Direct Emission of Strong Radio Pulses during Magnetar Flares

arXiv:2209.11136

C. Thompson   CITA

Relativistic outflow dominated by large-scale (B_ϕ, E_θ)

$$\omega\,\Delta t \sim 4\times10^6(\nu/\mathrm{GHz})(\Delta t/\mathrm{msec})$$

$\Longrightarrow$ Small-scale structure in expanding EM field:

Shock   Frozen Turbulence   High-order tearing

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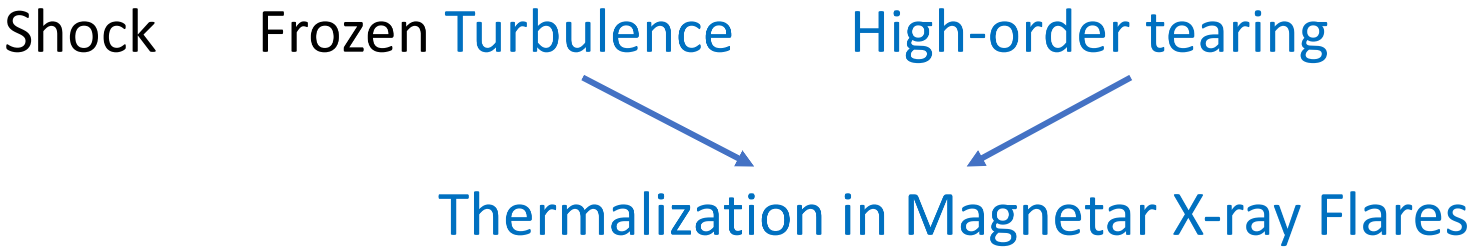

Shock   Frozen Turbulence   High-order tearing

Thermalization in Magnetar X-ray Flares

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Shock + Frozen Turbulence / Current Sheets

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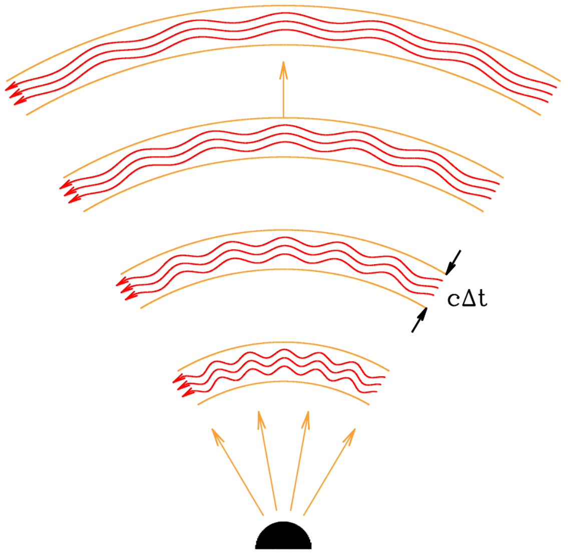
$$\omega\,\Delta t \sim 4{\times}10^6(\nu/\mathrm{GHz})(\Delta t/\mathrm{msec})$$

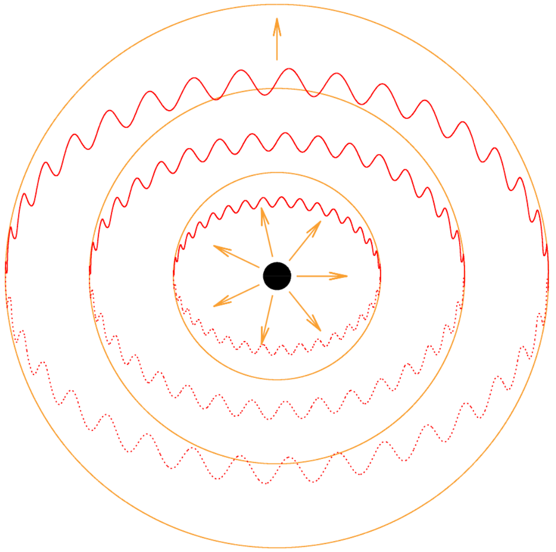
$\Longrightarrow$ Small-scale structure in expanding EM field:

Indirect emission: particle bunching + maser emission

Direct emission: magnetic islands → X-mode

frozen turbulence + shock → O-mode (X-mode)





$\omega \to 0$ comoving $\quad \delta \boldsymbol{J} \sim \dfrac{c}{4\pi}\Big(k_r \delta B_\theta - k_\theta \delta B_r\Big)\hat{\phi}$

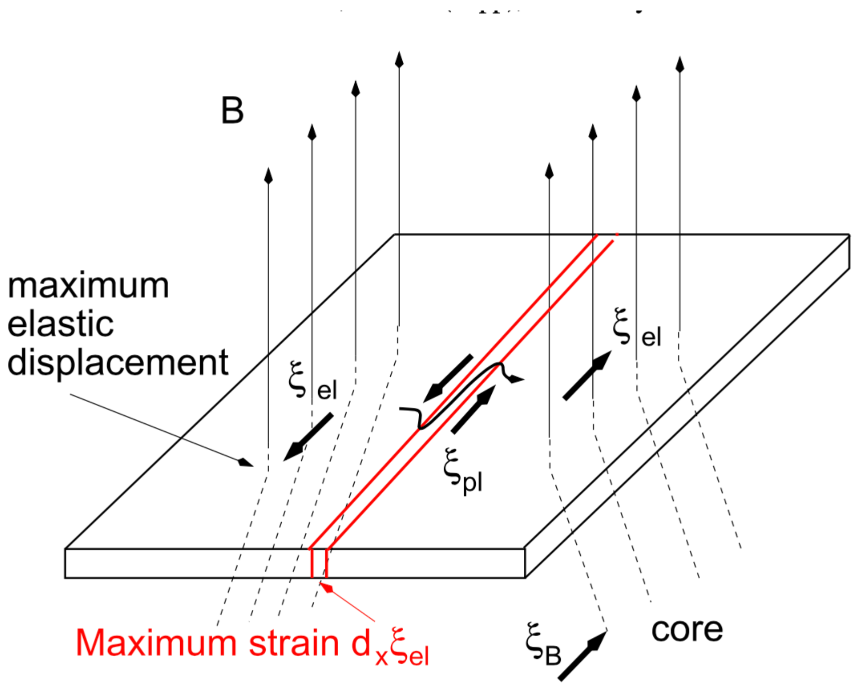
Favored Polarization: $\omega_{\rm obs} = c k_r \sim {\rm const}; \quad \delta B_\theta \sim 1/r$

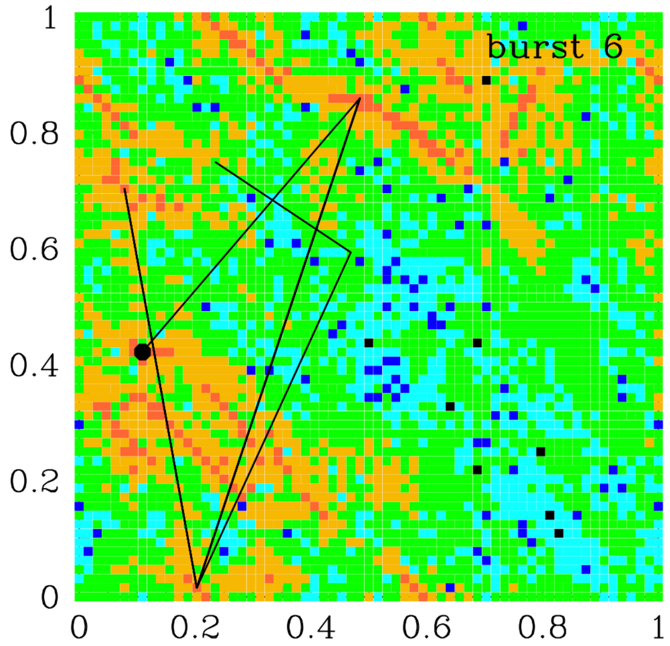
$k_\phi(r) = \left(\dfrac{r_0}{r}\right) k_{\phi 0},$ $\quad$ Freezing: $\quad k_\phi < k_\phi^F(r) = \dfrac{\Gamma}{r}$

