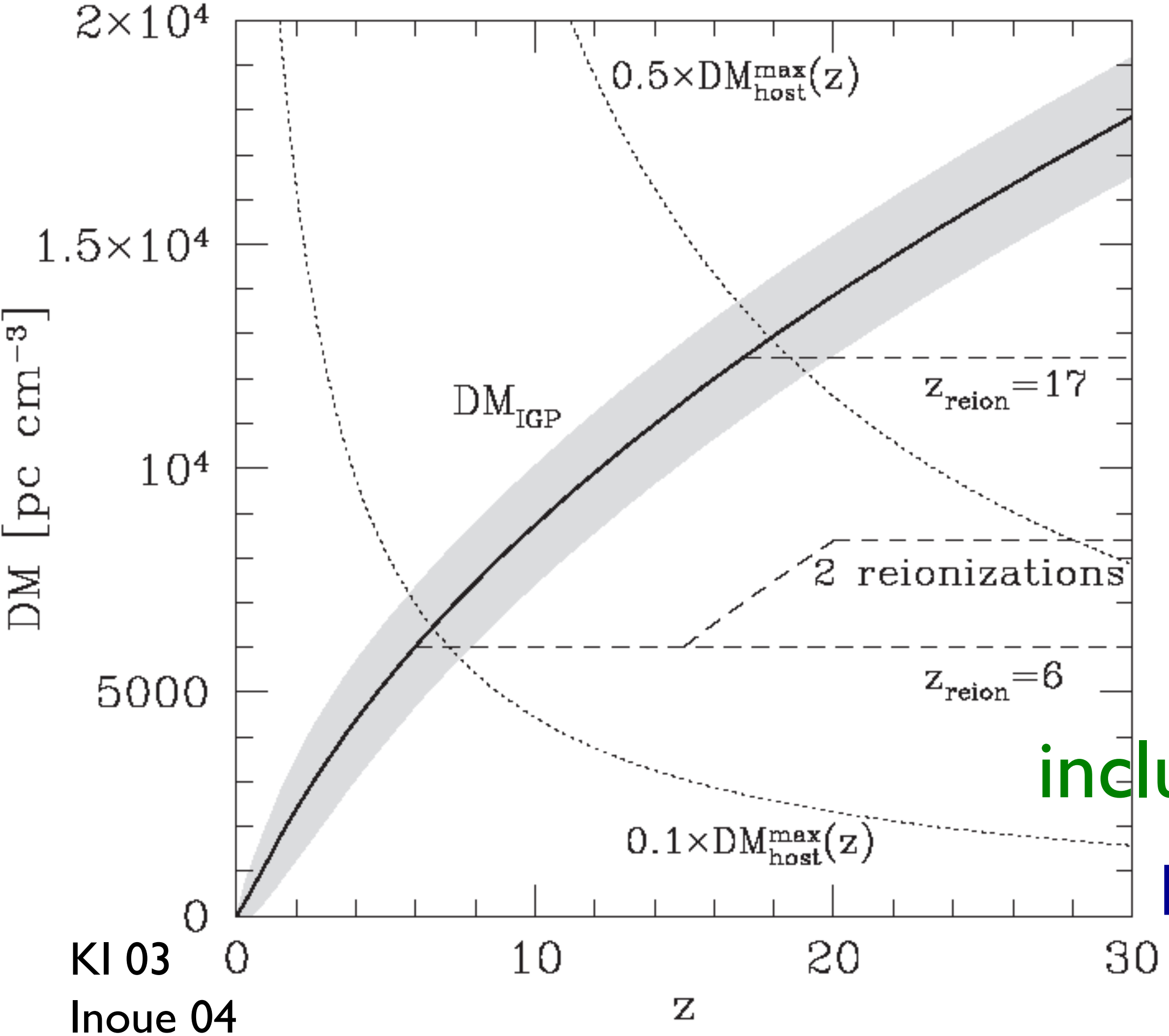


	$\tau_{\pm 0}$	γ'	δ'	ϵ'	ζ'	A
RD/O-mode/lL	$n_{\pm}(T_0, B_0)\sigma_{\mathrm{T}}r_0$	0	$-17/4$	0	$-1/4$	$\tilde{r}^{3/2}-1$
RD/O-mode/hL	$n_{\pm}(T_0)\sigma_{\mathrm{T}}r_0$	0	$-11/4$	0	$5/4$	$\tilde{r}^{3/2}-1$
MD/O-mode/lL	$n_{\pm}(T_0, B_0)\sigma_{\mathrm{T}}r_0$	$-7/6$	$-5/2$	$5/6$	$-3/2$	$\eta^{1/3}\tilde{r}-1$
MD/O-mode/hL	$n_{\pm}(T_0)\sigma_{\mathrm{T}}r_0$	$-3/2$	$-1/2$	$1/2$	$1/2$	$\eta^{1/3}\tilde{r}-1$
RD/E-mode/lL	$(4\pi^2T_0^2B_0^{-2}/5)n_{\pm}(T_0, B_0)\sigma_{\mathrm{T}}r_0$	0	$-5/4$	0	$11/4$	$\tilde{r}^{3/2}-1$
MD/E-mode/lL	$(4\pi^2T_0^2B_0^{-2}/5)n_{\pm}(T_0, B_0)\sigma_{\mathrm{T}}r_0$	$-11/6$	$3/2$	$1/6$	$5/2$	$\eta^{1/3}\tilde{r}-1$


X-ray
$e^{\pm}$ outflow
Breakout
FRB
Wave
Expanding fireball
X-ray
$e^{\pm}$ outflow
Magnetar
Trapped fireball

KI 20

Cosmic DM



KI 03
Inoue 04

$$\Delta t=\int_0^z dz\,\frac{dt}{dz}\,\frac{1}{2}\,\frac{\left(1+z\right)\nu_p^2}{\left[\left(1+z\right)\nu\right]^2}$$

$$=\frac{e^2}{2\pi m_e c}\,\frac{1}{\nu^2}\times$$

$$\underline{\frac{cn_0}{H_0}\int_0^z\frac{\left(1+z\right)dz}{\left[\Omega_m\left(1+z\right)^3+\Omega_\Lambda\right]^{1/2}}}$$

$\mathrm{DM}_{\mathrm{cosmic}}$

including missing baryon

$\mathrm{DM}_{\mathrm{Galaxy}}$~30-$10^3$ pc cm^{-3}