$\delta B^2 \propto |k_\perp|^{1-\alpha}$

$$\begin{aligned} \Gamma_{\rm max} &> (\omega \Delta t)^{\alpha - 1} \\ &\sim (\omega \Delta t)^{1/2} = 2.5 \times 10^3 \, \nu_9 (\Delta t_{-3})^{1/2}. \quad (\alpha = 3/2) \end{aligned}$$

T, Yang, Ortz 2017



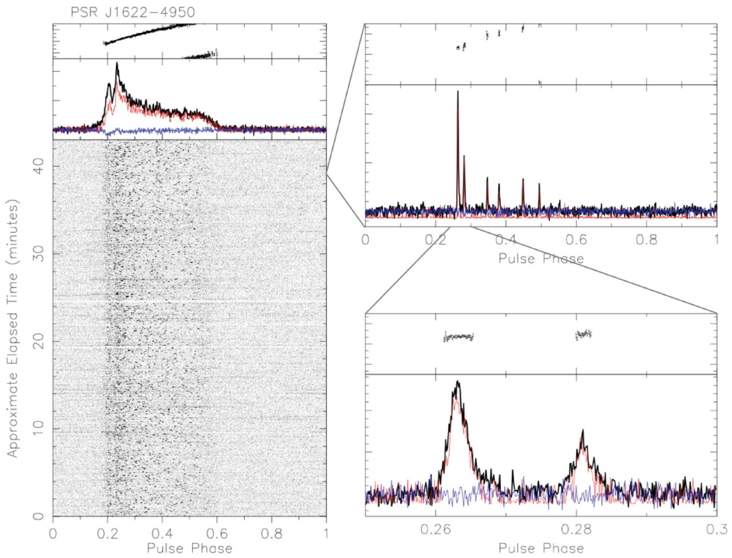


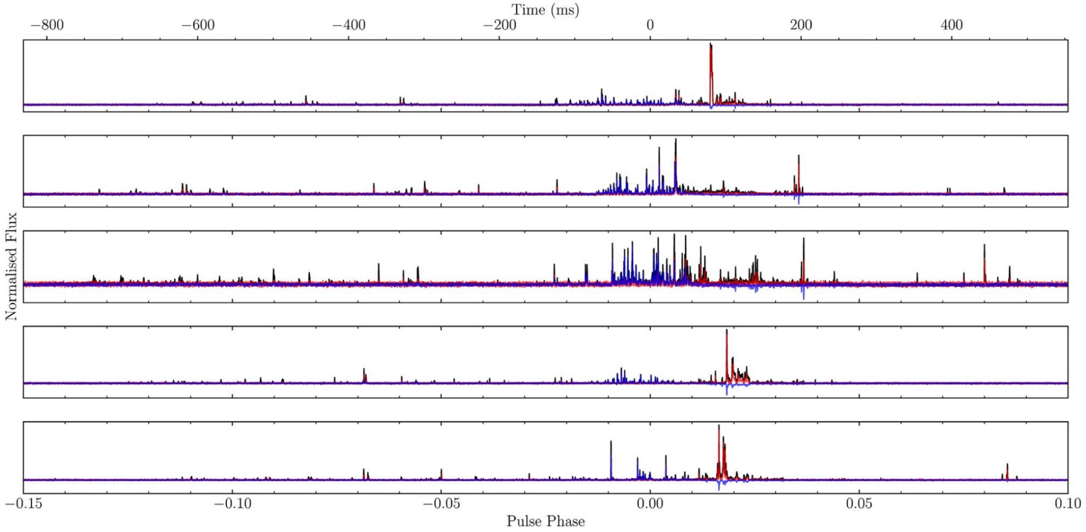
$$\Omega_{\rm eff} = \frac{2\pi\rho c}{B} = \frac{1}{2}\partial_y v_{\rm pl} \sim \frac{v_{\rm pl}}{\Delta}.$$

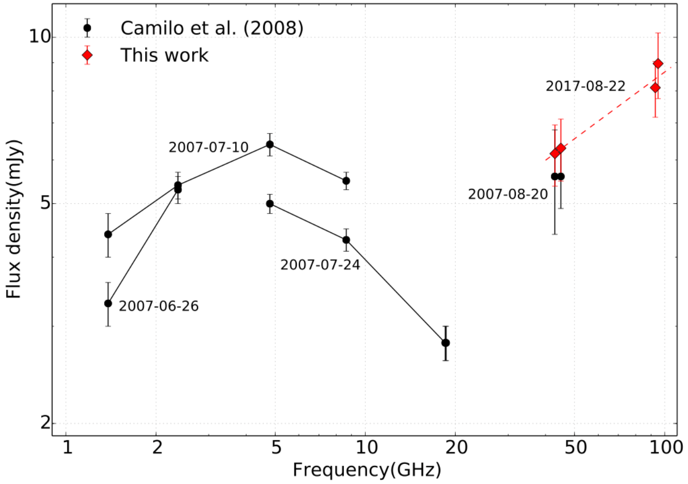
$$\begin{aligned} L_{\rm P} &\sim \left(\frac{1}{8}-\frac{1}{4}\right)\left(\frac{\Omega_{\rm eff}R}{c}\right)^4 B^2R^2c \\ &= (0.9-1.8)\times 10^{43}\, R_6^6 \left(\frac{B}{10\,B_{\rm Q}}\right)^2 \frac{\varepsilon^4_{{\rm pl},-1}}{\Delta^4_{4.5}} \quad {\rm erg\ s}^{-1} \end{aligned}$$

Radio-Emitting Magnetars – Implications

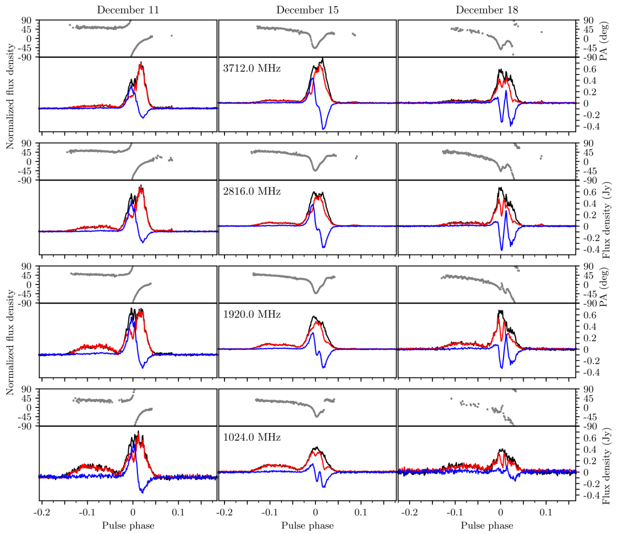
PSR J1622-4950   Levin et al. 2012







XTE J1810-197
Dai et al. 2019



1E 1547.0-5408
Chu et al. 2021

B0031-07

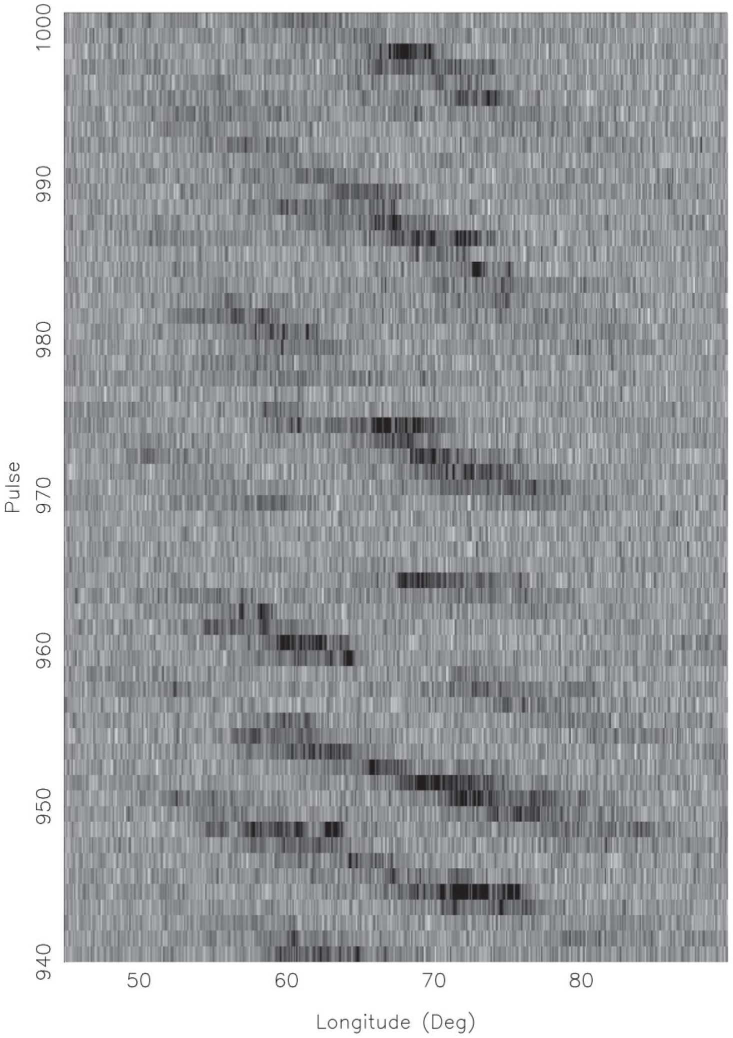
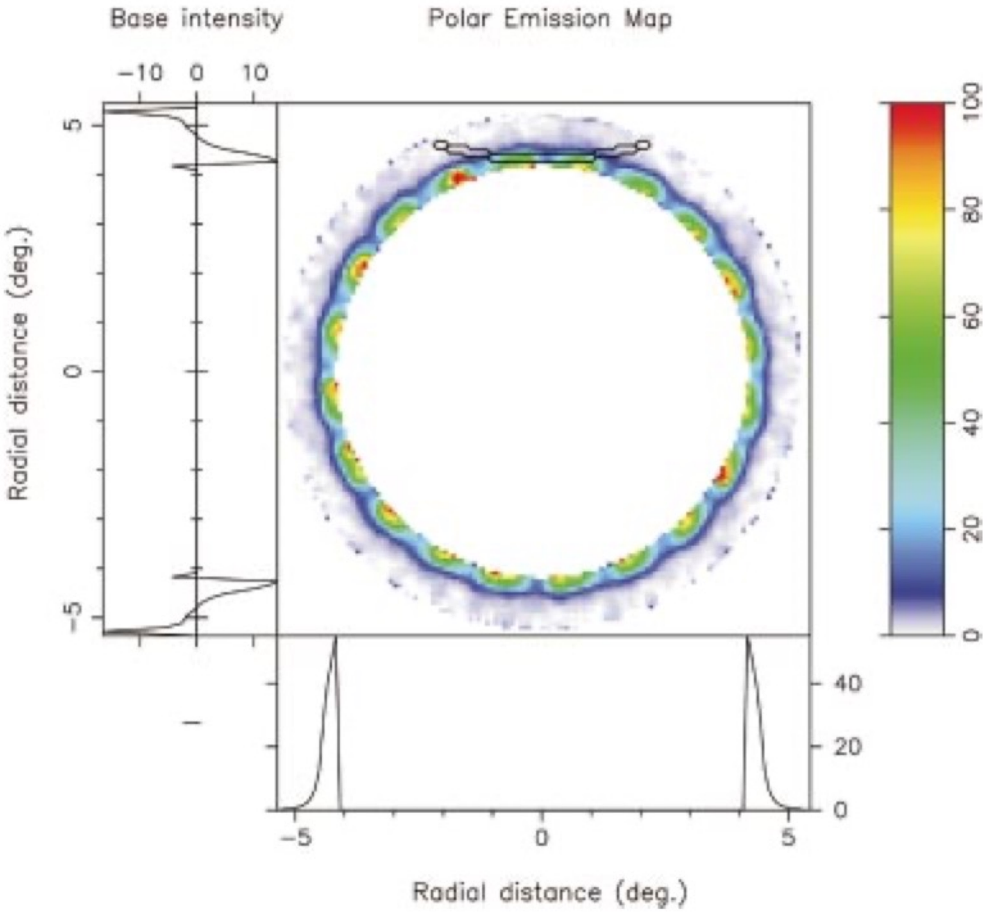


Fig. 20.1 Sequences of drifting sub-pulses are clearly visible at 328 MHz in stacked pulse observations of PSR B0031-07 (pulse period 0.943 s) taken with the Westerbork Synthesis Radio Telescope and the PuMa backend (courtesy of Dr. Roy Smits). The horizontal axis runs from 60 to 120° in 'longitude'

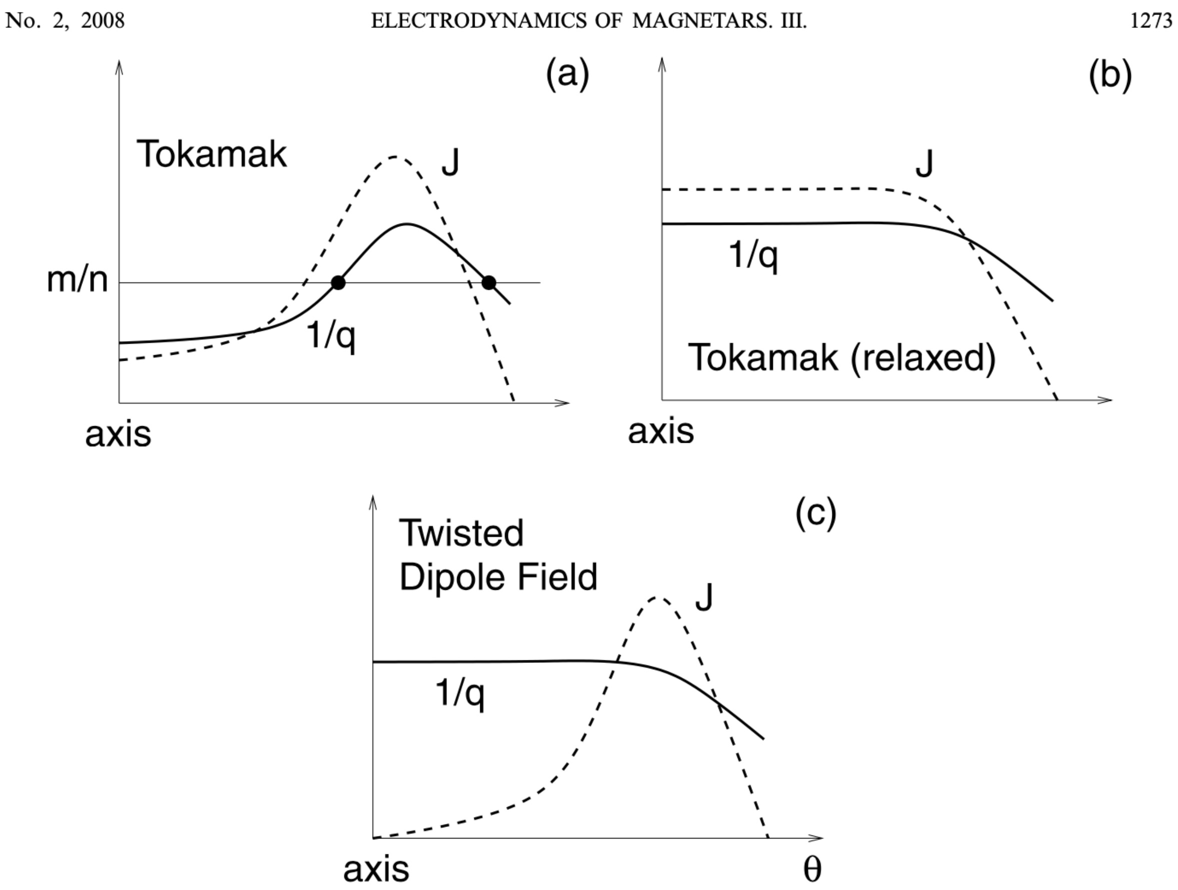
Kuijpers 2009

0943+10

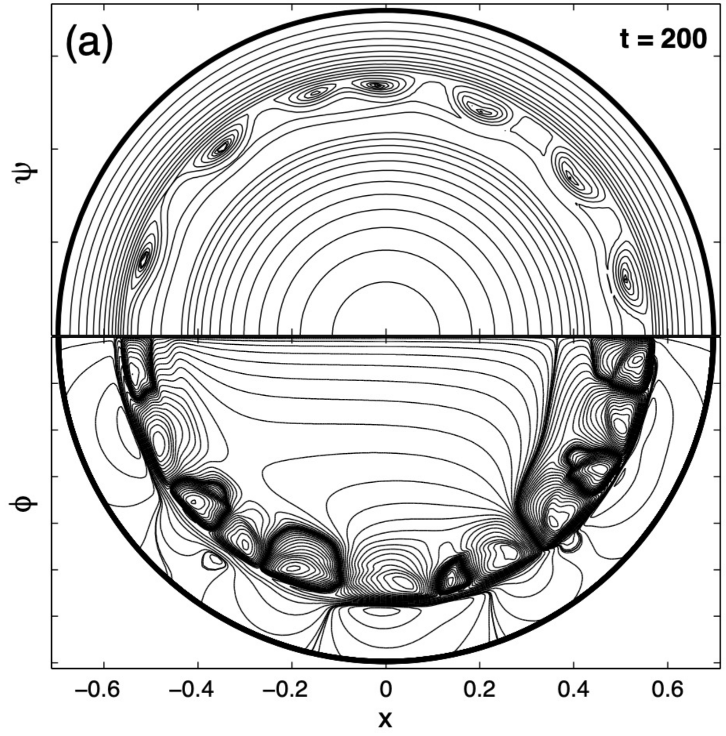


Rankin & Deshpande 2002

Internal High-Order Reconnection (Tokamak)



No. 2, 2008 ELECTRODYNAMICS OF MAGNETARS. III. 1273



Bierwage et al. 2005

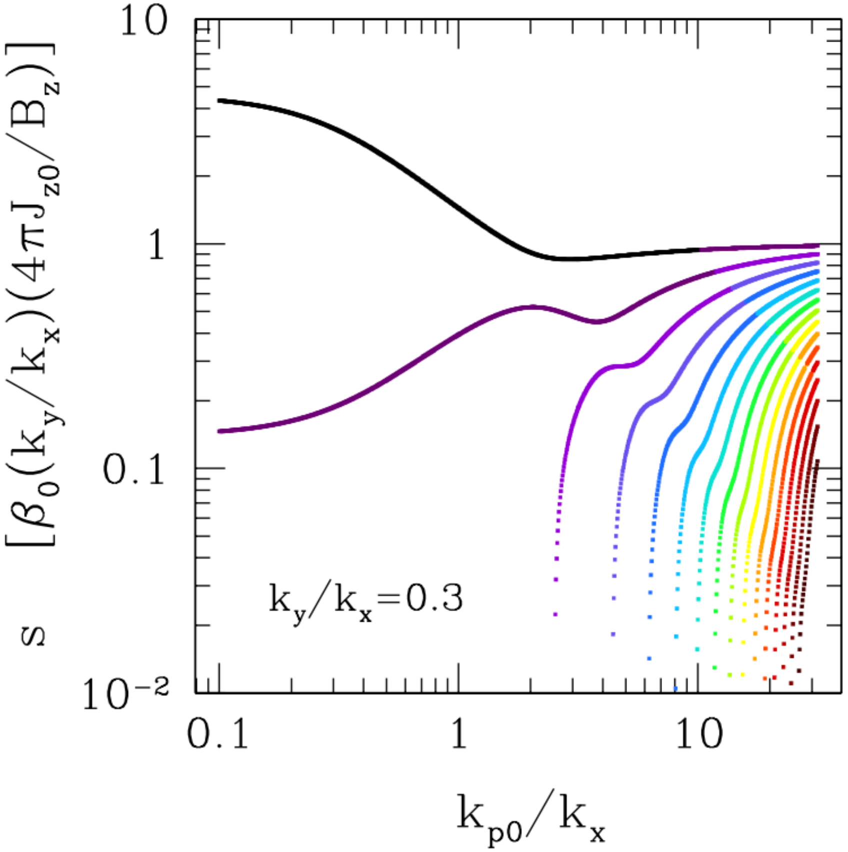
T 2022, ApJ

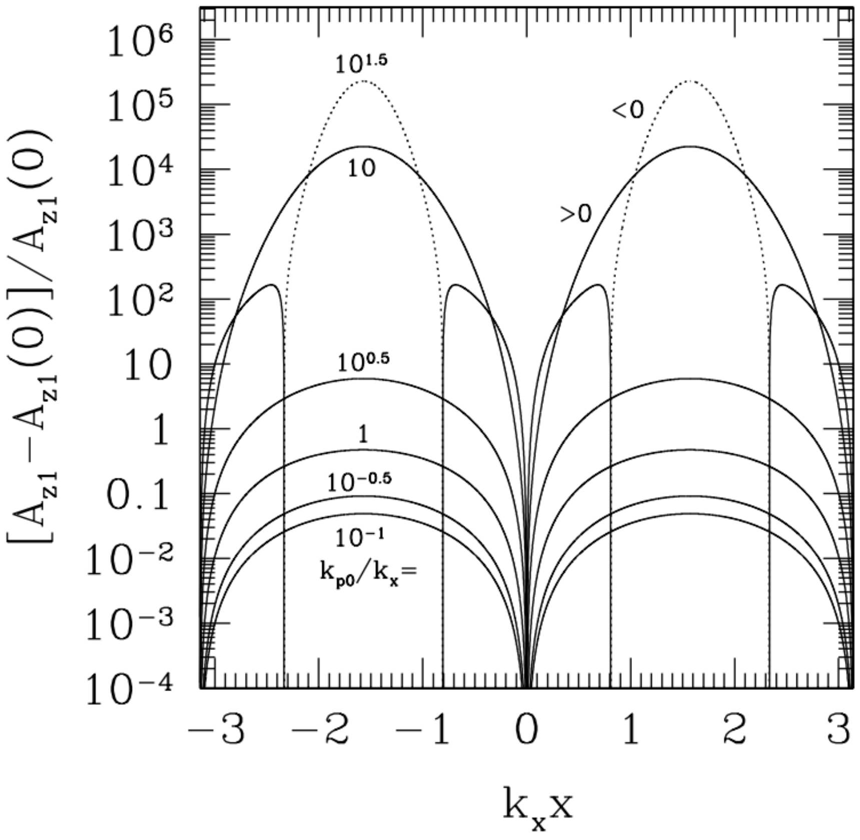
$$J_{z0} \propto \cos(k_x x)$$

modes $\sim e^{(-i\omega+s)t+ik_y y} f(x)$

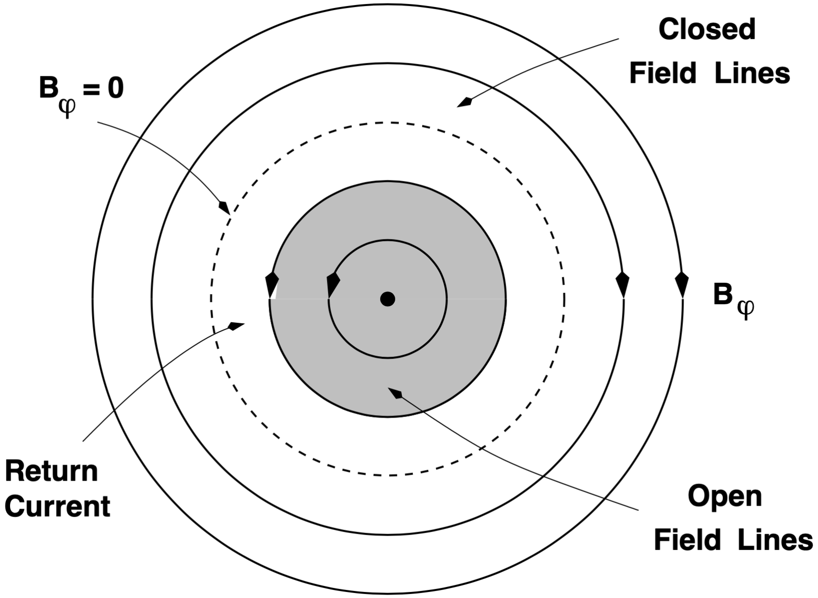
$$n_+ = n_- \qquad \omega = 0$$

$$s \sim 4\pi(k_y/k_x)J_{z0}/B_{z0} \sim 2(k_y/k_z)\Omega$$

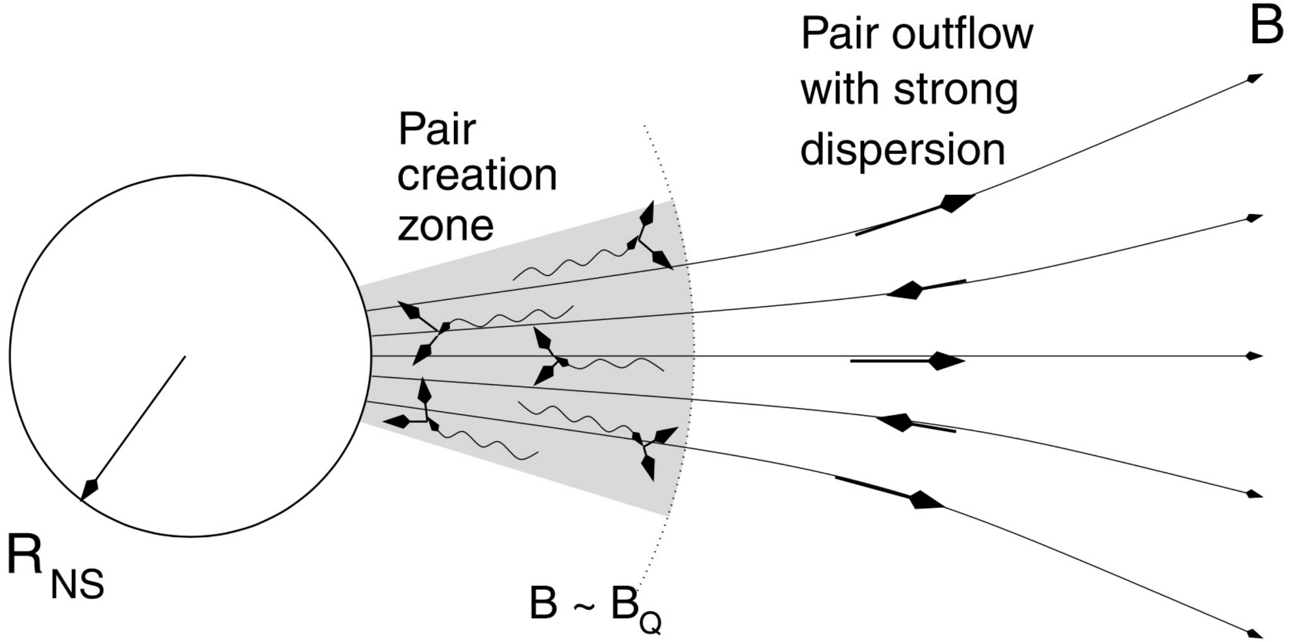




T, Lyutikov, Kulkarni 2002, ApJ



T 2008, ApJ



Radio emission feeds off mismatch in B_ϕ at separatrix –
– dynamic current, charge starvation, longitudinal e^{+-} excitation

Alfvenic cascade:
Charge Starvation and Landau Damping

T 2008, ApJ
T + R. Gill 2014, ApJ
Nattila & Beloborodov 2022

$$\left(\frac{|\boldsymbol{k}_\perp|_{\max}\, c}{\omega_p}\right)^2 \sim \frac{\bar{\lambda}_c}{r}\left(\frac{3\tau_{\mathrm{T}}}{2\alpha_{\mathrm{em}}}\right)^{(1+\alpha)/(3-\alpha)} \times \left[\frac{B(r)}{B_{\mathrm{Q}}}\right]^{-4/(3-\alpha)}\left(\frac{\delta B_0}{B}\right)^{-2}$$

$$\delta B^2 \propto |\boldsymbol{k}_\perp|^{1-\alpha}$$

Landau damping @ $k_\perp \gtrsim \omega_p/c$

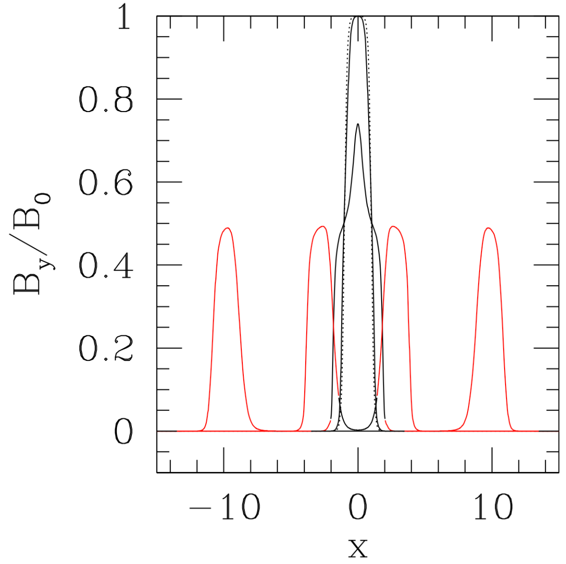
$$\frac{|\boldsymbol{k}_\perp|_{\max} c}{\omega_p} \sim 0.6\, \frac{(\Omega_{\mathrm{P}} R_6)^{1/2}\, \tau_{\mathrm{T},1}^{5/6}}{L_{\mathrm{P},42}^{1/2}} \times \left(\frac{B_p}{10\, B_{\mathrm{Q}}}\right)^{-1/3}\left(\frac{r}{30\, R}\right)^{3/2}$$

$$v_A = \frac{c}{\sqrt{1 + k_\perp^2 c^2/\omega_p^2}}$$

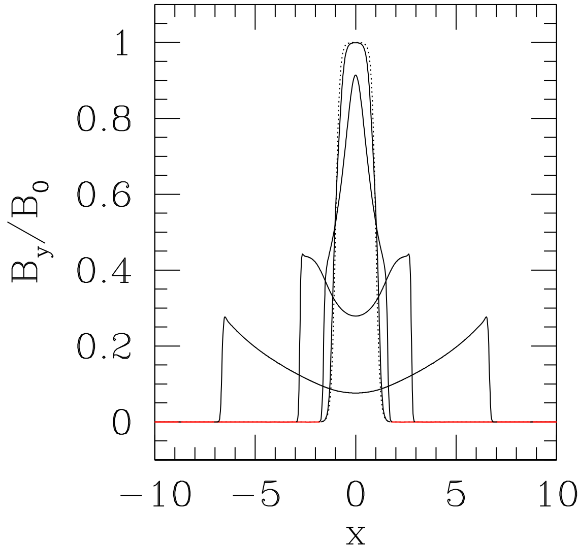
Green Function Solution for Expanding Thin EM Shell

T 2017, ApJ

$\omega_p \Delta r / c \quad \downarrow$



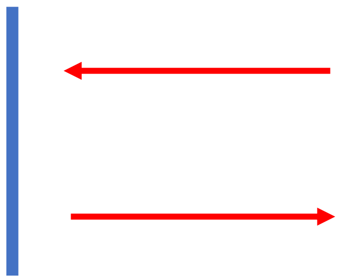
$\omega_p \Delta r / c \quad \text{const}$



1. Linear transformation
   Subluminal → Superluminal

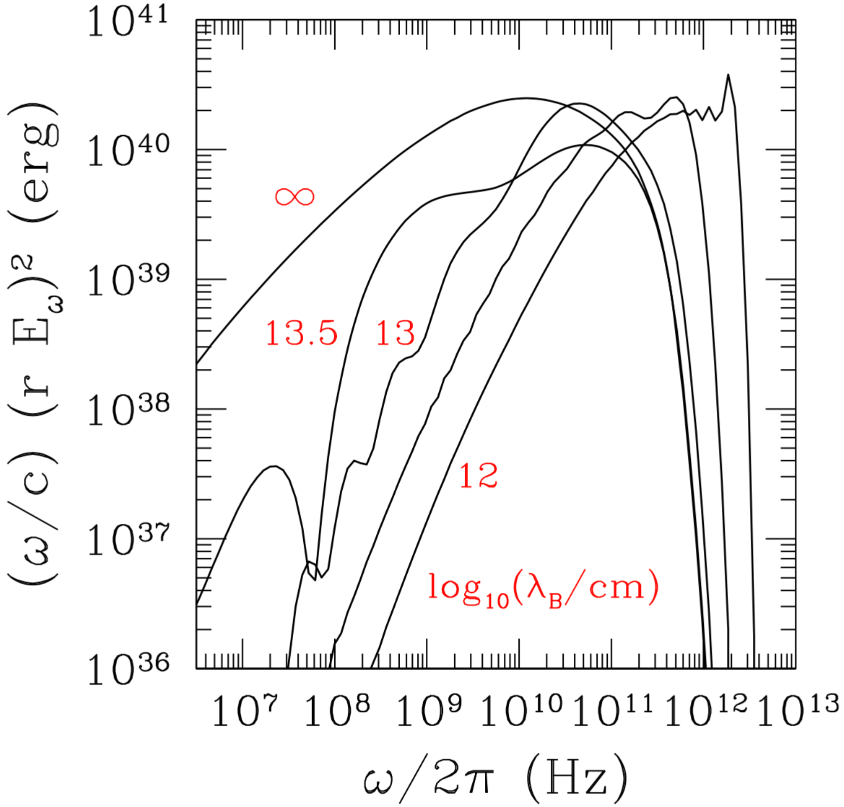
$$\omega_p \Delta r / c \quad \downarrow$$

2. Reflection of (variable) ambient B-field

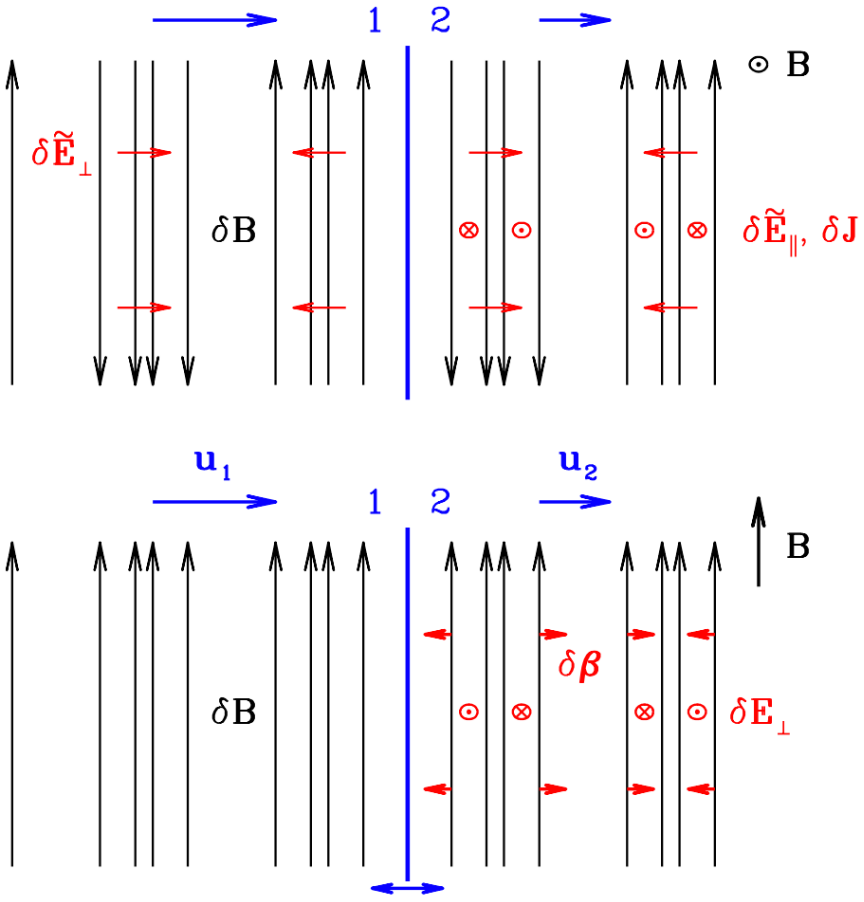


$E = B_{ex}$

$E = - B_{ex}$



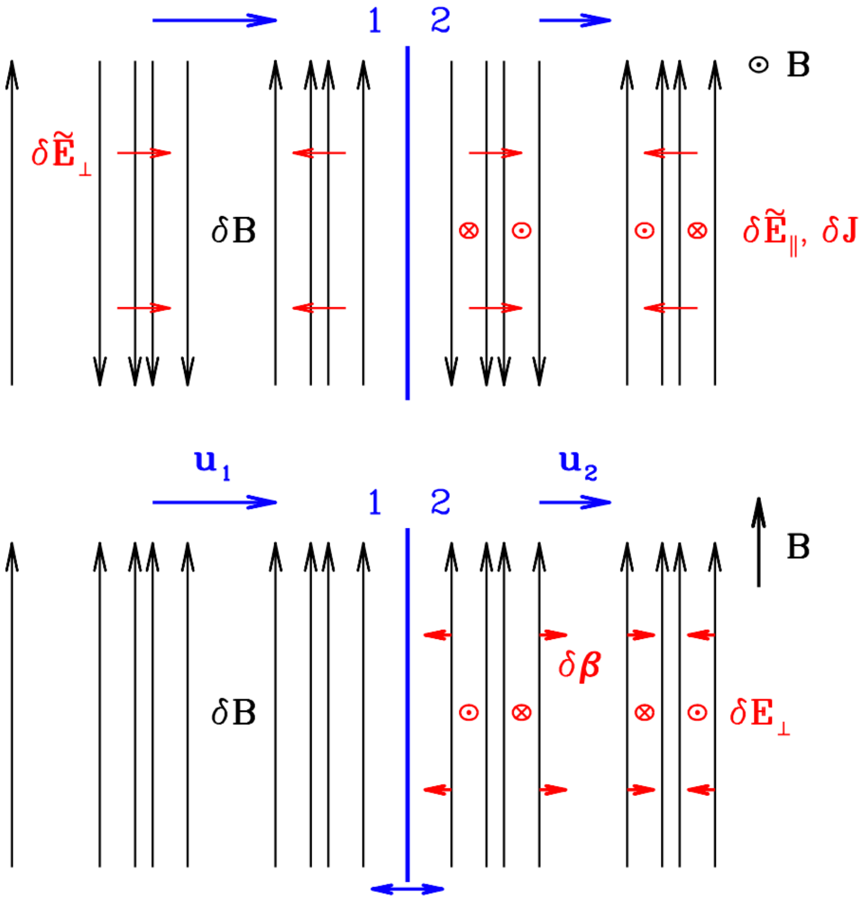
Linear Interaction of Zero-Frequency Modes with Shock



$$A[\widetilde{\omega}_1 \to 0] \xrightarrow{\text{shock}} O[\widetilde{\omega}_2 > \omega_{p2}]$$

$$I[\widetilde{\omega}_1 = 0] \xrightarrow{\text{shock}} I[\widetilde{\omega}_2 = 0] \, + \, X[\widetilde{\omega}_2]$$

Linear Interaction of Zero-Frequency Modes with Shock



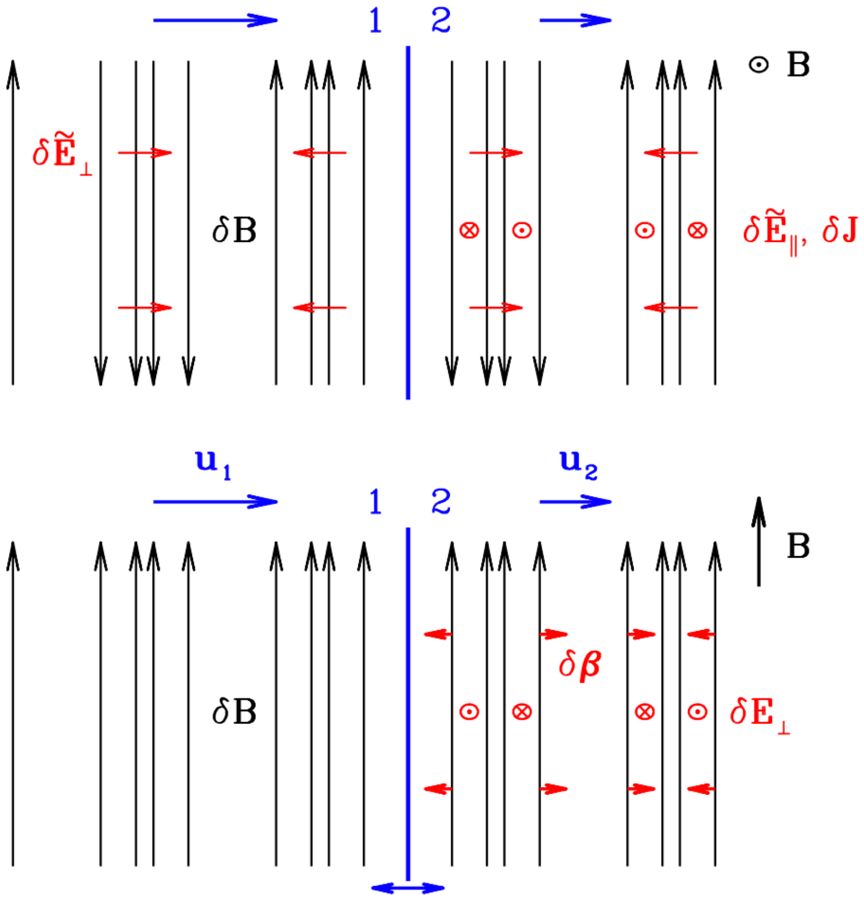
$$\delta\gamma \;\rightarrow\; \delta\gamma - u\,\delta\beta_s; \qquad \delta u \;\rightarrow\; \delta u - \gamma\,\delta\beta_s$$

$$\frac{1}{\gamma_2^2} = \frac{1}{\sigma_1} - \frac{1}{2\gamma_1^2}. \qquad (\sigma_1 \gg 1)$$

$$\gamma_1 > \gamma_{\phi,\mathrm{x}}(\sigma_1) = \left(\frac{3\sigma_1}{2}\right)^{1/2}$$

1. $\boldsymbol{E}_1 = \boldsymbol{E}_2;$
2. $\gamma_1 n_1 = \gamma_2 n_2;$
3. $w_1\gamma_1^2\beta_{\perp,1} = w_2\gamma_2^2\beta_{\perp,2};$
4. $w_2 = w_1 + \dfrac{B_1^2}{8\pi}\left(\dfrac{1}{\gamma_2^4} - \dfrac{1}{\gamma_1^4}\right);$
5. $\dfrac{1}{\gamma_2^2} = \dfrac{1}{\sigma_1} - \dfrac{1}{2\gamma_1^2}.$

Linear Interaction of Zero-Frequency Modes with Shock



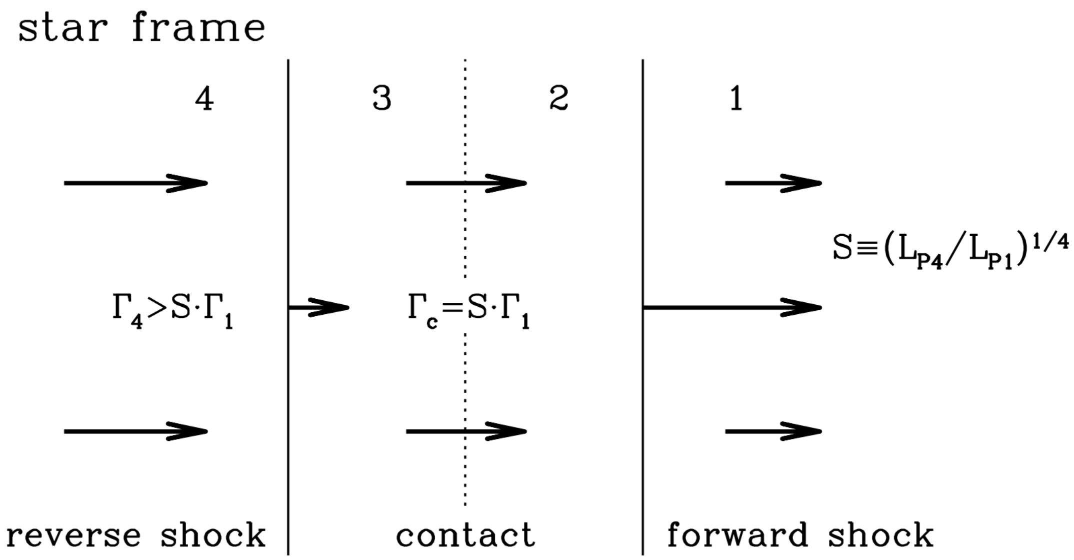
$$\frac{\delta B_{1,\mathrm{X}}}{\delta B_{2,\mathrm{X}}} \lesssim \frac{1}{24\sigma_1^2}$$

$$A \to A + O$$

$$\boxed{\delta B_{2,\mathrm{O}} \simeq \left(1 - \frac{\gamma_2}{\gamma_1}\right) \delta B_{1,\mathrm{A}}}$$

$$I \to I + X$$

$$\boxed{\begin{aligned} \frac{\delta B_{2,\mathrm{I}}}{\delta B_{1,\mathrm{I}}} &= \frac{1 - 3\sigma_1^2/4\gamma_1^4}{1 - \sigma_1/2\gamma_1^2}; \\ \frac{\delta B_{2,\mathrm{X}}}{\delta B_{1,\mathrm{I}}} &= \frac{(1 - 3\sigma_1/2\gamma_1^2)(1 - \sigma_1/\gamma_1^2)}{1 - \sigma_1/2\gamma_1^2} \end{aligned}}$$

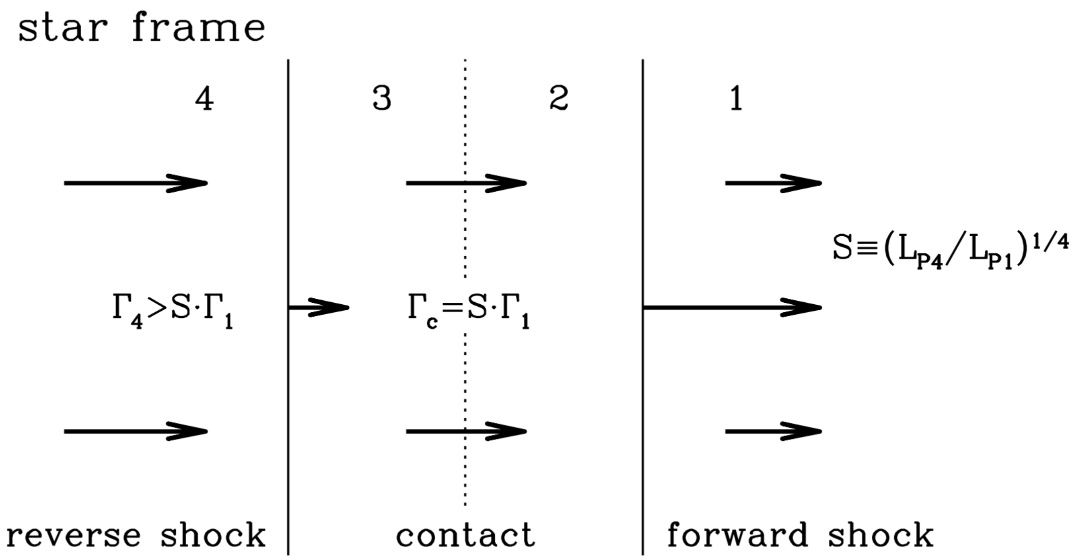


$$\mathcal{S} \equiv \left(\frac{L_{\mathrm{P},4}}{L_{\mathrm{P},1}} \right)^{1/4}$$

$$\Gamma_{1,\mathrm{fs}} = \sqrt{\sigma_1 \left(\frac{1}{2} + \mathcal{S}^2 \right)}; \qquad \Gamma_{\mathrm{fs},c} = \frac{\Gamma_{1,\mathrm{fs}}}{\mathcal{S}}.$$

Collision of two uniform shells

(Limitation: shell expansion strongly inhomogeneous if $L_{P4} >> L_{P1}$
peak Lorentz factor underestimated)



$$\mathcal{S} \equiv \left(\frac{L_{\mathrm{P},4}}{L_{\mathrm{P},1}}\right)^{1/4}$$

$$\begin{aligned} F^{\mathrm{O}}_{\mathrm{shock}}(\mathcal{S}) &= 1 - \frac{1}{\mathcal{S}}; \\ F^{\mathrm{X}}_{\mathrm{shock}}(\mathcal{S}) &= \frac{(\mathcal{S}^2 - 1)(2\mathcal{S}^2 - 1)}{\mathcal{S}^2(2\mathcal{S}^2 + 1)} \end{aligned}$$

Modes Excited Downstream of Forward Shock

$$\begin{aligned} \delta B_2 &= \Gamma_{\mathrm{fs}}[(\delta B_2)_{\mathrm{fs}} + \beta_{\mathrm{fs}}(\delta E_2)_{\mathrm{fs}}] \\ &\simeq \mathcal{S}^2\, F_{\mathrm{shock}}(\mathcal{S}) \left[\frac{1 - |\widetilde{\beta}_{g,2}|}{1 + |\widetilde{\beta}_{g,2}|} + \frac{1}{4\Gamma_c^2}\right] \cdot \delta B_{\mathrm{seed}}. \end{aligned}$$

$$\omega_2 \simeq \mathcal{S}^2 \left[\frac{1 - |\widetilde{\beta}_{g,2}|}{1 + |\widetilde{\beta}_{g,2}|} + \frac{1}{4\Gamma_c^2}\right] \cdot \Gamma_1 c \widetilde{k}_{\mathrm{seed}}$$

Comparison with Electromagnetic Maser Shock Instability (X-mode)

$$\frac{F_{\mathrm{P,X}}}{F_{\mathrm{P,O}}} \simeq \frac{4\varepsilon_{\mathrm{maser}}}{\sigma_1 \mathcal{S}^2} \left[\frac{1-|\widetilde{\beta}_{g,2}|}{1+|\widetilde{\beta}_{g,2}|}\right]^{-2} (\omega \Delta t)^{\alpha-1}.$$

$$\simeq 2\, \frac{\nu_9^{1/2} (\Delta t_{-3})^{1/2}}{(\sigma_1/10)(L_{\mathrm{P},4}/10^2 L_{\mathrm{P},1})^{1/2}}$$

$$\frac{\Gamma \omega_p}{2\pi} = 190\, \frac{(\Delta t_{-3})^{1/2}}{r_{13}} \left(\frac{\tau_{\mathrm{T},0}}{10}\right)^{1/2} \quad \mathrm{MHz.}$$

Energy-dependence:
frozen modes linearly damped
at smaller *r* in higher-*E* bursts

$$\frac{\delta J_\phi}{e n_\pm c} \propto \frac{\mathcal{E}^{1/2}}{\tau_{\mathrm{T},0}} \frac{r}{\Gamma}.$$